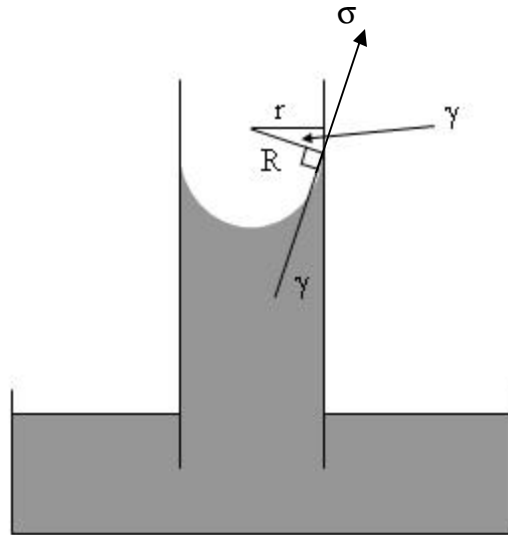


**Problem 1:**



With a contact angle  $\gamma$ , the surface tension parallel to the glass tube wall is  $\sigma \cos \gamma$ . The total upward force circumferencing the inner glass tube wall is thus

$$F_t = 2pr \cdot \sigma \cos \gamma \tag{b1}$$

The total downward force is the pull of gravity to the column of water:

$$F_g = pr^2 h r g \tag{b2}$$

At equilibrium, (b1) = (b2), or

$$2pr \cdot \sigma \cos \gamma = pr^2 h r g \tag{b3}$$

Rearrange and simplify, one has

$$P_c = rgh = \frac{2\sigma \cos \gamma}{r} \tag{b4}$$

Because  $R \cos \gamma = r$ , one proves

$$P_c = \frac{2\sigma}{R} \tag{b5}$$

**Problem 2**

Rearrange equation (b.4), one has the relation between capillary rise and equivalent pore radius:

$$r = \frac{2s \cos \theta}{\rho g h} \tag{b6}$$

For the napkin, we have an average from two measurements of  $(10.3 + 9)/2 = 9.65$  cm. Given that  $\sigma = 72.2$  dynes/cm,  $\theta = 15^\circ$ ,  $\rho = 1$  g/cm<sup>3</sup> and  $g = 981$  cm/s<sup>2</sup>, one has the equivalent pore radius =

$$r_{napkin} = \frac{2 * 72.2 * \cos(15^\circ)}{1 * 981 * 9.65} \frac{\text{dynes / cm}}{\text{g / cm}^3 * \text{cm / s}^2 * \text{cm}} = 0.015 \text{ cm}$$

Similarly for the notebook paper, we have an average from two observations of  $(0.3 + 0.7)/2 = 0.5$  cm. Using equation (b6) again, one has

$$r_{notebok} = \frac{2 * 72.2 * \cos(15^\circ)}{1 * 981 * 0.5} \frac{\text{dynes / cm}}{\text{g / cm}^3 * \text{cm / s}^2 * \text{cm}} = 0.284 \text{ cm}$$

With a contact angle of  $15^\circ$ , the estimated pore radius of the napkin is smaller than that of a notebook paper. Further examining the texture of the materials, one has to conclude that a notebook paper has a finer feel than a napkin. Therefore, the contact angle used for both type of materials may not be correct. In order for the estimated pore radius of the notebook paper to be smaller, the contact angle for the notebook paper must be much bigger than that of the napkin. This may also suggest that the napkin is made from a material (cotton, for example) that is different from that of the notebook paper (wood fiber, for example).

**Problem 3**

Using the equation (b4), one has the following capillary rise and equation as a function of tube radius:

$$h = \frac{2s}{r \rho g} \tag{b7}$$

With  $\rho = 1.0$  g/cm<sup>3</sup>,  $g = 981$  cm/s<sup>2</sup>,  $\sigma = 72.2$  erg/cm<sup>2</sup> =  $72.2$  (g•cm/s<sup>2</sup>•cm)/cm<sup>2</sup>, one may obtain the following table:

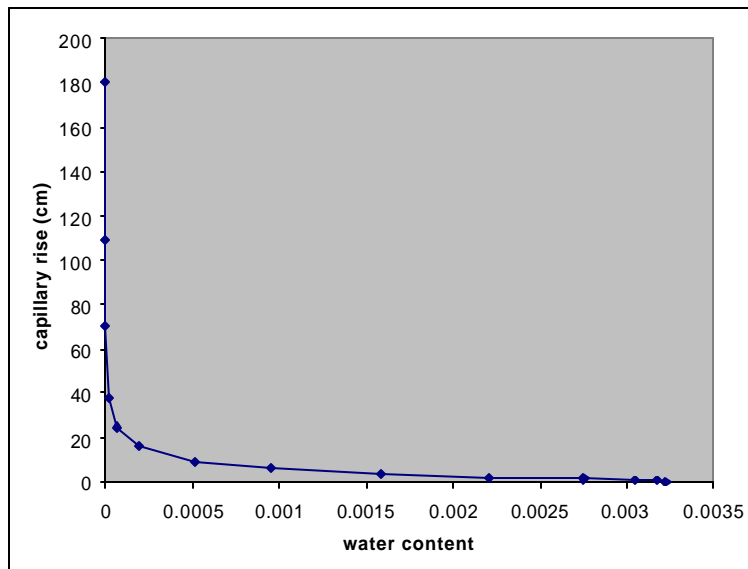
radius (cm)	frequency	area cm <sup>2</sup>	capillary rise cm	Water volume cm <sup>3</sup>	water content cm <sup>3</sup> /cm <sup>3</sup>	saturation %
0.0008185	1	2.10459E -06	181.0867268	0.000381113	7.62227E -08	1.90494E -05
0.0013559	4	2.31032E -05	109.3110482	0.002525437	5.8131E -07	1.4528E -04
0.0021006	16	0.00022179	70.56014982	0.015649525	3.71122E -06	9.27497E -04
0.0039242	44	0.002128655	37.7697466	0.080398746	1.9791E -05	4.94611E -03

0.0059515	91	0.010126018	24.90414767	0.252179849	7.02269E-05	0.0175509
0.0093489	149	0.040912434	15.85388898	0.648621181	0.000199951	0.0499713
0.0175649	191	0.185128636	8.438207839	1.56215391	0.000512382	0.1280532
0.0248351	191	0.370096922	5.96800643	2.208740812	0.00095413	0.2384537
0.0450413	149	0.949637777	3.29067111	3.124945597	0.001579119	0.3946494
0.0736017	91	1.548699476	2.013758822	3.118707232	0.002202861	0.5505333
0.1346353	44	2.50565171	1.100870723	2.75839861	0.00275454	0.6884076
0.198646	16	1.983487098	0.746131949	1.479943095	0.003050529	0.7623804
0.312416	4	1.226524969	0.474419066	0.58188683	0.003166906	0.7914651
0.613039	1	1.180663489	0.241772705	0.285452205	0.003223997	0.805733
total	992	10.00330418				
x-section area (cm <sup>2</sup> )		25				
porosity		0.400132167				
total pore volume (cm <sup>3</sup> )		2000.660836				
total volume (cm <sup>3</sup> )		5000				
total water volume (cm <sup>3</sup> )		16.11998414				
water content		0.003223997				
degree of saturation		0.805733%				

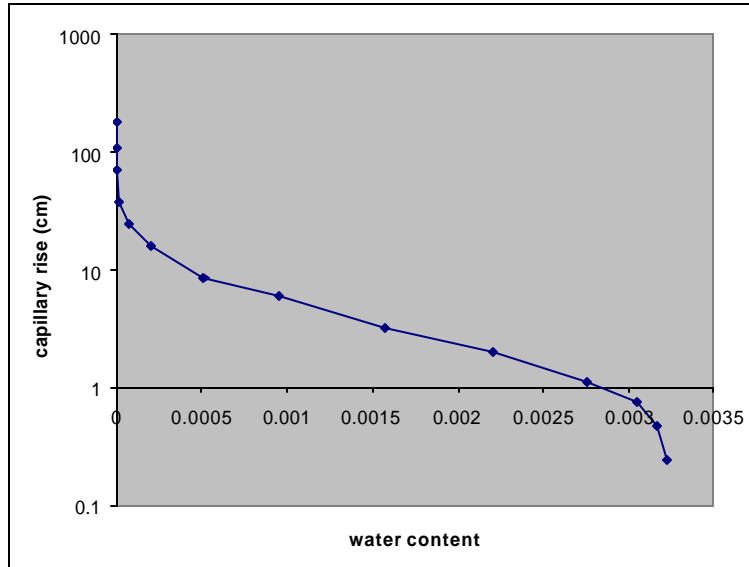
With that above table, one concludes that:

1. The total volume of the void inside the tubes is about 2000 cm<sup>3</sup>. The pipe's cross sectional area is 25 cm<sup>2</sup> and its length is 200 cm. The porosity is  $2000 \text{ cm}^3 / (25 \text{ cm}^2 * 200 \text{ cm}) = 0.4$ .
2. The following two figures are the capillary rise and water content relationships:

(a) linear



(b) logarithmic



3. According to the figure, the likely residual water content is in the order of  $10^{-6}$ .