MATH221

quiz #2, 10/13/15

Total 100

Solutions

Show all work legibly. **Name**:______________________

1. (20) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. True or False? The vectors are linearly independent.

   Solution. $-\mathbf{v}_1 + \mathbf{v}_2 - 3\mathbf{v}_3 = 0$.

   Mark one and explain.

   □ True □ False

2. (40) Let $T : \mathbf{R}^3 \to \mathbf{R}^1$ be a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = 1$, $T\left(\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\right) = 2$, and $T\left(\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}\right) = 3$. Find $T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$.

   Solution. Note that
$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix},$$
   hence $T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = -\frac{1}{3} \times 1 + \frac{1}{3} \times 2 = \frac{1}{3}$.

3. (20) Let T be a linear transformation that leaves $\mathbf{e}_1$ unchanged and maps $\mathbf{e}_2$ to $\mathbf{e}_2 - \mathbf{e}_1$.

   - (10) Find the standard matrix A of the linear transformation T.

     Solution. $A = [T(\mathbf{e}_1)\ T(\mathbf{e}_2)] = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.

   - (10) True or False? T is a one to one transformation.

     Solution. If $T(\mathbf{x}) = A\mathbf{x} = 0$, then $x_2 = 0$ and $x_1 = 0$.

     Mark one and explain.

     □ True □ False

- (10) True or False? T is an onto transformation.

  Solution. If $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, then $\mathbf{x}$ that solves $A\mathbf{x} = \mathbf{b}$ is $\mathbf{x} = \begin{bmatrix} b_1 + b_2 \\ b_2 \end{bmatrix}$.

  Mark one and explain.

  ▫ True ▫ False

- (10) If possible define the linear transformation S so that $S(T(\mathbf{x})) = \mathbf{x}$ for each $\mathbf{x} \in \mathbf{R}^2$ and find the standard matrix B of S.

  Solution. Since T is one to one and onto the inverse exists, and $B = A^{-1}$, i.e.

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

  Hence $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, and $S(\mathbf{x}) = B\mathbf{x}$.

4. (20) Suppose A is invertible and E_1, E_2, and E_3 are elementary matrices that reduce A to the identity matrix, i.e. $E_3E_2E_1A = I$. Use E_i and E_i^{-1} to produce elementary matrices that reduce A^{-1} to I.

   Solution. Since $E_3E_2E_1A = I$ one has $I = (E_3E_2E_1A)^{-1} = A^{-1}(E_3E_2E_1)^{-1}$ and

$$A = (E_3E_2E_1)^{-1} = E_1^{-1}E_2^{-1}E_3^{-1}, \text{ finally } I = AA^{-1} = E_1^{-1}E_2^{-1}E_3^{-1}A^{-1}.$$

5. (20) Suppose A and B are $n \times n$ matrices. True or False? If AB is invertible, then B is also invertible.

   Solution. Note that $I = (AB)^{-1}AB = \left[(AB)^{-1}A\right]B$, and $(AB)^{-1}A = B^{-1}$.

   Mark one and explain.

   ▫ True ▫ False