Selected Solutions

Homework #2

Section 1.3

10. $$x_1\begin{bmatrix}3\\-2\\5\end{bmatrix} + x_2\begin{bmatrix}-2\\-7\\4\end{bmatrix} + x_3\begin{bmatrix}4\\5\\-3\end{bmatrix} = \begin{bmatrix}3\\1\\2\end{bmatrix}$$

14. Write as augmented matrix

$$\left[\begin{array}{ccc|c}1&0&5&2\\-2&1&-6&-1\\0&2&8&6\end{array}\right] \to \left[\begin{array}{ccc|c}1&0&5&2\\0&1&4&3\\0&1&4&3\end{array}\right] \to \left[\begin{array}{ccc|c}1&0&5&2\\0&1&4&3\\0&0&0&0\end{array}\right] \quad \begin{array}{l}x_1 = 2-5x_3\\x_2 = 3-4x_3\\x_3 = \text{free}\end{array}$$

Since all three variables (x_1, x_2, x_3) exist, b is a linear combination of the vectors formed from the columns of A.

23. c. True. We can write $\frac{1}{2}v_1$ as $\frac{1}{2}v_1 + 0v_2$ which has the same value and is a linear combination of v_1 and v_2.

d. True. The equation $x_1a_1 + x_2a_2 + x_3a_3 = b$ can be written in augmented matrix form as $[a_1\ a_2\ a_3\ b]$. Since they are the same, they have the same solution set.

e. False. Suppose the vectors u and v are linearly dependent (a linear combination), then the span would be visualized as a line, not a plane.

29. Recall total mass of system: $m = m_1 + \dots + m_k$

center of gravity or mass of system: $\bar{v} = \frac{1}{m}(m_1v_1 + \dots + m_kv_k)$

$m = 4+2+3+5 = 14$

$$\bar{v} = \frac{1}{14}\left(4\begin{bmatrix}2\\-2\\4\end{bmatrix} + 2\begin{bmatrix}-4\\2\\3\end{bmatrix} + 3\begin{bmatrix}4\\0\\-2\end{bmatrix} + 5\begin{bmatrix}1\\-6\\0\end{bmatrix}\right) = \frac{1}{14}\left(\begin{bmatrix}8\\-8\\16\end{bmatrix} + \begin{bmatrix}-8\\4\\6\end{bmatrix} + \begin{bmatrix}12\\0\\-6\end{bmatrix} + \begin{bmatrix}5\\-30\\0\end{bmatrix}\right)$$

$$= \frac{1}{14}\begin{bmatrix}8-8+12+5\\-8+4+0-30\\16+6-6+0\end{bmatrix} = \frac{1}{14}\begin{bmatrix}17\\-34\\16\end{bmatrix} = \begin{bmatrix}17/14\\-17/7\\8/7\end{bmatrix}$$

Section 1.4

2. Undefined because there are fewer columns of matrix A than entries in vector x. $3\times 1 \cdot 2\times 1$

4. $\begin{bmatrix} 1 & 3 & -4 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1(1)+3(2)+-4(1) \\ 3(1)+2(2)+1(1) \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$

9. vector eqn | matrix eqn

$x_1\begin{bmatrix} 5 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_3\begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$ | $\begin{bmatrix} 5 & 1 & -3 \\ 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$

22. $\begin{bmatrix} 0 & 0 & 4 \\ 0 & -3 & -2 \\ -3 & 9 & -6 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & -3 & -2 \\ -1 & 3 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 3 & -2 \\ 0 & -3 & -2 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The matrix, which is now in reduced echelon form, exhibits a pivot in every row. According to theorem 4, since every row has a pivot, the vectors span $\mathbb{R}^3$.

35. Recall that a system $Ay=z$ is consistent if z can be written as a linear combination of A. We are given $Ay=z$.

Suppose: $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \end{bmatrix}$ $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ $z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix}$

So, $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ \vdots & \ddots & \vdots \\ a_{51} & a_{52} & a_{53} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} z_1 \\ \vdots \\ z_5 \end{bmatrix}$ Rewritten as: $y_1\begin{bmatrix} a_{11} \\ \vdots \\ a_{51} \end{bmatrix} + y_2\begin{bmatrix} a_{12} \\ \vdots \\ a_{52} \end{bmatrix} + y_3\begin{bmatrix} a_{13} \\ \vdots \\ a_{53} \end{bmatrix} = \begin{bmatrix} z_1 \\ \vdots \\ z_5 \end{bmatrix}$

This shows z is a linear combination of A.

Is $Ax=5z$ consistent? 5 is a scalar

$5Ay=5z$ ← both sides multiplied by a scalar

$A(5y)=5z$ ↳ this doesn't change a system's consistency

$Ax=5z$ Yes, $Ax=5z$ is consistent.