ENCE 621/489G, Homework 6, Solution Sheet

Problem 1: Storage Coefficients of Variably Saturated Soil

1. Storage Coefficients

At pressure head 10 cm, the soil is saturated, the water content is $\theta = \theta_s$ and is a constant.

Therefore, $\frac{d\theta}{dh} = 0$. Therefore the storage coefficient is thus:

```
> theta[s] := 0.357; theta[r] := 0.15; rho[w] := 1000.0; g :=
  0.981; beta[p] := 6.9e-8; beta[w] := 4.8e-10; dtOdh := 0.0;
  alpha := 0.28e-2; N := 1.35; m := 1.0 - 1.0/N;
```

$$\theta_s := 0.357$$

$$\theta_r := 0.15$$

$$\rho_w := 1000.0$$

$$g := 0.981$$

$$\beta_p := 0.69\ 10^{-7}$$

$$\beta_w := 0.48\ 10^{-9}$$

$$dtOdh := 0.$$

$$\alpha := 0.0028$$

$$N := 1.35$$

$$m := 0.2592592593$$

```
> S[p] := rho[w]*g*beta[p]*1.0; S[w] :=
  rho[w]*g*beta[w]*theta[s]; S[s10] := S[p]+S[w]+dtOdh;
```

$$S_p := 0.00006768900000$$

$$S_w := 0.1681041600\ 10^{-6}$$

$$S_{s10} := 0.00006785710416$$

with a unit of 1/m.

At a pressure head of 0 cm, the soil is also saturated and the storage coefficient is the same as that at 10 cm pressure head.

At a pressure head of h = -100 cm and using the van Genuchten water retention relationship, one has:

```
> psi := abs(-100.0); theta[100] := theta[r] +
  (theta[s]-theta[r])/(1+(alpha*psi)^N)^m;
```

$$\psi := 100.0$$

$$\theta_{100} := 0.3483342915$$

One also needs to calculate the derivative $\frac{d\,\theta}{d\,h}$ as follows:

```
> dtOdh :=
(theta[s]-theta[r])*m*(alpha*psi)^N*N/((1+(alpha*psi)^N)^(m+1
)*psi);
```

$$dtOdh := 0.0001055582711$$

The storage coefficient at this pressure head may thus be calculated as:

```
> S[p] := rho[w]*g*beta[p]*theta[100]/theta[s]; S[w] :=
rho[w]*g*beta[w]*theta[100]; S[s100] := S[p]+S[w]+dtOdh;
```

$$S_p := 0.00006604593798$$

$$S_w := 0.1640236512\ 10^{-6}$$

$$S_{s100} := 0.0001717682327$$

with a unit of 1/m. This storage coefficient is about three times of that at saturation, largely caused by dewatering of the soil.

At a pressure head of h = -1000000 cm and using the van Genuchten water retention relationship, one has:

```
> psi := abs(-1000000.0); theta[1000000] := theta[r] +
(theta[s]-theta[r])/(1+(alpha*psi)^N)^m;
```

$$\psi := 0.10000000\ 10^{7}$$

$$\theta_{1000000} := 0.1628665625$$

With the derivative $\frac{d\,\theta}{d\,h}$ as follows:

```
> dtOdh :=
(theta[s]-theta[r])*m*(alpha*psi)^N*N/((1+(alpha*psi)^N)^(m+1
)*psi);
```

$$dtOdh := 0.4503196932\ 10^{-8}$$

the storage coefficient (in 1/m) may be calculated as:

```
> S[p] := rho[w]*g*beta[p]*theta[1000000]/theta[s]; S[w] :=
rho[w]*g*beta[w]*theta[1000000]; S[s1000000] :=
S[p]+S[w]+dtOdh;
```

$$S_p := 0.00003088032143$$

$$S_w := 0.7669060695\ 10^{-7}$$

$$S_{s1000000} := 0.00003096151524$$

which is only a half of that at saturation.

2.Relative Hydraulic Conductivity

At pressure heads 0 and 10 cm, the water content θ is equal to θ_s. Thus, $\Theta = 1$, which suggests that $K_r = 1$. Therefore, the hydraulic conductivity is equal to $K_s = .35\ X\ 10^{(-4)}$ cm/s.

At pressure head -100 cm, one has the following:

```
> Theta[100] := (theta[100] - theta[r])/(theta[s] - theta[r]);
```

$$\Theta_{100} := 0.9581366739$$

```
> K[r100] := Theta[100]^0.5*(1.0-(1.0-Theta[100]^(1/m))^m)^2;
```

$$K_{r100} := 0.1460956666$$

```
> K[s] := 0.35e-4; K[100] := K[s]*K[r100];
```

$$K_s := 0.000035$$

$$K_{100} := 0.5113348331\ 10^{-5}$$

with the units of the hydraulic conductivity in cm/s.

Similarly, at a pressure head of -1000000 cm, one has the following:

```
> Theta[1000000] := (theta[1000000] - theta[r])/(theta[s] -
  theta[r]);
```

$$\Theta_{1000000} := 0.06215730676$$

```
> K[r1000000] :=
  Theta[1000000]^0.5*(1.0-(1.0-Theta[1000000]^(1/m))^m)^2;
```

$$K_{r1000000} := 0.8258135932\ 10^{-11}$$

```
> K[s] := 0.35e-4; K[1000000] := K[s]*K[r1000000];
```

$$K_s := 0.000035$$

$$K_{1000000} := 0.2890347576\ 10^{-15}$$

3. Flow Rates

At pressure heads equal or less than zero, the pressure heads over the entire length of the soil column is uniform. The hydraulic head is thus $H = h\_ + z$, with $h\_$ the uniform pressure head. Using the bottom of the soil column as reference datum, or $H_b = h\_$, the hydraulic head at the top of the soil column is thus $H_t = h\_ + 10$ cm. The hydraulic gradient, using the upward direction as positive z, is thus $H_t - H_b = 1$ cm/cm. Using the hydraulic conductivities calculated above one has the specific discharge at 0, -100, and -1000000 (q_0, q_{100}, $q_{1000000}$, respectively) in cm/s as follows:

```
> dHdz := 1.0; q[0] := -K[s]*dHdz; q[100] := -K[100]*dHdz;
  q[1000000] := -K[1000000]*dHdz;
```

$$dHdz := 1.0$$

$$q_0 := -0.0000350$$

$$q_{100} := -0.5113348331\ 10^{-5}$$

$$q_{1000000} := -0.2890347576\ 10^{-15}$$

The volumetric flow rates are the product of specific discharge and cross sectional area of the soil column, which are:

```
> A := 3.14159*5.0^2; Q[0] := q[0]*A; Q[100] = q[100]*A;
  Q[1000000] = q[1000000]*A;
```

$$A := 78.5397500$$

$$Q_0 := -0.002748891250$$

$$Q_{100} = -0.0004016010996$$

$$Q_{1000000} = -0.2270071760\ 10^{-13}$$

The pore water velocity is the velocity through the water saturated portion of the soil, or specific discharge divided by water contents:

```
> v[0] := q[0]/theta[s]; v[100] := q[100]/theta[100]; v[1000000]
  := q[1000000]/theta[1000000];
```

$$v_0 := -0.00009803921569$$

$$v_{100} := -0.00001467942851$$

$$v_{1000000} := -0.1774672180\ 10^{-14}$$

Problem 2: Flow Net

Flow rate along a flow tube: according to the Darcy's law, the volumetric flow rate is the product of specific discharge and the cross sectional area. In the context of the flow net, assume the particular area per unit length into the page is *dA*, the distance between two equipotential line of a particular cell is *dl*, and the equipotential contour interval is *dH*, one thus has:

$dQ = \frac{dA\ K\ dH}{dl}$ or using dA = dm*unit length into the page and the relation that dm ~ dl, one has $dQ = K\ dH$. Because dH = H/n, where n is the n+1 is the number of equipotential lines, one has $dQ = \frac{K\ H}{n}$, with K as the hydraulic conductivity. The total head drop from upstream to downstream of the dam, in m, is:

```
> h[1] := 160; h[2] := 70; H := h[1] - h[2];
```

$$h_1 := 160$$

$$h_2 := 70$$

$$H := 90$$

Because there are seven equipotential lines, we have n = 6. The volumetric flow rate within individual flow tube is thus:

```
> n := 6; dH := H/n; K := 1e-4; dQ := K*dH;
```

$$n := 6$$

$$dH := 15$$

$$K := 0.0001$$

$$dQ := 0.0015$$

Because there are four (m) flow tubes, the total volumetric flow rate may thus be obtained by multiplying *dQ* by m:

```
> m := 4; Q := m*dQ;
```

$$m := 4$$

$$Q := 0.0060$$

Problem 3: Estimating Transimissivity with Flow Net

In the vicinity of pumping center B, we have 26 flow tubes converging. With the equipotential lines 30 to 60 between four equipotential lines, we have m = 26, n = 3, and the following equation to determine the transimissivity:

$$Q = \frac{T\,H\,m}{n},$$

given Q = 3 x 10^4 m/d. Numerically, one has:

```
> H := (60.0 - 30.0)*0.305; n := 3; Q := 3e4; m:= 26; T :=
  Q*n/(H*m);
```

$$H := 9.1500$$

$$n := 3$$

$$Q := 30000.$$

$$m := 26$$

$$T := 378.3102144$$

```
>
```

in the unit of m^2/d.