MATH221

quiz #1, 02/27/20

Total 100

By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal.

Show all work legibly. **Name**:______________________

1. (40) This problem consists of two parts dealing with systems of linear equation $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = 0$.

   (a) (20) Solve the system

$$\begin{array}{ccccccc} x_1 & & -x_3 & & -2x_5 & = & 1 \\ & x_2 & +3x_3 & & -x_5 & = & 2 \\ 2x_1 & & -2x_3 & +x_4 & -3x_5 & = & 0 \end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \quad\quad\quad \\ \\ \\ \\ \\ \end{bmatrix}$$

(b) (20) Consider the homogeneous system of linear equations

$$\begin{array}{rrrrrcl} x_1 & & -x_3 & & -2x_5 & = & 0 \\ & x_2 & +3x_3 & & -x_5 & = & 0 \\ 2x_1 & & -2x_3 & +x_4 & -3x_5 & = & 0 \end{array}$$

Identify vectors that span the solution set of the system.

The vectors are:

2. (20) Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be a set of linearly independent vectors, and $\mathbf{w}_1 = \mathbf{u}_1$, $\mathbf{w}_2 = \mathbf{u}_2$, $\mathbf{w}_3 = \mathbf{u}_3 + \mathbf{u}_1$. True or False? The vectors $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ are linearly independent.

Mark one and explain.

□ True □ False

3. (20) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation that reflects a vector with respect to the (x, y) plane, and then rotates it 90^o clockwise. Find the standard matrix A of T.

$A =$

4. (20) Let $A = [\mathbf{a}_1, \mathbf{a}_2] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(a) (15) Find vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ so that $A\mathbf{b}_1 = \mathbf{e}_1$, and $A\mathbf{b}_2 = \mathbf{e}_2$.

$\mathbf{b}_1 = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

(b) (5) Let $B = [\mathbf{b}_1, \mathbf{b}_2]$. Compute $B\mathbf{a}_1$ and $B\mathbf{a}_2$.

$B\mathbf{a}_1 = \begin{bmatrix} \quad\quad\quad \end{bmatrix}$ and $B\mathbf{a}_2 = \begin{bmatrix} \quad\quad\quad \end{bmatrix}$

5. (20) Let $A = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3] = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. True or False? The columns of A are linearly independent.

Mark one and explain.

□ True □ False