ENCE 621/489G Solution Sheet, Homework 4

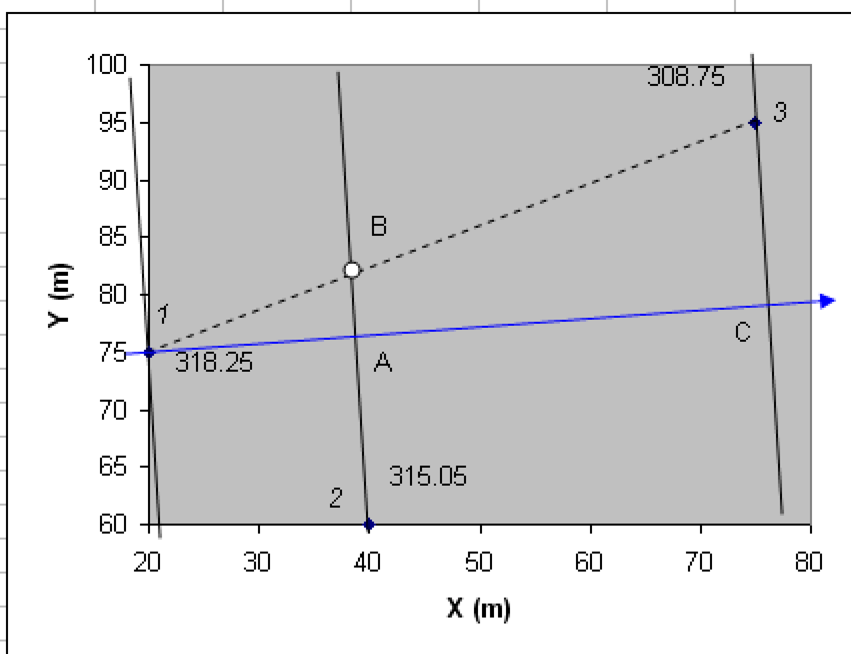
1. Water level contour

To calculate various head potentials, one use the following relationships:

Pressure Head = Depth to Piezometer - Depth to Top of Water Column
Elevation Head = Surface Elevation - Depth to Piezometer
Hydraulic head = Elevation Head + Pressure Head = Surface Elevation - Depth to Top of Water Column

	A	B	C	D	E	F	G	H	I
1	*Piezometer*	*X*	*Y*	*S Elevation*	*Depth*	*Depth_to_Water*	*p head*	*e head*	*h head*
2	1	20	75	320	5	1.75	3.25	315	318.25
3	2	40	60	316	3.5	0.95	2.55	312.5	315.05
4	3	75	95	310	2.5	1.25	1.25	307.5	308.75

The contour map can be obtained by interpolating on the line connecting point 1 and 3. The point at which the hydraulic head is around 315.05 is about 33.4% of the length of the line from point 1, marked as point B in the following figure.



From point 1 to point A, the distance is about 19.2 m and that from A to C is about 37.2 m. The gradient from point 1 to A is thus

```
> G1 = (315.05-318.25)/19.2;
```

$$G1 = -0.1666666667$$

Given a hydraulic conductivity K = 5 x $10^{(-7)}$ m/s, one has the Darcy velocity as

```
> q = -5.0E-7*(-0.1667);
```

$$q = 0.83350\ 10^{-7}$$

in m/s. A similar set of values (gradient and Darcy velocity) are also obtained between points A and C.

2. Hydraulic conductivity of multiple strata

The volumetric flow rate perpendicular to the layering can be represented by using the Darcy's equation as follows:

$$Q = -\frac{K_z\,\Delta H\,A}{m_T} \qquad (2.1)$$

where A is an area perpendicular to the flow direction and m_T is the total thickness of the strata, ΔH is the total hydraulic head difference between the top and the bottom of the strata, and K_z is the equivalent hydraulic conductivity perpendicular to the layering. Because the flow is perpendicular to the layering, one may also express the volumetric flow rate across each layer as

$$Q_1 = -\frac{K_1\,\Delta H_1\,A}{m_1},\ Q_2 = -\frac{K_2\,\Delta H_2\,A}{m_2},\ \ldots,\ Q_i = -\frac{K_i\,\Delta H_i\,A}{m_i},\ \ldots,\ Q_n = -\frac{K_n\,\Delta H_n\,A}{m_n},$$

where m_i is the thickness of layer i. However, we know that the total hydraulic head difference is equal to the sum of hydraulic head difference of individual layers, or

$$\Delta H_T = \sum_{i=1}^{n} \Delta H_i \qquad (2.2)$$

Also, the volumetric flow rates should all be identical or $Q_T = Q_1 = Q_2 = \ldots = Q_i = \ldots = Q_n$, or

$$\Delta H_1 = -\frac{Q\,m_1}{A\,K_1},\ \Delta H_2 = -\frac{Q\,m_2}{A\,K_2},\ \ldots,\ \Delta H_i = -\frac{Q\,m_i}{A\,K_i},\ \ldots,\ \Delta H_n = -\frac{Q\,m_n}{A\,K_n},\ \text{and}$$

$$\Delta H_T = -\frac{Q\,m_T}{A\,K_z}$$

Substitute these relations into equation (2.2), one has

$$-\frac{Q\,m_T}{A\,K_z} = \sum_{i=1}^{n}\left(-\frac{Q\,m_i}{A\,K_i}\right)$$

Eliminate the common terms on both sides of the above equation and rearrange. One thus has

$$K_z = \frac{m_T}{\sum_{i=1}^{n}\left(-\frac{m_i}{K_i}\right)} = \frac{\sum_{i=1}^{n} m_i}{\sum_{i=1}^{n}\left(-\frac{m_i}{K_i}\right)},$$ or using the notation of the problem statement, we have

$$\text{or } K_z = \frac{\sum_{i=1}^{n} \Delta T_i}{\sum_{i=1}^{n} \left(-\frac{\Delta T_i}{K_i} \right)}$$

Problem 3. Anisotropy

The hydraulic conductivity tensor in the rotated coordinate system may be calculated as follows:

```
> Kx := 10; Kz := 1; theta := 15; pp := 3.14159;
```

$$Kx := 10$$

$$Kz := 1$$

$$\theta := 15$$

$$pp := 3.14159$$

```
> Kzz := 0.5*(Kx + Kz) - 0.5*(Kx - Kz)*cos(2*pp*theta/180);
```

$$Kzz := 1.602884688$$

```
> Kxx := 0.5*(Kx + Kz) + 0.5*(Kx - Kz)*cos(2*theta/180*pp);
```

$$Kxx := 9.397115312$$

```
> Kxz := -0.5*(Kx - Kz)*sin(2*theta/180*pp);
```

$$Kxz := -2.249998276$$

or the hydraulic conductivity tensor may be expressed in the matrix form as follows:

```
> K2 := linalg[matrix](2,2,[Kxx,Kxz,Kxz,Kzz]);
```

$$K2 := \begin{bmatrix} 9.397115312 & -2.249998276 \\ -2.249998276 & 1.602884688 \end{bmatrix}$$

Because the equipotential lines (contour lines) are parallel to the z' axis, there is only gradient along the x' direction. Given the distance between contour lines $\Delta l = 25$ cm, the hydraulic gradient is uniform and can be calculated as follows:

```
> Dl := 25.0; Gxp := 1/Dl; Gzp := 0.0;
```

$$Dl := 25.0$$

$$Gxp := 0.04000000000$$

$$Gzp := 0.$$

The groundwater velocity along the negative x' direction (note the contour increases in the positive x' direction) is thus the product of hydraulic conducitivity and the gradient, or:

```
> qxp := -Kx*Gxp - 0.0*0.0; qzp := -0.0*0.0 - Kz*0;
```

$$qxp := -0.4000000000$$

$$qzp := 0$$

In a more compact form, one may multiply the hydraulic conductivity tensor with the hydraulic gradient vector as follows:

```
> K1 := linalg[matrix](2,2,[Kx,0.0,0.0,Kz]);
```

$$K1 := \begin{bmatrix} 10 & 0. \\ 0. & 1 \end{bmatrix}$$

```
> Gpv := linalg[vector]([Gxp, 0.0]);
```

$$Gpv := [\, 0.04000000000, 0. \,]$$

```
> qp := -linalg[multiply](K1,Gpv);
```

$$qp := -[\, 0.4000000000, 0. \,]$$

The distance between equipotential lines in the rotated (x, z) coordinate system may be calculated as: $\Delta x = \frac{\Delta l}{\cos(\theta)}$ and $\Delta z = \frac{\Delta l}{\sin(\theta)}$ and they are:

```
> Dx := Dl/cos(theta/180*pp); Dz := Dl/sin(theta/180*pp);
```

$$Dx := 25.88190298$$

$$Dz := 96.59266235$$

The hydraulic gradient vector in rotated (x, z) coordinate system is thus:

```
> Gv := linalg[vector]([1.0/Dx, -1.0/Dz]);
```

$$Gv := [\, 0.03863703534, -0.01035275326 \,]$$

With the hydraulic conductivity tensor in the (x, z) coordinate (K2) one may again do a matrix-vector multiplication:

```
> q := -linalg[multiply](K2, Gv);
```

$$q := -[\, 0.3863703534, -0.1035275326 \,]$$

The velocity component projecting back to x' direction is $q_x \cos(\theta) - q_z \sin(\theta)$, or

```
> qxp := round((-0.3863703*cos(theta/180.0*pp) -
  0.1035275*sin(theta/180.0*pp))*10)/10.0;
```

$$qxp := -0.4000000000$$

Similary, the velocity component projecting back to the z' direction is $q_x \sin(\theta) + q_z \cos(\theta)$, or

```
> qzp := round(-0.3863703*sin(theta/180*pp) +
  0.1035275*cos(theta/180*pp));
```

$$qzp := 0$$