Building An Applied and Computational Math Degree Program from the Ground Up

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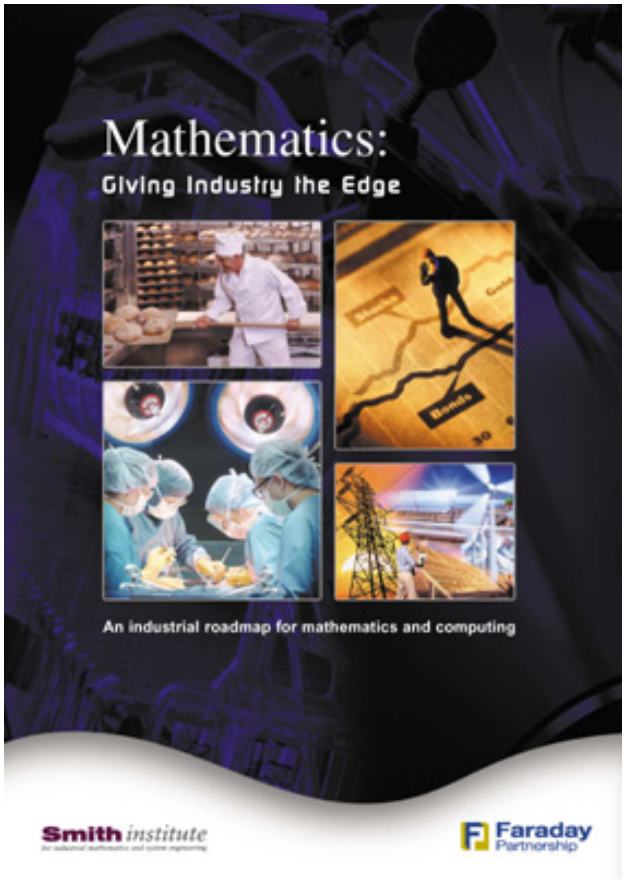
Engage to Excel

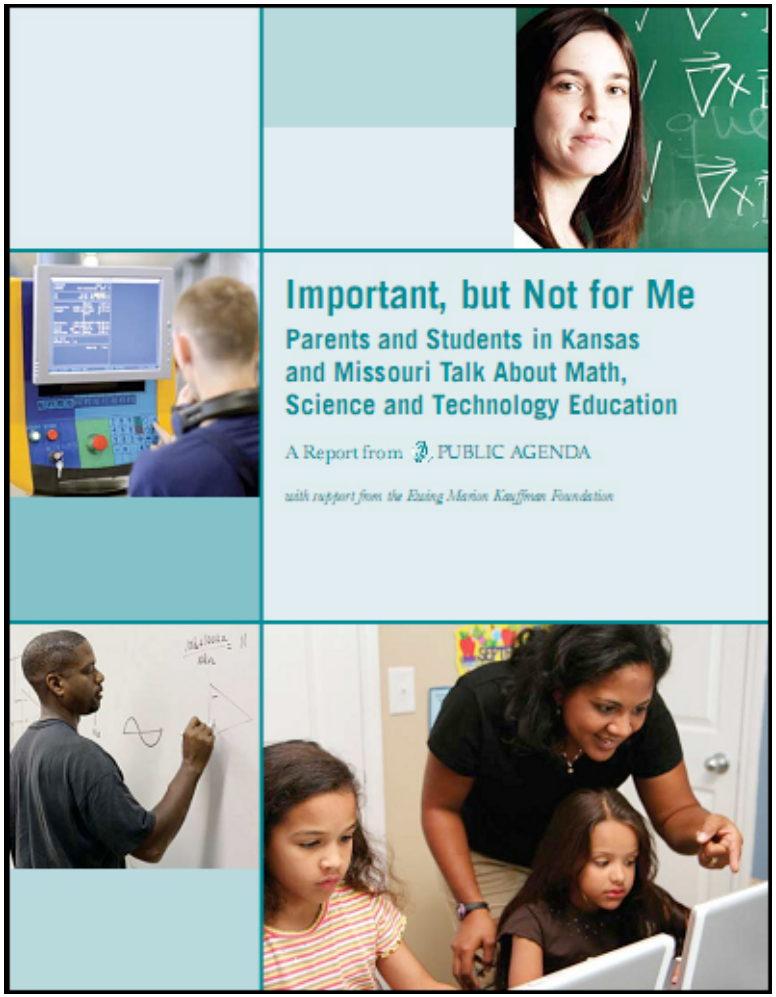
- Whitehouse (PCAST) initiative to produce 1M additional STEM graduates in the next 10 years
- This is a 34% increase
- SMART Grants were discontinued…need to find better ways of attracting and retaining students into STEM fields.
- Math/Science is "important, but not for me"
- Math is a major obstacle for why students avoid STEM
- SIAM Community has an opportunity to lead—applied and computational math will scale better than other disciplines

Problems of the 21st Century
(rethinking the curriculum)

- A need for better modeling and simulation methods and technologies
- A workforce more capable of interdisciplinary design.
- Better tools and understanding for analyzing uncertainty and risk
- Greater capabilities for dealing with large data sets.
- New methods for coping with complex systems.
- Improved capabilities for predicting and understanding market behavior.



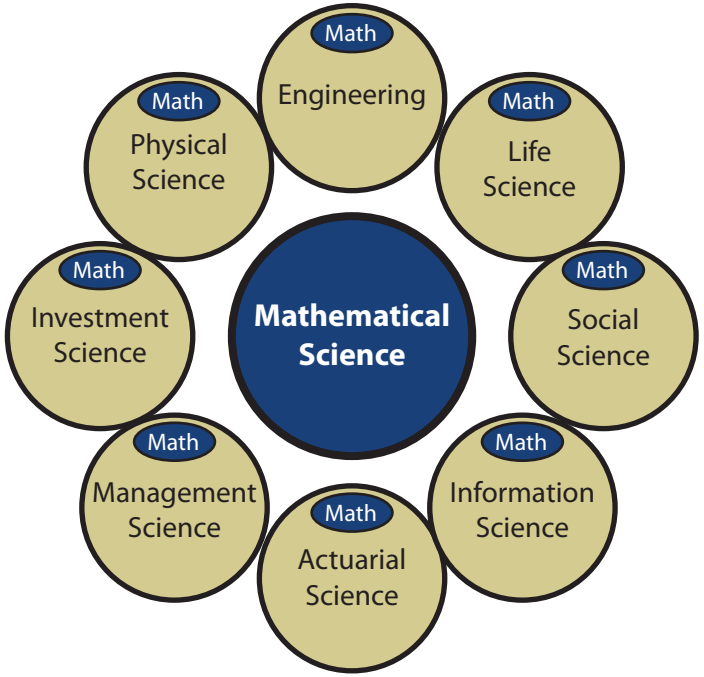
Problem: Lack of Household Participation in Math & Science



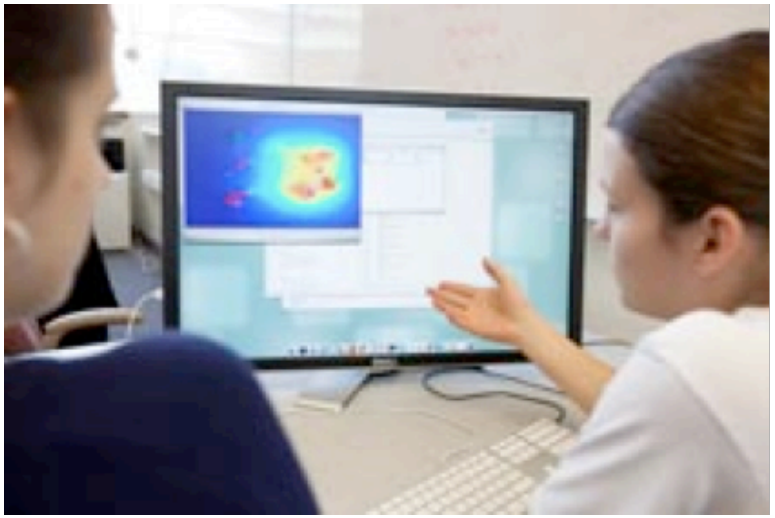
Problem: Mathematicians usually don't play well with others.

Disconnected Disciplines

- When math sciences is disconnected from the scientific community
  - Jargon-laden communities form
  - Disciplines develop their own specialized in-house expertise
  - Reinvention of the wheel
  - Progress suffers



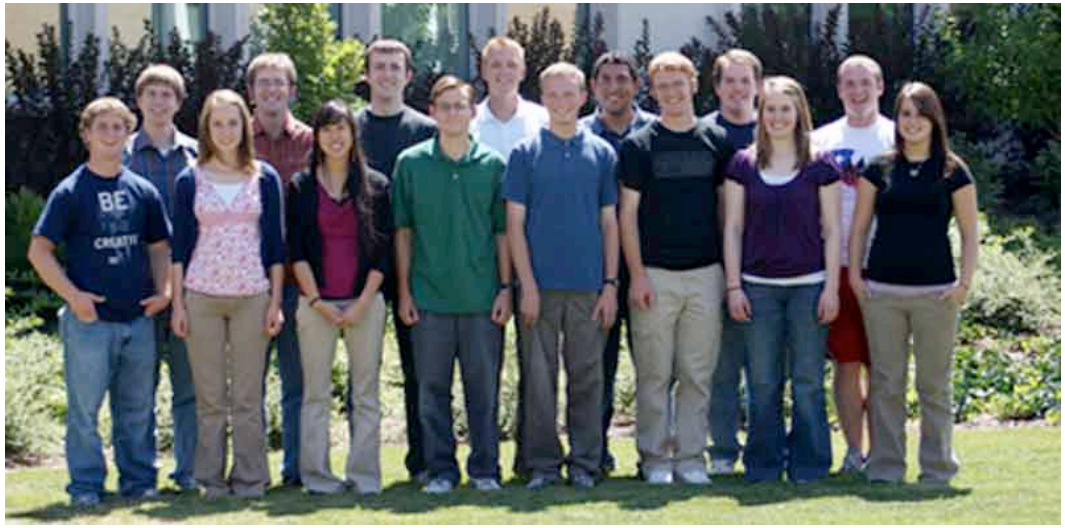
Observation: Need New Models for STEM Research and Education

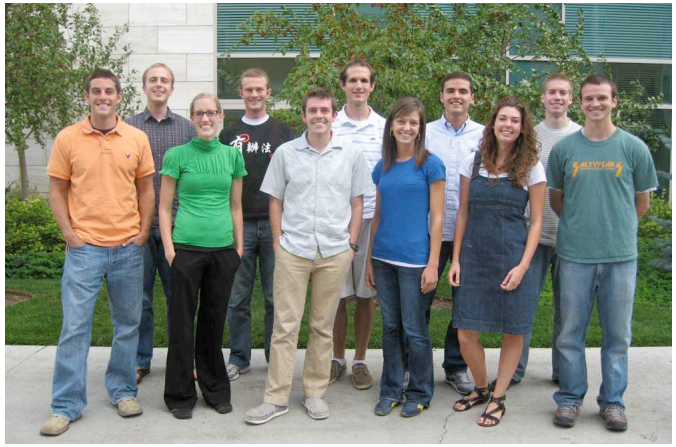


NSF-CSUMS: Computational Science Training for Undergraduates in the Mathematical Sciences

- Seeking new models for research and education in math sciences with integrated strengths in computation.
- Cohort structure of 6 or more students
- Year long research experience
- Roughly 10k/student
- Cohort-level activities, seminars, etc.
- Strengthen research infrastructure, capacity

IMPACT: Interdisciplinary Mentoring Program in Analysis, Computation, & Theory





Program Outline

- Summer Bootcamp (8 weeks)
  - Learn advanced linear algebra, optimization, signal processing, dynamical systems, control theory, statistics…
  - Long days, ask students to put in a minimum of 60 hours/week
  - Computer labs integrate with instruction.
- School Year
  - Break out into research groups
  - Weekly meetings
  - Local MAA conference in March
- Spring Wrap-up
  - Work on finished product

Progress To Date

- Over 55 participants
- Roughly 1/3 from under-represented groups
- Several papers, conference talks, and posters
- Excellent graduate school placements
- 3 NSF Graduate Fellowships
- Great collaborations and friends across campus
- Currently funded by private industry
- Blueprints for a new degree program

Let's Design a new major in Applied and Computational Mathematics

Could also be used for an integrated MS program

Attract and Retain Students into the Mathematical Sciences through:

- Interdisciplinary Research
  - Attracts students with diverse interests into math
  - Connects with faculty in different departments
- Modernized Curriculum
  - Cuts through jargon, eliminates barriers, reduces redundancy
  - Makes mathematics, statistics, and computation the common core in pure and applied science
- Socialization and Team Building
  - Vertically and horizontally integrated research groups
  - Socialization provides a safety net, reduces attrition
- Industrial Cooperation
  - Interesting problems, data, additional funding.
  - Provides opportunities for internships, employment, entrepreneurship

A Few Big Ideas: Accessible to Undergrads

- Orthonormal Sets
- Spectral Theory
- Matrix Decompositions and Special Matrices
- Fixed-Point Theorems
- Convexity
- Central Limit Theorem
- Other?

Orthonormal Sets

- Orthogonal Projections, Least Squares
- Regression, Curve Fitting, Estimation (BLUE/Gauss-Markov)
- Fourier Coefficients
- Fourier Series
- Fourier Transforms
- Gram Schmidt, QR
- Orthogonal Polynomials
- Wavelets

Spectral Theory

- Decomposition of Matrices & Operators
- Spectral Theorem for Self-Adjoint & Normal Matrices
- Jordan Form
- Cayley Hamilton Theorem
- Peron Frobenius Theorem
- Differential Equations
  - Linear Theory
  - Quantum Mechanics
  - Sturm-Liouville

Matrix Decompositions and Special Matrices

- SVD
  - Polar Decompositions
  - Generalized Inverse
  - PCA and friends
  - Low rank approximations
- QR, LU, QZ
- Hessenberg
- Toeplitz
- Hankel

Fixed-Point Theorems

- Contraction Mapping Principle
  - Newton's Method(s)
    - Inverse and Implicit Function Theorems
    - Optimization: BFGS, Interior-Point, etc.
    - Continuation Methods
    - Kalman Filtering, State Estimation
    - Algorithms to Solve Many/Most Inverse Problems
  - Successive Approximations: Existence and Uniqueness of Solutions to Myriad Problems.
  - Small Gain Theorem
- Brouwer's Fixed Point Theorem
  - Perron's Theorem
  - Hartman-Grobman
  - Equilibrium proofs in Game Theory, Economics.

Convexity

- Jensen's Inequality
  - Most inequalities (Young, AGM, Holder, etc.)
  - Information Theory (Gibbs inequality)
  - Probability (Transforms, Expectation)
  - Statistics (Rao-Blackwell)
- Separation Theorems
  - Hahn-Banach Theorem
  - Important in Finance, Economics, etc.
- Convex Optimization
  - Important class of problems
  - Curve fitting, regression, estimation, etc.

Central Limit Theorem(s)

- Obvious Importance in Probability & Statistics
  - The basis of sampling, inference
  - The core of filtering, noise canceling
  - Brownian motion, stochastic processes
- High-Dimensional Algorithms
  - Monte Carlo Methods
  - High-Dimensional Integration
  - Johnson Lindenstrauss Theorem
  - Compressed Sensing

Other "Undergraduate" Ideas

- Bayes Theorem
- Chinese Remainder Theorem
- Interpolation & Splines
- Cauchy Integral Formula
- Maximum Modulus Theorem
- Lie Groups Symmetries
- Fundamental Theorem of Algebra, Liouville's Theorem
- Stability
- Conditioning
- Variation of Constants Formula
- Euler-Lagrange Equation
- Numerical Range (Field of Values)
- Pseudospectra
- Sylvester and Riccati Equations
- Euler Characteristic

Degree Requirements

Pre Core

- Calculus I, II, & III
- ODEs & Linear Algebra
- Intro Proof Class
- Single Variable Real Analysis
- Intro C++

26 Credits

Core

- Adv. Calc + Adv. Matrix Theory (6+2=8 credits)
- Computation & Optimization (6+2=8 credits)
- Probability & Statistics (6+2=8 credits)
- Differential & Integral Equations (6+2=8 credits)

Concentration

12 credits

Growing list of Concentrations

- Biology
- Business Management
- Chemical Engineering
- Chemistry
- Civil Engineering: Geotechnical
- Civil Engineering: Structures and structural mechanics
- Civil Engineering: Transportation
- Civil Engineering: Water Resources and Environmental
- Computer Science
- Economics
- Electrical and Computer Engineering: Circuits
- Electrical and Computer Engineering: Electromagnetics
- Electrical and Computer Engineering: Signals and Systems
- Financial Markets
- Geological Sciences
- Manufacturing Systems Design
- Mathematical Biology
- Mathematical Theory
- Mechanical Engineering: Dynamic Systems
- Mechanical Engineering: Fluids and Thermodynamics
- Physics
- Political Science
- Statistics
- Statistics: Actuarial Science
- Statistics: Biostatistics

Math Science Core: The Foundations of Applied Mathematics

- Linear & Nonlinear Analysis
- Computation & Optimization
- Probability & Statistics
- Differential & Integral Equations

Each core course has a computer lab where students learn computation and applications

Junior Core: Fall Term

Analysis I

- Abstract Vector Spaces
- Linear Transformations
- Inner Product Spaces
- Spectral Theory
- Metric Space Topology
- Differentiation
- Contraction Mappings & Applications
- Convex Analysis

Computation & Optimization I

- Combinatorics & Graphs
- Complexity & Data
- Approximation Theory
- Analysis of Algorithms
- Introduction to Optimization
- Linear Optimization
- Unconstrained Optimization
- Constrained Optimization

Junior Core: Winter Term

Analysis II

- Riemann-Darboux Integration
- Line & Surface Integration
- Complex Integration
- Exterior Algebra & Differential Forms
- Advanced Spectral Theory
- Generalized Inverses
- Perturbation of Linear Operators
- Matrix Groups and Permutations

Computation & Optimization II

- Dyn Opt (Finite Horizon)
- Dyn Opt (Infinite Horizion)
- Dyn Opt (Uncertain Stopping)
- Dyn Opt (Overlapping Generations)
- Discrete Transforms
- Advanced Algorithms
- Advanced Complexity
- Conditioning & Stability

Senior Core: Fall Term

Probability & Statistics I

- Random Spaces and Variables
- Distributions
- Expectation
- Limit Theorems
- Markov Processes
- Poisson, Queuing, Renewal
- Information Theory
- Martingales, Diffusion

Differential and Integral Equations I

- Linear Dynamical Systems
- Nonlinear Dynamical Systems
- Bifurcation Theory
- Control Theory
- Modeling PDE
- Hyperbolic PDE
- Parabolic PDE
- Elliptic PDE

Senior Core: Winter Term

Probability & Statistics II

- Estimation
- Likelihood
- Regression
- Hypothesis Testing
- Multivariate Statistics
- Bayesian Statistics
- State Estimation
- Time Series Analysis & System Identification

Differential and Integral Equations II

- Integral Equations
- Calculus of Variations
- Optimal Control I
- Optimal Control II
- Stochastic Calculus
- Stochastic Differential Equations
- Stochastic Optimal Control
- Applications

Final Talking Points

- Lock-step approach is efficient
  - Cohort approach—students in all the same classes.
  - Students apply to the program (e.g., Junior core, Senior core).
  - Classes would be full as a result.
- Comes at a cost of 2 FTES (8 courses/year), could graduate 40 majors/year.
- More opportunities for funding
- More opportunities for collaboration