ENCE 621/489G Midterm 3 Solution Sheet

1. To determine the volumetric flow rate, one needs to use the following equation:

$$Q = \frac{K \Delta H m}{n}$$

where K is hydraulic conductivity, ΔH is the total hydraulic head difference, m is the number of flow tubes and n the number of equipotential intervals. At the top of the soil, the pressure head h = 100 cm = 1 m. At the drain, the pressure head is 0. The elevations at the top and the drain are 100 m and 50 m, respectively. Therefore, we have the hydraulic heads at the top and drain of 101 m and 50 m, respectively. We thus have ΔH = 101 - 50 = 51 m. From the flow net, we count 5 equipotential intervals and 6 flow tubes, or n = 5 and m = 6. We thus have the volume of water collected per hour as

```
> K := 5e-6*60*60; DeltaH := 101.0 - 50.0; m := 6.0; n := 5.0; dQ
  := K*DeltH/n; Q := K*DeltH*m/n;
```

$$K := 0.018000$$
$$DeltaH := 51.0$$
$$m := 6.0$$
$$n := 5.0$$
$$dQ := 0.1836000000$$
$$Q := 1.101600000$$

2. To determine the storage coefficient, one needs to calculate the water capacity of the soil. The water capacity at 0 cm is zero, because when h = 0 > h_d the water content is always the same as θ_s. For h = -10 cm and -1000 cm, the corresponding water capacity values are:

```
> theta[r] := 0.03; theta[s] := 0.41; N := 0.75; m := 4.25; hd :=
  -4.0; h := -10.0; WC10 := -N*(theta[s]-theta[r])/h*(hd/h)^N;
```

$$\theta_r := 0.03$$
$$\theta_s := 0.41$$
$$N := 0.75$$
$$m := 4.25$$
$$hd := -4.0$$
$$h := -10.0$$
$$WC10 := 0.01433474110$$

```
> h := -1000; WC1000 := -N*(theta[s]-theta[r])/h*(hd/h)^N;
```

$$h := -1000$$
$$WC1000 := 0.4533043155\ 10^{-5}$$

With the given water and soil matrix compressibility, one has the storage coefficient at h = 0 cm as:

```
> rho[w] := 1000; g := 9.81; beta[w] := 4.8e-10; beta[p] :=
```

```
6.9e-8;S[s0] := rho[w]*g*(beta[p]*1.0+beta[w]*theta[s])+0.0;
```

$$\rho_w := 1000$$

$$g := 9.81$$

$$\beta_w := 0.48\ 10^{-9}$$

$$\beta_p := 0.69\ 10^{-7}$$

$$S_{s0} := 0.0006788206080$$

At h = -10 cm, we have $\frac{d\theta}{dh}$ = WC10 as calculated above, therefore:

```
> h := -10; theta[10] := theta[r] + (theta[s] -
theta[r])*(hd/h)^N; S[s10] :=
rho[w]*g*(beta[p]*theta[10]/theta[s]+beta[w]*theta[s])+WC10*100
;
```

$$h := -10$$

$$\theta_{10} := 0.2211298813$$

$$S_{s10} := 1.433841115$$

At h = -1000 cm, we have $\frac{d\theta}{dh}$ = WC1000 as calculated above, therefore:

```
> h := -1000; theta[1000] := theta[r] + (theta[s] -
theta[r])*(hd/h)^N; S[s1000] :=
rho[w]*g*(beta[p]*theta[1000]/theta[s]+beta[w]*theta[s])+WC1000
*100;
```

$$h := -1000$$

$$\theta_{1000} := 0.03604405754$$

$$S_{s1000} := 0.0005147419042$$

When the soil column is unsaturated and if we use the upward z as the positive z direction, the hydraulic gradient $\frac{dH}{dz} = 1$, given that the pressure head is uniformly at -10 cm over the length of the soil column. We may thus calculate the hydraulic conductivity, specifice discharge and vlumetric flow rate as follows:

```
> h := -10; K[s] := 0.24; K[10] := K[s]*(hd/h)^m; q :=
-K[10]*(1.0); r := 5.0; A := 3.14159*r^2; Q := q*A;
```

$$h := -10$$

$$K_s := 0.24$$

$$K_{10} := 0.004886143358$$

$$q := -0.004886143358$$

$$r := 5.0$$

$$A := 78.5397500$$

$$Q := -0.3837564778$$

with the negative signs of q and Q indicating the direction of the flow or downwards.

Using the tangent defraction low, the relationship between hydraulic conductivity and defraction angle is:

$$\frac{K_1}{K_2} = \frac{\tan(\theta_1)}{\tan(\theta_2)}.$$

Given the following values for the top clay layer:

```
> K[1] := 5e-6; theta[1] := 5.0;
```

$$K_1 := 0.5\ 10^{-5}$$

$$\theta_1 := 5.0$$

one may calculate the angle between the interface of the clay and sand layers, or θ_{clay} as:

```
> K[2] := 4e-9; theta[2] :=
  arctan(K[2]/K[1]*tan(theta[1]*3.14159/180.0))/3.14159*180;
  theta[clay] := 90.0 - theta[2];
```

$$K_2 := 0.4\ 10^{-8}$$

$$\theta_2 := 0.004010184917$$

$$\theta_{clay} := 89.99598982$$

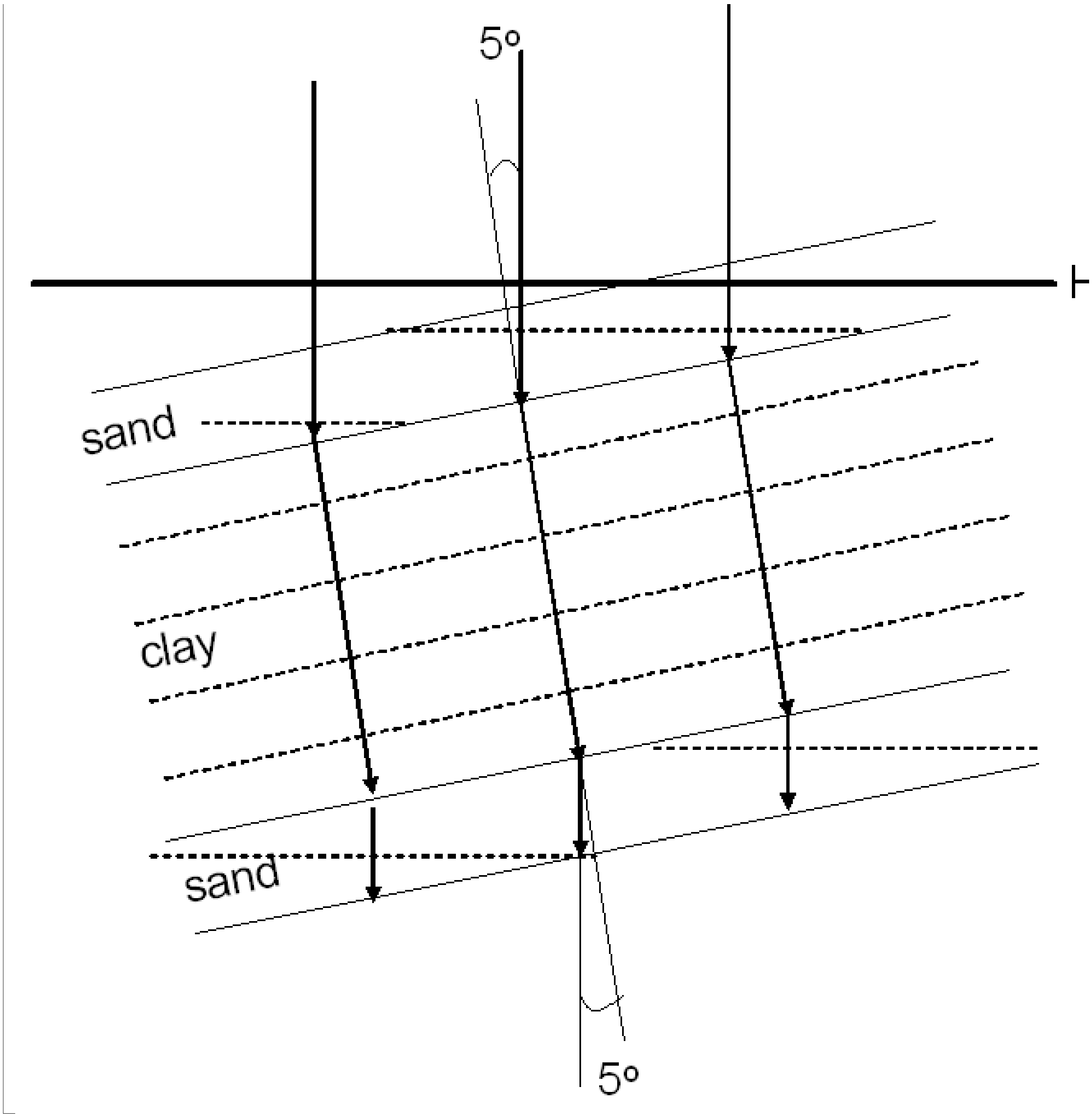
The water enter the bottom sand layer with angle as calculated in the following:

```
> theta[3] :=
  arctan(K[1]/K[2]*tan(theta[2]*3.14159/180.0))/3.14159*180;
  theta[sand] := 90.0 - theta[3];
```

$$\theta_3 := 5.000000001$$

$$\theta_{sand} := 85.00000000$$

which suggests that the angles of entering and existing the clay layer are the same. The flow lines and equipotential lines may be plotted as follows:



5°
sand
clay
sand
5°