ENCE 489G/621, Homework #3, Solution Sheet

1. For the constant permeameter, we have the following
   a. Pressure head at the bottom of the soil core = h + d_2 + L
   b. Elevation head at the bottom of the soil core = z_1 + d_1
   c. Hydraulic head at the bottom of the soil core = h + d_2 + L + z_1 + d_1
   d. Hydraulic head at the top of the soil core = d_2 + L + z_1 + d_1
   e. Hydraulic gradient across the length of the soil core = [(d_2 + L + z_1 + d_1) – (h + d_2 + L + z_1 + d_1)]/L = -h/L
   f. Specific discharge q = -K•hydraulic gradient = -K•(-h/L); volumetric flow rate Q = q•A = A•K•h/L; therefore K = QL/Ah
   g. A = πr^2 = 3.14159*5^2 = 78.54 cm^2; Q = 0.5 *l*/min = 0.5 × 1000 cm^3/min = 500 cm^3/min; K = QL/Ah = 500•10/(78.54•30) cm^3/min•cm/(cm^2•cm) = 2.12 cm/min.
2. On the basis of mass balance, the volumetric rate of water removed from the standing pipe at an instant t or adh/dt and the volumetric rate of water flowing through the soil core or A(-Kh/L) should be equal, or

$$a\frac{dh}{dt} = -\frac{AKh}{L} \tag{2.1}$$

   Move the second term to the right hand side and integrate over the period from t_0 to t_1,

$$\int_{h_0}^{h_1}\frac{dh}{h} = -\int_{t_0}^{t_1}\frac{AKdt}{aL} \tag{2.2}$$

   or

$$\ln h\Big|_{h_0}^{h_1} = -\frac{AK}{aL}t\Big|_{t_0}^{t} \Rightarrow \ln h_1 - \ln h_0 = -\frac{AK}{aL}(t_1 - t_0) \tag{2.3}$$

   Finally, we have

$$K = \frac{aL}{A(t_1 - t_0)}\ln\left(\frac{h_0}{h_1}\right) = 2.3\frac{aL}{A(t_1 - t_0)}\log_{10}\left(\frac{h_0}{h_1}\right) \tag{2.4}$$

3. The equation derived by Darcy is

$$\ln(h + e) = \ln(h_0 + e) - \frac{K}{e}(t - t_0) \tag{3.1}$$

   Rearranging for K, one has

$$K = \frac{e}{t - t_0} \ln\left(\frac{h_0 + e}{h + e}\right) \qquad (3.2)$$

Given the following:

e =	2.25	m =	225	cm	
h =	5	cm	t =	60	min
h_0 =	10	cm	t_0 =	0	min

One has

$$K = \frac{225}{60} \ln\left(\frac{10 + 225}{5 + 225}\right) \frac{cm}{\min} \ln\left(\frac{cm}{cm}\right) = 8.06 \times 10^{-2} \, cm/\min \; = 1.34 \times 10^{-3} \, cm/s$$

With hydraulic conductivity obtained above, one may use equation (3.1) to calculate the value of h at a particular time t. With this value, one may then calculate the hydraulic gradient at time t and the specific discharge or Darcy's velocity as well. Given the effective porosity 0.25, ρ = 1 g/cm^3, μ = 0.001 Kg/ms = 0.1 g/cm-s, and the average grain size of 1.10 mm or d = 0.11 cm, one obtains the following table:

time (min)	height of water (cm)	hydraulic gradient	pore water velocity (cm/min)	Reynold's No.
0	10.00	-1.0444	0.3369	6.1771E-02
10	9.16	-1.0407	0.3357	6.1550E-02
20	8.32	-1.0370	0.3345	6.1329E-02
30	7.49	-1.0333	0.3333	6.1110E-02
60	5.00	-1.0222	0.3298	6.0456E-02

Apparently, the calculated Reynold's No.'s are all less than 1, indicating that the student's experiment design is fine and that Darcy's law is applicable.