MATH221

quiz #4, 12/01/15

Total 100

Solutions

Show all work legibly. **Name**:______________________

1. (40) Compute $\det A = \det \begin{bmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 6 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{bmatrix}$.

Solution.

$$\det \begin{bmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 6 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{bmatrix} = \det \begin{bmatrix} -1 & 2 & 3 & 0 \\ 0 & 10 & 15 & 0 \\ 0 & 14 & 21 & 6 \\ 0 & 10 & 16 & 3 \end{bmatrix} = \det \begin{bmatrix} -1 & 2 & 3 & 0 \\ 0 & 10 & 15 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$= -10 \times \det \begin{bmatrix} 0 & 6 \\ 1 & 3 \end{bmatrix} = -10 \times -6 = 60.$$

$\det A =$

2. (80) Let $A = \begin{bmatrix} 8 & 4 \\ 4 & 8 \end{bmatrix}$

   (a) (20) If possible find eigenvalues λ_1 and λ_2 of the matrix A.

   Solution.

$$0 = \det(A - \lambda I) = \det \begin{bmatrix} 8-\lambda & 4 \\ 4 & 8-\lambda \end{bmatrix} = (8-\lambda)^2 - 16,$$

   and $\lambda_1 = 4$, $\lambda_2 = 12$.

   $\lambda_1 =$ $\qquad\qquad$ $\lambda_2 =$

   (b) (20) Find eigenvector $\mathbf{x}_1 = \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix}$ that corresponds to λ_1.

   Solution.

$$0 = (A - \lambda_1 I)\mathbf{x} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}\mathbf{x}, \text{ hence } x_1 + x_2 = 0, \text{ and } \mathbf{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

   $\mathbf{x}_1 =$

(c) (20) Find eigenvector $\mathbf{x}_2 = \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix}$ that corresponds to λ_2.

Solution.

$$0 = (A - \lambda_1 I)\mathbf{x} = \begin{bmatrix} -4 & 4 \\ 4 & -4 \end{bmatrix} \mathbf{x}, \text{ hence } -x_1 + x_2 = 0, \text{ and } \mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$\mathbf{x}_2 =$

(d) (20) Compute A^{10}.

Solution. Let $\mathbf{v}_1 = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$, and $V = [\mathbf{v}_1, \mathbf{v}_2]$. Then $V^{-1} = V^T$, and $V^T A V = \begin{bmatrix} 4 & 0 \\ 0 & 12 \end{bmatrix}$. Finally $A = V \begin{bmatrix} 4 & 0 \\ 0 & 12 \end{bmatrix} V^T$, and $A^{10} = V \begin{bmatrix} 4^{10} & 0 \\ 0 & 12^{10} \end{bmatrix} V^T$.

$$\frac{1}{2} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4^{10} & 0 \\ 0 & 12^{10} \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 4^{10} + 12^{10} & 12^{10} - 4^{10} \\ 12^{10} - 4^{10} & 4^{10} + 12^{10} \end{bmatrix} = \frac{4^{10}}{2} \begin{bmatrix} 3^{10} + 1 & 3^{10} - 1 \\ 3^{10} - 1 & 3^{10} + 1 \end{bmatrix}.$$

$A^{10} =$