MATH 221, Spring 2016 - Homework 9 Solutions

Due Tuesday, April 26

Section 6.1

Page 336, Problem 2:

- $\mathbf{w} \cdot \mathbf{w} = 3(3) + -1(-1) + -5(-5) = 9 + 1 + 25 = 35$
- $\mathbf{x} \cdot \mathbf{w} = 6(3) + -2(-1) + 3(-5) = 18 + 2 - 15 = 5$
- $\frac{\mathbf{x} \cdot \mathbf{w}}{\mathbf{w} \cdot \mathbf{w}} = \frac{5}{35} = \frac{1}{7}$

Page 336, Problem 7:

- $\|\mathbf{w}\| = \sqrt{\mathbf{w} \cdot \mathbf{w}} = \sqrt{35}$

Page 336, Problem 10:

- First, compute the norm of the vector: $\sqrt{-6(-6) + 4(4) + -3(-3)} = \sqrt{36 + 16 + 9} = \sqrt{61}$
- Then, normalize the vector (multiply by the scalar $\frac{1}{\sqrt{61}}$: $\begin{bmatrix} -6/\sqrt{61} \\ 4/\sqrt{61} \\ -3/\sqrt{61} \end{bmatrix}$

Page 336, Problem 14:

- First find $\mathbf{u} - \mathbf{z} = \begin{bmatrix} 4 \\ -4 \\ -6 \end{bmatrix}$
- use the formula $\text{dist}(\mathbf{u}, \mathbf{z}) = \|\mathbf{u} - \mathbf{z}\| = \sqrt{(\mathbf{u} - \mathbf{z}) \cdot (\mathbf{u} - \mathbf{z})} = \sqrt{4(4) + -4(-4) + -6(-6)} = \sqrt{68} = 2\sqrt{17}$

Page 336, Problem 16:

- Vectors are orthogonal if the dot product of the vectors equals zero.
- Compute $\mathbf{u} \cdot \mathbf{v} = 12(2) + 3(-3) + -5(3) = 0$. So, **the vectors are orthogonal**.

Page 336, Problem 17:

- $\mathbf{u} \cdot \mathbf{v} = 3(-4) + 2(1) + -5(-2) + 0(6) = 0$. So, **the vectors are orthogonal**.

Page 337, Problem 20a:

True or False: $\mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} = 0$

TRUE: By Theorem 1, $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$, so by substitution $\mathbf{u} \cdot \mathbf{v} - \mathbf{u} \cdot \mathbf{v} = 0$.

Page 337, Problem 20b:

True or False: For any scalar c, $\|c\mathbf{v}\| = c\,\|\mathbf{v}\|$

FALSE: As stated on page 331, $\|c\mathbf{v}\| = |c|\,\|\mathbf{v}\|$.

Page 337, Problem 20c:

True or False: If $\mathbf{x}$ is orthogonal to every vector in a subspace W, then $\mathbf{x}$ is in $W^{\perp}$.

TRUE: This statement follows from the definition of Orthogonal Complements on page 334 (here, the set that spans W is W itself).

Page 337, Problem 20d:

True or False: For any scalar c, $\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 = \|\mathbf{u}+\mathbf{v}\|^2$, then $\mathbf{u}$and $\mathbf{v}$ are orthogonal.

TRUE: This statement is part of Theorem 2 in this section (the Pythagorean Theorem).

Page 337, Problem 20e:

True or False: For an $m \times n$ matrix A, vectors in the null space of A are orthogonal to vectors in the row space of A.

TRUE: This statement is part of Theorem 3 of this section.

Page 337, Problem 23:

- $\mathbf{u} \cdot \mathbf{v} = 2(-7) + -5(-4) + -1(6) = 0$
- $\|\mathbf{u}\|^2 = \mathbf{u} \cdot \mathbf{u} = 2(2) + -5(-5) + -1(-1) = 30$
- $\|\mathbf{v}\|^2 = \mathbf{v} \cdot \mathbf{v} = -7(-7) + -4(-4) + 6(6) = 101$
- $\|\mathbf{u}+\mathbf{v}\|^2 = (\mathbf{u}+\mathbf{v}) \cdot (\mathbf{u}+\mathbf{v}) = -5(-5) + -9(-9) + 5(5) = 131$

Page 337, Problem 31:

Suppose $\mathbf{x}$ is in both W and $W^{\perp}$. Because W spans W and $\mathbf{x} \in W$, $\mathbf{x}$is orthogonal to every vector in W (by definition of orthogonal complements). Because $\mathbf{x}$ is orthogonal to **every** vector in W, that means $\mathbf{x} \cdot \mathbf{x} = 0$, which implies $\mathbf{x} = 0$ (by Theorem 1).

Section 6.2

Page 344, Problem 3:

- To determine if the set is orthogonal, compute the dot product of each pair of vectors:
- $\begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} = -6(3) + -3(1) + 9(-1) = -18 - 3 - 9 = -30 \neq 0$
- Because at least one of the pairs of vectors is not orthogonal, the set of vectors is not orthogonal.

Page 345, Problem 8:

- First, compute $\mathbf{u}_1 \cdot \mathbf{u}_2 = 3(-2) + 1(6) = -6 + 6 = 0$, which shows $\{\mathbf{u}_1, \mathbf{u}_2\}$ is orthogonal.
- Because neither $\mathbf{u}_1$ or $\mathbf{u}_2$ are nonzero and $\{\mathbf{u}_1, \mathbf{u}_2\}$ is orthogonal, $\{\mathbf{u}_1, \mathbf{u}_2\}$ is linearly independent, and hence a basis for $\mathbb{R}^2$ (by Theorem 4 of this section)
- To express $\mathbf{x}$ as a linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$, we can use Theorem 5 to find the coefficients of $\mathbf{u}_1$ and $\mathbf{u}_2$:
- $c_1 = \frac{\mathbf{x} \cdot \mathbf{u}_1}{\mathbf{u}_1 \cdot \mathbf{u}_1} = \frac{-6(3)+3(1)}{3(3)+1(1)} = \frac{-18+3}{9+1} = \frac{-15}{10} = -\frac{3}{2}$
- $c_1 = \frac{\mathbf{x} \cdot \mathbf{u}_2}{\mathbf{u}_2 \cdot \mathbf{u}_2} = \frac{-6(-2)+3(6)}{-2(-2)+6(6)} = \frac{12+18}{4+36} = \frac{30}{40} = \frac{3}{4}$
- So, $\mathbf{x} = -\frac{3}{2}\mathbf{u}_1 + \frac{3}{4}\mathbf{u}_2$

Page 345, Problem 12:

- Let $\mathbf{y} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$. Use the formula on page 340 to compute the orthogonal projection of $\mathbf{y}$ onto $\mathbf{u}$:
- $\hat{\mathbf{y}} = \frac{\mathbf{y}\cdot\mathbf{u}}{\mathbf{u}\cdot\mathbf{u}}\mathbf{u} = \frac{1(-1)+-1(3)}{-1(-1)+3(3)}\mathbf{u} = \frac{-4}{10}\mathbf{u} = -\frac{2}{5}\mathbf{u} = \begin{bmatrix} 2/5 \\ -6/5 \end{bmatrix} = \begin{bmatrix} 0.4 \\ -1.2 \end{bmatrix}$

Page 345, Problem 13:

- This problem is like Example 3 in the book. First, find the orthogonal projection of $\mathbf{y}$ onto $\mathbf{u}$: $\hat{\mathbf{y}} = \frac{\mathbf{y}\cdot\mathbf{u}}{\mathbf{u}\cdot\mathbf{u}}\mathbf{u} = \frac{2(4)+3(-7)}{4(4)+-7(-7)}\mathbf{u} = \frac{-13}{65}\mathbf{u} = \begin{bmatrix} -56/65 \\ 91/65 \end{bmatrix} = \begin{bmatrix} -4/5 \\ 7/5 \end{bmatrix}$
- The component of $\mathbf{y}$ orthogonal to $\mathbf{u}$ is: $\mathbf{y} - \hat{\mathbf{y}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} -4/5 \\ 7/5 \end{bmatrix} = \begin{bmatrix} 14/5 \\ 8/5 \end{bmatrix}$
- Therefore, $\mathbf{y} = \hat{\mathbf{y}} + (\mathbf{y} - \hat{\mathbf{y}}) = \begin{bmatrix} -4/5 \\ 7/5 \end{bmatrix} + \begin{bmatrix} 14/5 \\ 8/5 \end{bmatrix}$

Page 345, Problem 16:

- This problem is like Example 4 in the book. First, find the orthogonal projection of $\mathbf{y}$ onto $\mathbf{u}$: $\hat{\mathbf{y}} = \frac{\mathbf{y}\cdot\mathbf{u}}{\mathbf{u}\cdot\mathbf{u}}\mathbf{u} = \frac{-3(1)+9(2)}{1(1)+2(2)}\mathbf{u} = \frac{15}{5}\mathbf{u} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$
- The component of $\mathbf{y}$ orthogonal to $\mathbf{u}$ is: $\mathbf{y} - \hat{\mathbf{y}} = \begin{bmatrix} -3 \\ 9 \end{bmatrix} - \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \end{bmatrix}$
- The distance from $\mathbf{y}$ to the line containing $\mathbf{u}$ and the origin is the distance of this orthogonal component: $\|\mathbf{y} - \hat{\mathbf{y}}\| = \sqrt{(-6)^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$

Page 345, Problem 20:

- It is clear that the set of vectors is orthogonal because $\begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix} \cdot \begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix} = -\frac{2}{3} + \frac{2}{3} + 0 = 0.$
- The **SET is NOT ORTHONORMAL** because the length of the second vector is $\sqrt{(\frac{1}{3})^2 + (\frac{2}{3})^2} = \sqrt{\frac{5}{9}} \neq 1$
- The first vector is orthonormal because its length is $\sqrt{(-\frac{2}{3})^2 + (\frac{1}{3})^2 + (\frac{2}{3})^2} = \sqrt{\frac{9}{9}} = 1.$
- To normalize the first vector, simply multiply by $\frac{1}{\sqrt{\frac{5}{9}}} = \frac{3}{\sqrt{5}}$ to produce the orthonormal set: $\left\{ \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \\ 0 \end{bmatrix} \right\}$

Page 345, Problem 23a:

True or False: Not every linearly independent set in $\mathbb{R}^n$ is an orthogonal set.

TRUE: Any counterexample such as the set $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$, which is linearly independent in $\mathbb{R}^2$ but not orthogonal.

Page 345, Problem 23e:

True or False: If L is a line through $\mathbf{0}$ and if $\hat{\mathbf{y}}$ is the orthogonal projection of $\mathbf{y}$ onto L, then $\|\hat{\mathbf{y}}\|$ gives the distance from $\mathbf{y}$ to L.

FALSE: As it was used in the previous exercises and shown in Example 4 of the text, the distance is $\|\mathbf{y} - \hat{\mathbf{y}}\|$.

Page 345, Problem 24a:

True or False: Not every orthogonal set in $\mathbb{R}^n$ is linearly independent.

TRUE: Every nonzero orthogonal set in $\mathbb{R}^n$ is linearly independent (Theorem 4).

Page 345, Problem 24b:

True or False: If a set $S = \{\mathbf{u}_1, \ldots, \mathbf{u}_p\}$ has the property that $\mathbf{u}_i \cdot \mathbf{u}_j = 0$ whenever $i \neq j$, then S is an orthonormal set.

FALSE: This is the definition of an **orthogonal** set (in order to be an orthonormal set, the vectors must be unit vectors).

Page 345, Problem 24c:

True or False: If the columns of an $m \times n$ matrix A are orthonormal, then the linear mapping $\mathbf{x} \mapsto A\mathbf{x}$ preserves lengths.

TRUE: See the paragraph after Theorem 7, which states that the linear mappin preserves lengths and orthogonality.

Page 345, Problem 24d:

True or False: The orthogonal projection of $\mathbf{y}$ onto $\mathbf{v}$ is the same as the orthogonal projection of $\mathbf{y}$ onto $c\mathbf{v}$ whenever $c \neq 0$.

TRUE: This is explicitly stated in the text on page 340 in the paragraph before the definition of the orthogonal projection.

Page 345, Problem 24e:

True or False: An orthogonal matrix is invertible.

TRUE: By definition of an orthogonal matrix (page 344), it must be invertible.

Section 6.3

Page 352, Problem 4:

- To verify the vectors form an orthogonal set compute the dot product: $\begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix} = 3(-4) + 4(3) + 0(0) = 0$
- The orthogonal projection of $\mathbf{y}$ onto Span $\{\mathbf{u}_1, \mathbf{u}_2\}$ is $\hat{\mathbf{y}} = \frac{\mathbf{y}\cdot\mathbf{u}_1}{\mathbf{u}_1\cdot\mathbf{u}_1}\mathbf{u}_1 + \frac{\mathbf{y}\cdot\mathbf{u}_2}{\mathbf{u}_2\cdot\mathbf{u}_2}\mathbf{u}_2 = \frac{6(3)+3(4)+-2(0)}{3(3)+4(4)+0(0)}\mathbf{u}_1 + \frac{6(-4)+3(3)+-2(0)}{-4(-4)+3(3)+0(0)}\mathbf{u}_2$.
  So, $\hat{\mathbf{y}} = \frac{30}{25}\mathbf{u}_1 + \frac{-15}{25}\mathbf{u}_2 = \frac{6}{5}\begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} + \frac{-3}{5}\begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}$

Page 352, Problem 5:

- To verify the vectors form an orthogonal set compute the dot product: $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} = 3(1) + -1(-1) + 2(-2) = 0$
- The orthogonal projection of $\mathbf{y}$ onto Span $\{\mathbf{u}_1, \mathbf{u}_2\}$ is $\hat{\mathbf{y}} = \frac{\mathbf{y}\cdot\mathbf{u}_1}{\mathbf{u}_1\cdot\mathbf{u}_1}\mathbf{u}_1 + \frac{\mathbf{y}\cdot\mathbf{u}_2}{\mathbf{u}_2\cdot\mathbf{u}_2}\mathbf{u}_2 = \frac{-1(3)+2(-1)+6(2)}{3(3)+-1(-1)+2(2)}\mathbf{u}_1 + \frac{-1(1)+2(-1)+6(-2)}{1(1)+-1(-1)+-2(-2)}\mathbf{u}_2$.
  So, $\hat{\mathbf{y}} = \frac{7}{14}\mathbf{u}_1 + \frac{-15}{6}\mathbf{u}_2 = \frac{1}{2}\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + \frac{-5}{2}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix}$

Page 352, Problem 8:

- To verify the vectors form an orthogonal set compute the dot product: $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix} = 1(-1) + 1(3) + 1(-2) = 0$

- The orthogonal projection of $\mathbf{y}$ onto Span $\{\mathbf{u}_1, \mathbf{u}_2\}$ is $\hat{\mathbf{y}} = \frac{\mathbf{y}\cdot\mathbf{u}_1}{\mathbf{u}_1\cdot\mathbf{u}_1}\mathbf{u}_1 + \frac{\mathbf{y}\cdot\mathbf{u}_2}{\mathbf{u}_2\cdot\mathbf{u}_2}\mathbf{u}_2 = \frac{-1(1)+4(1)+3(1)}{1(1)+1(1)+1(1)}\mathbf{u}_1 + \frac{-1(-1)+4(3)+3(-2)}{-1(-1)+3(3)+-2(-2)}\mathbf{u}_2$. So, $\hat{\mathbf{y}} = \frac{6}{3}\mathbf{u}_1 + \frac{7}{14}\mathbf{u}_2 = 2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 7/2 \\ 1 \end{bmatrix}$

- By the Orthogonal Decomposition Theorem, $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$ where $\mathbf{z} = (\mathbf{y} - \hat{\mathbf{y}}) = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} - \begin{bmatrix} 3/2 \\ 7/2 \\ 1 \end{bmatrix} = \begin{bmatrix} -5/2 \\ 1/2 \\ 2 \end{bmatrix}$ is orthogonal to W.

Page 352, Problem 9:

- To verify the vectors form an orthogonal set compute the dot product of each pair of vectors: $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \\ 1 \\ 2 \end{bmatrix} = 1(-1)+1(3)+0(1)+1(2) = 0$, $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = 1(-1)+1(0)+0(1)+1(1) = 0$, $\begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = -1(-1)+3(0)+1(1)+-2(1) = 0$

- The orthogonal projection of $\mathbf{y}$ onto Span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is $\hat{\mathbf{y}} = \frac{\mathbf{y}\cdot\mathbf{u}_1}{\mathbf{u}_1\cdot\mathbf{u}_1}\mathbf{u}_1 + \frac{\mathbf{y}\cdot\mathbf{u}_2}{\mathbf{u}_2\cdot\mathbf{u}_2}\mathbf{u}_2 + \frac{\mathbf{y}\cdot\mathbf{u}_3}{\mathbf{u}_3\cdot\mathbf{u}_3}\mathbf{u}_3 = \frac{4(1)+3(1)+3(0)+-1(1)}{1(1)+1(1)+0(0)+1(1)}\mathbf{u}_1 + \frac{4(-1)+3(3)+3(1)+-1(-2)}{-1(-1)+3(3)+1(1)+-2(-2)}\mathbf{u}_2 + \frac{4(-1)+3(0)+3(1)+-1(1)}{-1(-1)+0(0)+1(1)+1(1)}$. So, $\hat{\mathbf{y}} = \frac{6}{3}\mathbf{u}_1 + \frac{10}{15}\mathbf{u}_2 + \frac{-2}{3} = 2\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix} + \frac{-2}{3}\begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix}$

- By the Orthogonal Decomposition Theorem, $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$ where $\mathbf{z} = (\mathbf{y} - \hat{\mathbf{y}}) = \begin{bmatrix} 4 \\ 3 \\ 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \\ -1 \end{bmatrix}$ is orthogonal to W.

Page 352, Problem 11:

- First, verify the vectors form an orthogonal set: $\mathbf{v}_1 \cdot \mathbf{v}_2 = 3(1) + 1(-1) + -1(1) + 1(-1) = 0$

- Because $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal, the closest point to $\mathbf{y}$ in the subspace W spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$ is $\hat{\mathbf{y}} = \frac{\mathbf{y}\cdot\mathbf{v}_1}{\mathbf{v}_1\cdot\mathbf{v}_1}\mathbf{v}_1 + \frac{\mathbf{y}\cdot\mathbf{v}_2}{\mathbf{v}_2\cdot\mathbf{v}_2}\mathbf{v}_2$ (by the Best Approximation Theorem).

- Therefore, $\hat{\mathbf{y}} = \frac{3(3)+1(1)+5(-1)+1(1)}{3(3)+1(1)+-1(-1)+1(1)}\mathbf{v}_1 + \frac{3(1)+1(-1)+5(1)+1(-1)}{1(1)+-1(-1)+1(1)+-1(-1)}\mathbf{v}_2 = \frac{6}{12}\mathbf{v}_1 + \frac{6}{4}\mathbf{v}_2 = \begin{bmatrix} 3 \\ -1 \\ 1 \\ -1 \end{bmatrix}$

Page 352, Problem 12:

- First, verify the vectors form an orthogonal set: $\mathbf{v}_1 \cdot \mathbf{v}_2 = 1(-4) + -2(1) + -1(0) + 2(3) = 0$
- Because $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal, the closest point to $\mathbf{y}$ in the subspace W spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$ is $\hat{\mathbf{y}} = \frac{\mathbf{y}\cdot\mathbf{v}_1}{\mathbf{v}_1\cdot\mathbf{v}_1}\mathbf{v}_1 + \frac{\mathbf{y}\cdot\mathbf{v}_2}{\mathbf{v}_2\cdot\mathbf{v}_2}\mathbf{v}_2$ (by the Best Approximation Theorem).
- Therefore, $\hat{\mathbf{y}} = \frac{3(1)+-1(-2)+1(-1)+13(2)}{1(1)+-2(-2)+-1(-1)+2(2)}\mathbf{v}_1 + \frac{3(-4)+-1(1)+1(0)+13(3)}{-4(-4)+1(1)+0(0)+3(3)}\mathbf{v}_2 = \frac{30}{10}\mathbf{v}_1 + \frac{26}{26}\mathbf{v}_2 = \begin{bmatrix} -1 \\ -5 \\ -3 \\ 9 \end{bmatrix}$

Page 353, Problem 21a:

True or False: If $\mathbf{z}$ is orthogonal to $\mathbf{u}_1$ and to $\mathbf{u}_2$ and if $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$, then $\mathbf{z}$ must be in $W^\perp$.

TRUE: This is true by definition of orthogonal complements (see Section 6.1) and is discussed in Example 1 of this section.

Page 353, Problem 21b:

True or False: For each $\mathbf{y}$ and each subspace W, the vector $\mathbf{y} - \text{proj}_W\mathbf{y}$ is orthogonal to W.

TRUE: This is true by the Orthogonal Decomposition Theorem.

Page 353, Problem 21c:

True or False: The orthogonal projection $\hat{\mathbf{y}}$ of $\mathbf{y}$ onto a subspace W can sometimes depend on the orthogonal basis for W used to compute $\hat{\mathbf{y}}$.

FALSE: This contradicts the statement in the text following Theorem 8 at the bottom of page 348.

Page 353, Problem 21d:

True or False: If $\mathbf{y}$ is in a subspace W, then the orthogonal projection of $\mathbf{y}$ onto W is $\mathbf{y}$ itself.

TRUE: This property is explicitly stated on page 350.

Section 6.4

Page 336, Problem 5:

Use the process stated on page 355:

- $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -4 \\ 0 \\ 1 \end{bmatrix}$
- $\mathbf{v}_2 = \begin{bmatrix} 7 \\ -7 \\ -4 \\ 1 \end{bmatrix} - \frac{7(1)+-7(-4)+-4(0)+1(1)}{1(1)+-4(-4)+0(0)+1(1)} \begin{bmatrix} 1 \\ -4 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -7 \\ 4 \\ 1 \end{bmatrix} - \frac{36}{18} \begin{bmatrix} 1 \\ -4 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -4 \\ -1 \end{bmatrix}$
- Thus, an orthogonal basis for W is $\{\mathbf{v}_1, \mathbf{v}_2\}$

Page 336, Problem 11:

Just make the columns of the matrix into an orthogonal basis using the G-S Process:

- $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$

- $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 4 \\ -4 \\ 2 \end{bmatrix} - \frac{2(1)+1(-1)+4(-1)+-4(1)+2(1)}{1(1)+-1(-1)+-1(-1)+1(1)+1(1)} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \\ -4 \\ 2 \end{bmatrix} - \frac{-5}{5} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \\ -3 \\ 3 \end{bmatrix}$

- $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -4 \\ -3 \\ 7 \\ 1 \end{bmatrix} - \frac{5(1)+-4(-1)+-3(-1)+7(1)+1(1)}{1(1)+-1(-1)+-1(-1)+1(1)+1(1)} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} - \frac{5(3)+-4(0)+-3(3)+7(-3)+1(3)}{3(3)+0(0)+3(3)+-3(-3)+3(3)} \begin{bmatrix} 3 \\ 0 \\ 3 \\ -3 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ -3 \\ 7 \\ 1 \end{bmatrix} - 4 \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} - \frac{-1}{3} \begin{bmatrix} 3 \\ 0 \\ 3 \\ -3 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 2 \\ -2 \end{bmatrix}$

- Thus, an orthogonal basis for the columns space is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.