1. Compute and explain $1+3+5 --- +997+999 = 500^2 = 250000$

<u>logic</u>: The sum is based on arithmatic sequence

a.) Sum of series can be given as

$$S_n = \frac{n(a_1+a_n)}{2}$$

Where S_n = sum of series

a_1 = starting element in series

a_n = ~~ending~~ last (n^{th}) element in series

n = no. of elements

b.) No of elements in series can be found by

$$a_n = a_1 + (n-1)d$$

Where d = differnce between two consecutive elements.

<u>Solution</u>: a.) Let's find no. of terms in series

i.e $a_n = a_1 + (n-1)d$

$999 = 1 + (n-1)2$

$\therefore 999-1+2 = 2n$

$\therefore n = 1000/2 = 500$

b.) Let's calculate sum of given serien

$$S_n = \frac{n(a_1+a_n)}{2}$$

$= 500 (1+999)/2$

$= 500 \times 500$

$= 500^2$

$= \boxed{250000}$

2. Compute and explain $1+2+2^2+2^3+ --- +2^{20} = 20971$

<u>logic</u>: The sum is based on geometric series (finite

a.) $S = a\frac{(1-r^n)}{(1-r)}$

Where S = sum of series

r = ratio of two consecutive terms

n = numbers of elements (terms)

a = first element in series

Solution: a) Let's find sum of given series

$$S = \frac{a(1-r^n)}{(1-r)}$$

$$= 1\,\frac{(1-2^{21})}{(1-2)}$$

$$= -(1-2^{21})$$

$$= \boxed{2097151}$$

3. In how many ways the number 100 can written a sum of two positive integers x_1+x_2? (as, for example, 1+99, 47+53)

99 ways.

logic: $x_1+x_2=100$ —— (i)

Note: x_1 & x_2 are positive integers

	Maximum	Minimum
x_1 can be	99	1
x_2 can be	99	1

∴ x_1 varies from 1 to 99 where adding positive x_2 will give a total of 100.

∴ $\boxed{99 \text{ ways}}$

4. True or False? Twice the sum of two square always gives the sum of two squares.
(for example $2(5^2+4^2)=1^2+9^2$) TRUE

logic: $2(x^2+y^2) = (x+y)^2 + (x-y^2)$

R.H.S $(x+y)^2+(x-y)^2$

$$= x^2+2xy+y^2+x^2-2xy+y^2$$

$$= x^2+y^2+x^2+y^2$$

$$= \boxed{2(x^2+y^2) = L.H.S}$$

Hence twice sum of two squares always gives the sum of two squares

5. There are three buckets: one red, one blue & one yel
They each hold a maximum of 5 gallons. Liquid is measured carefully in whole gallons and poured into the buckets, a different number of gallons in each one. If the liquid in the red bucket was poured into the blue bucket, it would then contain the same amount of liquid as the yellow bucket. Half the contents of the yellow bucket is same as twice that in the red bucket.
How much liquid is there in each bucket?

<u>Solution</u>

Red → R, Blue − B, Yellow − Y

1. Maximum capacity = 5 gallons
2. Only whole gallons (~~integers~~) are poured in each bucket.

Let R contains ~~x~~ m gallons
B contains n gallons
Y contains q gallons

Then ~~to~~ given

$$m + n = q \rightarrow (i)$$

&

$$\frac{1}{2} q = 2m$$

$$\therefore \quad q = 4m \rightarrow (ii)$$

$$\therefore \quad m = 1 \quad \because \quad q = 4m \text{ and } q, m, n \le 5$$

$$\& \quad q = 4$$

$$n = 3 \quad \because \quad m + n = q$$

$$\boxed{\begin{array}{l} \text{Red} = 1 \text{ gallons} \\ \text{Blue} = 3 \text{ gallons} \\ \text{Yellow} = 4 \text{ gallons} \end{array}}$$