

## Solution HA9

Note! Each question carry 02 points.

P824 Q2 An old legend tells of a wizard who agreed to be save a kingdom provided that the king would agree to a "modest" reward. The wizard asked to be given merely as much grain as would put one kernel on the first square of a chessboard, two kernels on the second, four on the third, eight on the fourth, and so forth, up through the 64<sup>th</sup> square. The king agreed and the wizard saved the kingdom, but the king was completely unable to honor the agreement. Why?

[Hint: Notice that  $1 = 2^1 - 1$  and  $(1+2) = 2^2 - 1$  and  $(1+2+4) = 2^3 - 1$ ] Generalize to arrive at a total for the number of kernels. A kernel of rice is about a quarter of an inch long and about a 16<sup>th</sup> of an inch wide and a 16<sup>th</sup> of an inch high. So about a thousand kernels will fit in a cubic inch (you should verify this calculation). [Calculate the total volume of kernels]

Solution: Total number of kernels =  $1 + 2 + 4 + 8 + \dots + 2^{64} - 1$

$$\therefore \frac{2^{64} - 1}{2 - 1}$$

$$= 1.84467 \times 10^{19}$$

No of kernels in a cubic inch =  $\frac{1}{(0.25 \times 0.0625 \times 0.0625)}$

$$= 1024 \text{ kernels.}$$

Total volume of kernels = No of kernels  $\times$  volume of each kernels

$$= \frac{2^{64} - 1}{2 - 1} \times 0.25 \times \frac{1}{16} \times \frac{1}{16}$$

$$= 1.8014 \times 10^{16}$$

Pg 824 20(c) Suppose that you save for retirement by contributing the same amount each month from your 23<sup>rd</sup> birthday until your 65<sup>th</sup> birthday, in an amount that pays a steady 5% annual interest compounded monthly?

c) How much will be in your fund if you get a steady return of 10% compounded monthly?

Solution:

$$\begin{aligned}
 A &= d \left[ \frac{(1+i)^n - 1}{i} \right] \\
 &= d \left[ \frac{(1 + (0.05/12))^{42 \times 12} - 1}{(0.05/12)} \right] \\
 &= 7734.298 d
 \end{aligned}$$

∴ whatever is monthly contribution d

The final amount will be 7734.298 times d.

i.e.  $7734.298 \times d$ .

Pg 850 Q3 Your car dealer offers to finance a \$6000 used car loan at 3% to be repaid in four years of monthly payments. What is the monthly payment?

Solution:

$$\begin{aligned}
 A &= P(1+r)^n \\
 &= 6000(1+0.03)^4 \\
 &= \$6753.05
 \end{aligned}$$

$$\text{Monthly payment} = \frac{6753.05}{48}$$

$$= \$140.68 \quad \text{option A}$$

Pg 850 Q7 If a store credit amount charges 1.5% interest each month, what is the annual percentage rate?

Solution: Effective annual rate =  $(1+i)^n - 1$   
= 19.56% Option B

Pg 850 Q9 Your credit union offers to finance a \$6000 conventional loan at 4% to be repaid in four years of monthly payments. What is the monthly payment?

$$A \in (1+i)^n = d \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$6000 \times \left(1 + \frac{0.04}{12}\right)^{48} = d \left[ \frac{(1 + 0.4/12)^{48} - 1}{(0.4/12)} \right]$$

$$7038.07 = d \cdot 51.96$$

$$d = \$135.45 \quad \text{Option A.}$$