Name:________________________

MATH225

quiz 1

Sections 2.2–2.5

02/19/08

Total 100

Solutions

Show all work legibly.

In each problem involving a differential equation **identify** the equation.

1. (20) Solve $xy' + y - e^x = 0,\ y(1) = 1.$

   The equation is

   $$y' + \frac{1}{x}y = \frac{e^x}{x}. \qquad (*)$$

   The general solution Y of the homogeneous equation $y' + \frac{1}{x}y = 0$ is $Y(x) = \frac{c}{x}$. A particular solution $y_p(x)$ of the equation $(*)$ is $y_p(x) = \frac{e^x}{x}$. The general solution for the equation $(*)$ is $y(x) = y_p(x) + Y(x) = \frac{e^x}{x} + \frac{c}{x}$. If $y(1) = 1$, then $c = 1 - e$, and the solution to the initial value problem is $\frac{e^x + 1 - e}{x}$.

   (5) Mark one:
   ▫ Bernulli's equation, ▫ linear equation, ▫ separable equation, ▫ homogeneous equation, ▫ exact equation
   (15) the solution is: $\frac{e^x + 1 - e}{x}$.

2. (20) Solve $(e^x \sin y - 2y \sin x)dx + (e^x \cos y + 2\cos x)dy = 0.$

   $$M(x, y) = e^x \sin y - 2y \sin x, \text{ and } N(x, y) = e^x \cos y + 2\cos x.$$

   Since

   $$M_y = e^x \cos y - 2 \sin x = N_x$$

   the equation is exact, and

   $$\phi(x, y) = \int_0^x \left(e^t \sin 0 - 2 \cdot 0 \cdot \sin t\right) dt + \int_0^y \left(e^x \cos s + 2\cos x\right) ds = e^x \sin y + 2ycosx.$$

   (5) Mark one:
   ▫ Bernulli's equation, ▫ linear equation, ▫ separable equation, ▫ homogeneous equation, ▫ exact equation
   (15) the solution is: $e^x \sin y + 2y \cos x = c$.

3. (20) Solve the differential equation $(x+y)-(x-y)y'(x)=0$.

This is a homogeneous equation, and the substitution $y=xu$ leads to the separable equation $\frac{1-u}{1+u^2}du=\frac{dx}{x}$. Integration of both sides leads to $\ln|x|=\tan^{-1}u-\frac{1}{2}\ln(1+u^2)+c$, and $\ln|x|=\tan^{-1}\left(\frac{y}{x}\right)-\frac{1}{2}\ln\left[1+\left(\frac{y}{x}\right)^2\right]+c$.

(5) Mark one:

▫ Bernulli's equation, ▫ linear equation, ▫ separable equation, ▫ homogeneous equation, ▫ exact equation

(15) the solution is: $\ln|x|=\tan^{-1}\left(\frac{y}{x}\right)-\frac{1}{2}\ln\left[1+\left(\frac{y}{x}\right)^2\right]+c$.

4. (20) Solve the differential equation $y'(x)=e^{2x}+3y$.

The equation is

$$y'(x)-3y=e^{2x} \qquad (*)$$

The general solution for the homogeneous equation is $Y(x)=ce^{3x}$. A particular solution for the equation $(*)$ is $y_p(x)=-e^{2x}$, the general solution for $(*)$ is $-e^{2x}+ce^{3x}$.

(5) Mark one:

▫ Bernulli's equation, ▫ linear equation, ▫ separable equation, ▫ homogeneous equation, ▫ exact equation

(15) the solution is: $-e^{2x}+ce^{3x}$.

5. (20) Solve the differential equation $\frac{dy}{dx}=\frac{x+3y}{3x+y}$ using an appropriate substitution.

This is a homogemeous equation. The substitution $y=xu$ leads to the separable equation $\frac{3+u}{1-u^2}du=\frac{dx}{x}$. Integrating both sides one gets

$$\ln|x|=2\int\frac{du}{1-u}+\int\frac{du}{1+u}=\ln\left[(1-u)^2|1+u|\right]+c,$$

and, finally, $\ln|x|=\ln\left[(1-y/x)^2|1+y/x|\right]+c$.

(5) Mark one:

▫ Bernulli's equation, ▫ linear equation, ▫ separable equation, ▫ homogeneous equation, ▫ exact equation

(15) the solution is: $\ln|x|=\ln\left[(1-y/x)^2|1+y/x|\right]+c$.

6. (20) (**extra credit**) Solve the differential equation $x\frac{dy}{dx}-(1+x)y=xy^2$ using an appropriate substitution.

The equation $\dfrac{dy}{dx} - \dfrac{(1+x)}{x}y = y^2$ is Bernulli's equation with $n = 2$. The substitution $u = \dfrac{1}{y}$ leads to the linear equation

$$u' + -\frac{(1+x)}{x} = -1. \qquad (*)$$

The general solution for $(*)$ is $u(x) = -1+\frac{1}{x}+\frac{c}{x}e^{1-x}$, the general solution for the Bernulli's equation is $y(x) = \left[-1 + \dfrac{1}{x} + \dfrac{c}{x}e^{1-x}\right]^{-1}$.

(5) Mark one:

□ Bernulli's equation, □ linear equation, □ separable equation, □ homogeneous equation, □ exact equation

(15) the solution is: $y(x) = \left[-1 + \dfrac{1}{x} + \dfrac{c}{x}e^{1-x}\right]^{-1}$.