MATH221

quiz #1, 09/17/15

Total 100

Solutions

Show all work legibly. **Name**:______________________

1. (20) Solve the system

$$\begin{array}{rrrcl} 2x_1 & & -4x_3 & = & 0 \\ & x_2 & +3x_3 & = & 2 \\ x_1 & +5x_2 & +3x_3 & = & 0 \end{array}$$

Solution.

$$\begin{bmatrix} 2 & 0 & -4 & 0 \\ 0 & 1 & 3 & 2 \\ 1 & 5 & 3 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 2 \\ 1 & 5 & 3 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 5 & 5 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & -10 & -10 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$x_1 = 2,\ x_2 = -1,\ x_3 = 1$

2. (20) Determine values of h for which the system

$$2x_1 - 6x_2 = 4,\ -4x_1 + hx_2 = 2$$

is consistent.

Solution.

$$\begin{bmatrix} 2 & -6 & 4 \\ -4 & h & 2 \end{bmatrix} \to \begin{bmatrix} 2 & -6 & h \\ 0 & h-12 & 10 \end{bmatrix}$$

$h \neq 12$

3. (20) Let

$$A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}.$$

True or False? The system $A\mathbf{x} = 0$ has a non trivial solution.

Solution.

$$\begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 \\ 2 & 0 & 6 \\ -1 & 8 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 \\ 0 & 4 & 4 \\ 0 & 6 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}$$

Mark one and explain.

□ True □ False

4. (20) True or False? If A is 5×3 matrix, $\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 5 \\ 6 \\ 0 \\ 1 \end{bmatrix}$, and $A\mathbf{y} = \mathbf{b}$, then the equation $A\mathbf{x} = 5\mathbf{b}$ is consistent.

Solution. If $\mathbf{x} = 5\mathbf{y}$, then $A\mathbf{x} = A(5\mathbf{y}) = 5A\mathbf{y} = 5\mathbf{b}$.

Mark one and explain.

□ True, $\mathbf{x} =$ □ False

5. (20) Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -5 \\ -3 \\ 1 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

True or False? The vector $\mathbf{v}_3$ can be written as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

Solution. $0 \cdot \mathbf{v}_1 + 0 \cdot \mathbf{v}_2 = \mathbf{v}_3$.

Mark one and explain.

□ True □ False

6. (20) Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \text{ and the matrix } A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \end{bmatrix}.$$

(a) (10) True or False? $\mathbf{v}_1^T\mathbf{v}_3 = \mathbf{v}_2^T\mathbf{v}_3 = 0$.

Mark one and explain.

- ☐ True
- ☐ False

(b) (10) True or False? $A\mathbf{x} = 0$ has a nontrivial solution.

Mark one and explain.

- ☐ True, $\mathbf{x} = \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
- ☐ False