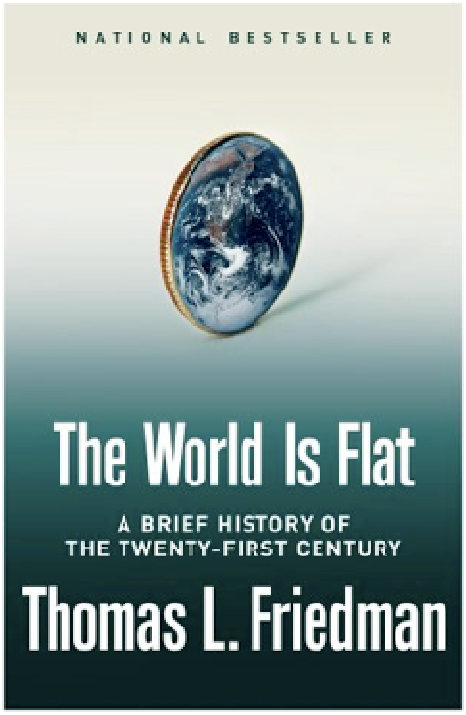
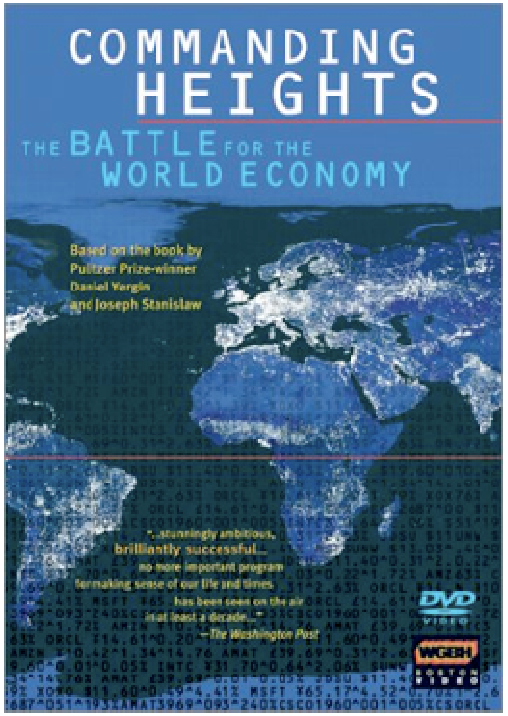
Undergraduate Foundations of Applied Mathematics

Jeffrey Humpherys
Brigham Young University

NSF Grant Nos.
DMS-CSUMS-0639328
CAREER-0847074

Concerns about Globalization and American Competitiveness



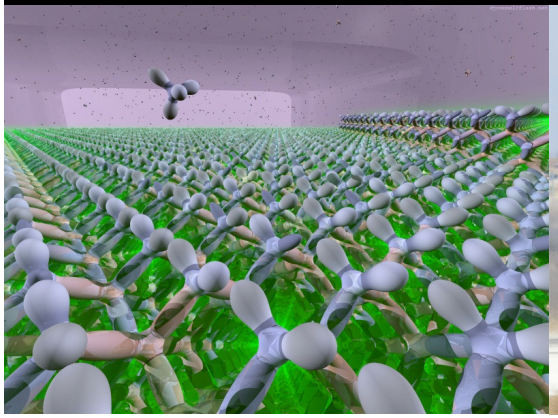


As seen in Manufacturing and Information Technology

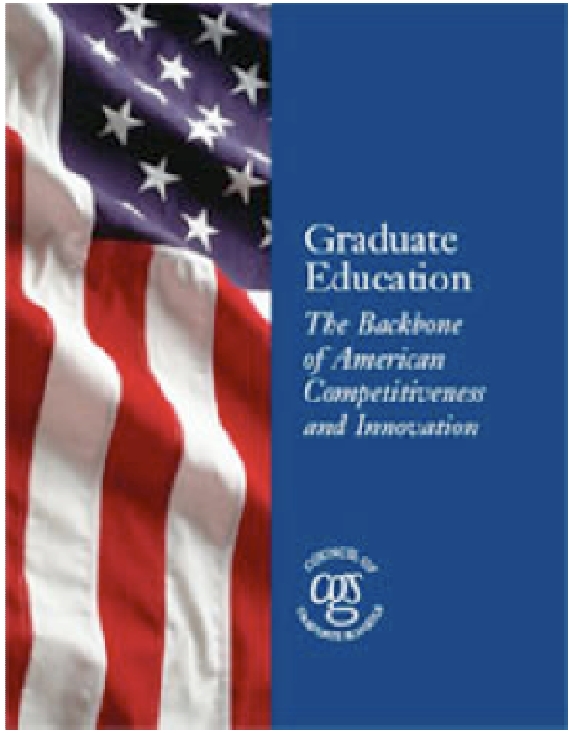
Also in Retail and Financial Markets

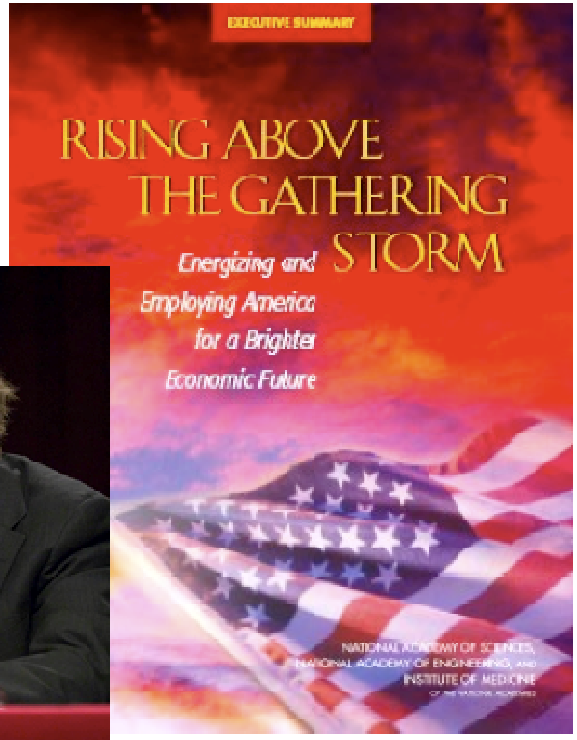
Innovation is Required for Long-Term Economic Growth



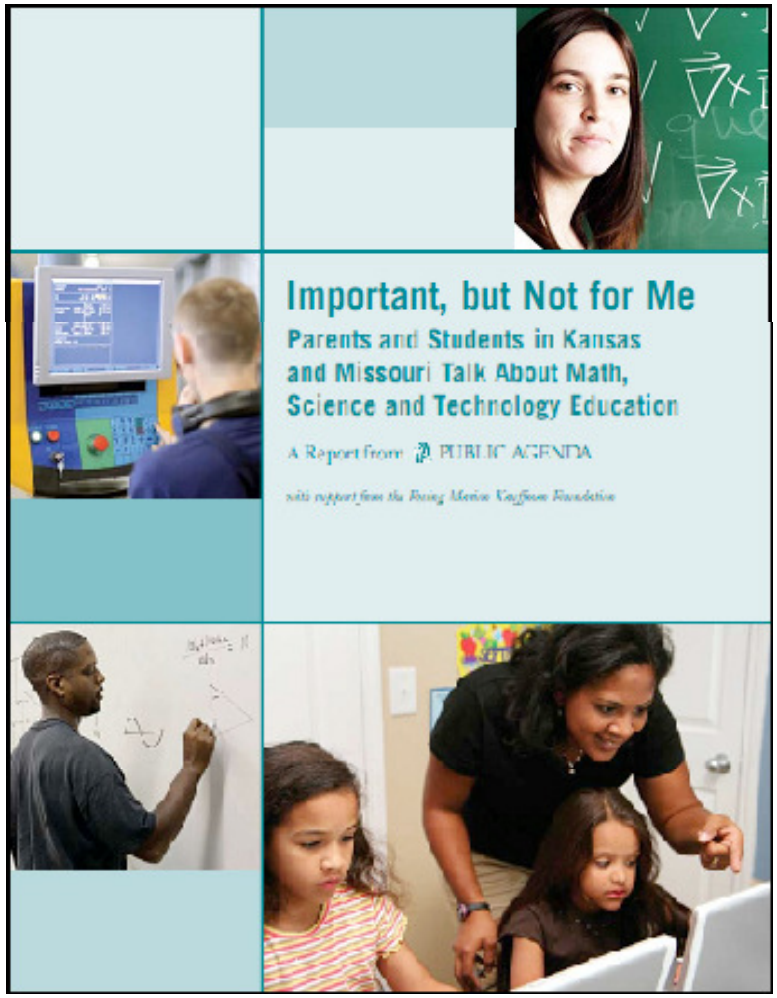


Observation: Innovation Requires a Strong STEM Workforce





Problem: Lack of Household Participation in Math & Science



Proposed Solution: Government Stimulating Growth in STEM Fields

Educate to Innovate

START HERE
GO FURTHER
FEDERAL STUDENT AID

Home | About Us | Contact Us | FAQs | Link to Us | Site Map | Survey

Search
GO
Advanced Search

The National Science & Mathematics Access to Retain Talent Grant (National SMART Grant)

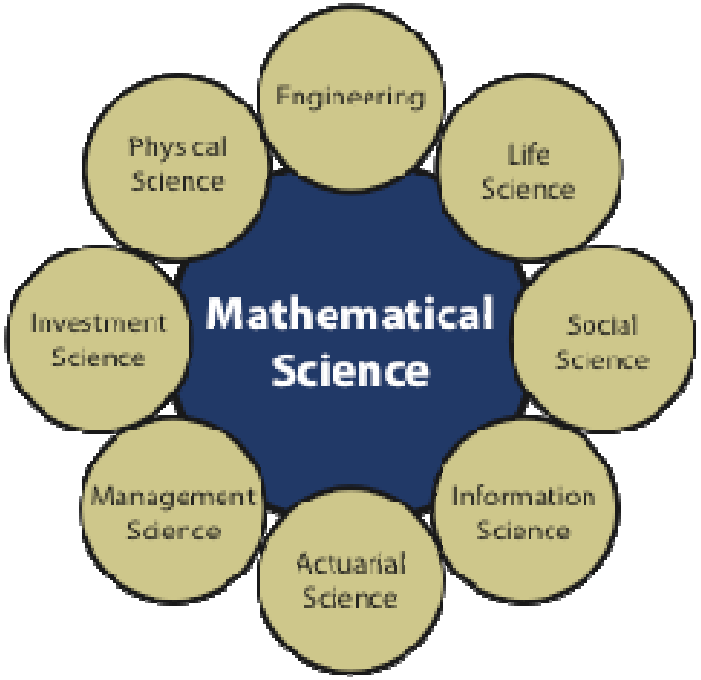
The National Science and Mathematics Access to Retain Talent Grant, also known as the National Smart Grant is available during the third and fourth

Math is the Language of Science

- Odom Report
  - Math sciences is the common language in the pure and applied sciences.
  - Allows for interdisciplinary interaction
  - US's leadership position is fragile
  - Greater investment is needed

- If you want to learn about nature…it is necessary to understand the language that she speaks in.

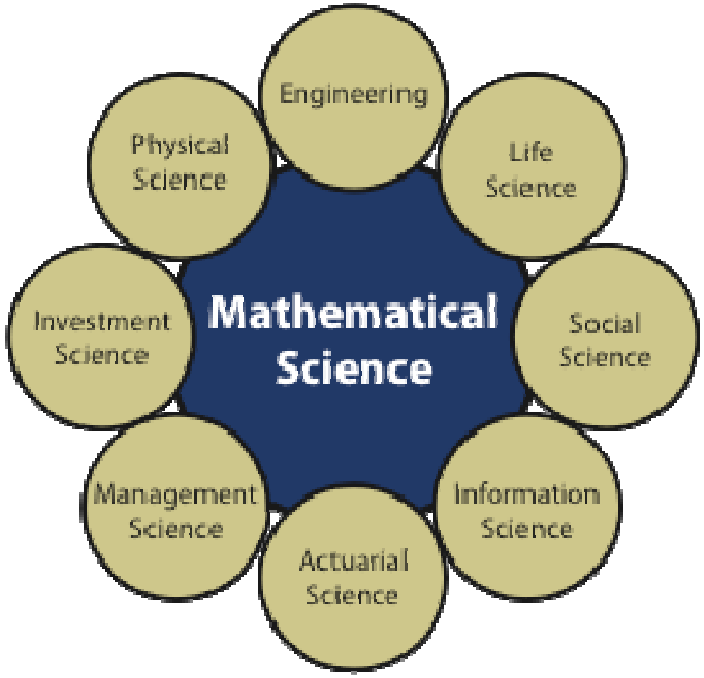
---Richard Feynman



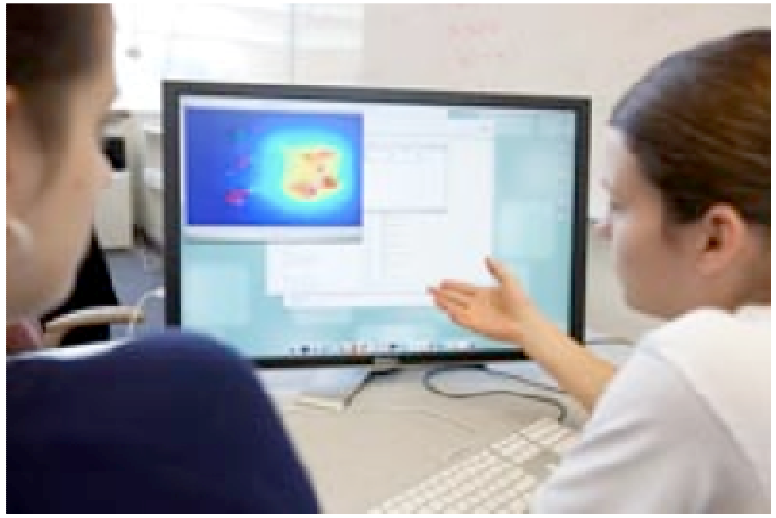
Math is the Language of Science

- Odom Report
  - Math sciences is the common language in the pure and applied sciences.
  - Allows for interdisciplinary interaction
  - US's leadership position is fragile
  - Greater investment is needed
- If you want to learn about nature…it is necessary to understand the language that she speaks in.

---Richard Feynman



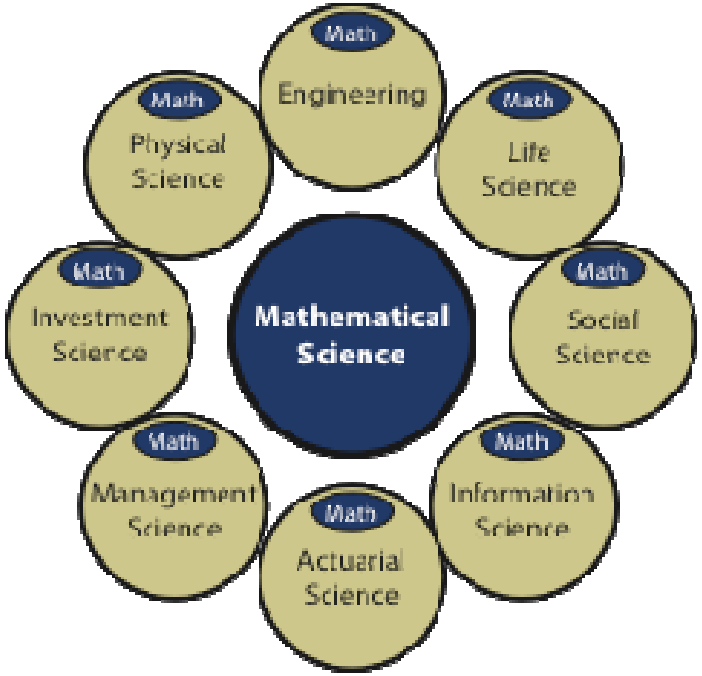
Observation: Need New Models for STEM Research and Education



Problem: Mathematicians usually don't play well with others.

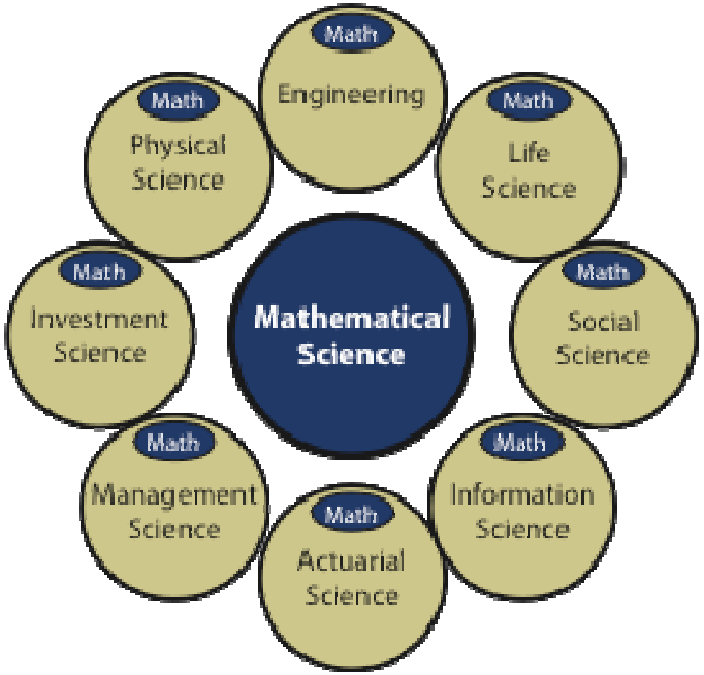
Disconnected Disciplines

- When math sciences is disconnected from the scientific community
- Disciplines develop their own specialized in-house expertise
- Jargon-laden communities form
- Reinvention of the wheel



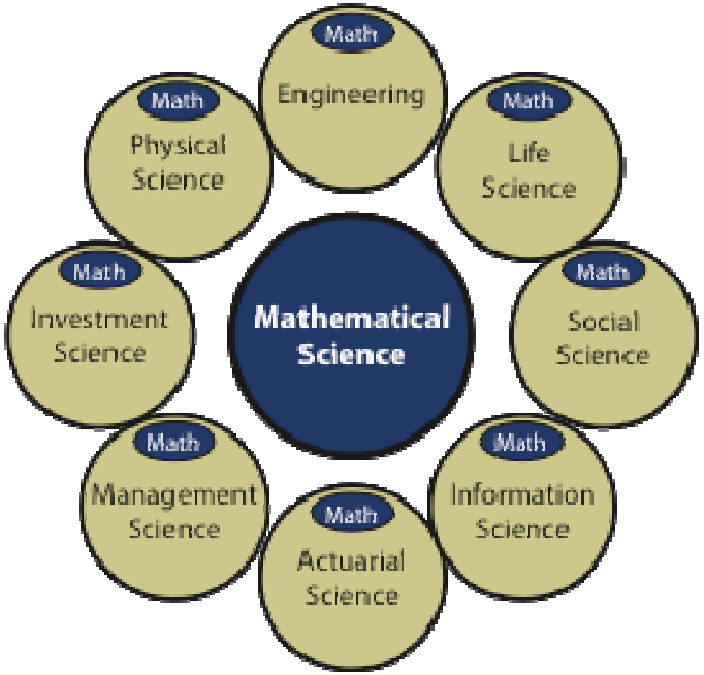
Disconnected Disciplines

- When math sciences is disconnected from the scientific community
- Disciplines develop their own specialized in-house expertise
- Jargon-laden communities form
- Reinvention of the wheel



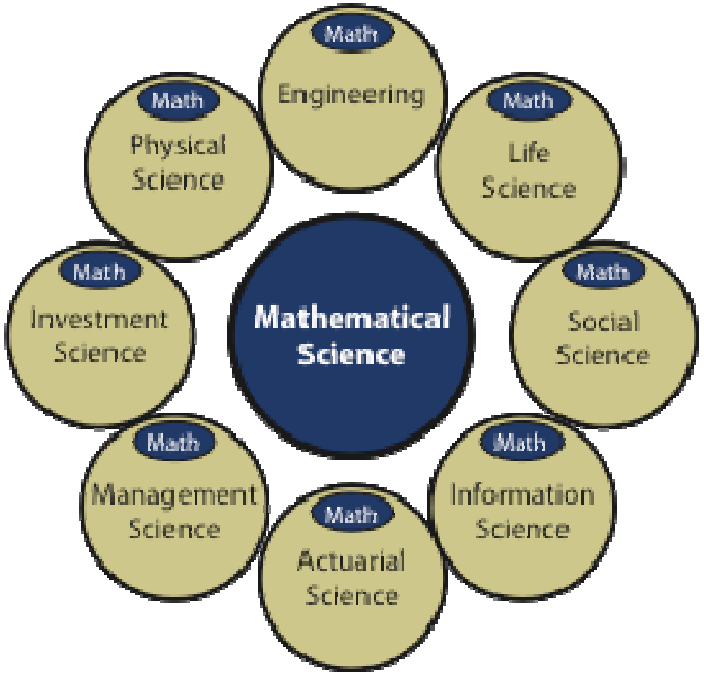
Disconnected Disciplines

- When math sciences is disconnected from the scientific community
- Disciplines develop their own specialized in-house expertise
- Jargon-laden communities form
- Reinvention of the wheel



Disconnected Disciplines

- When math sciences is disconnected from the scientific community
- Disciplines develop their own specialized in-house expertise
- Jargon-laden communities form
- Reinvention of the wheel

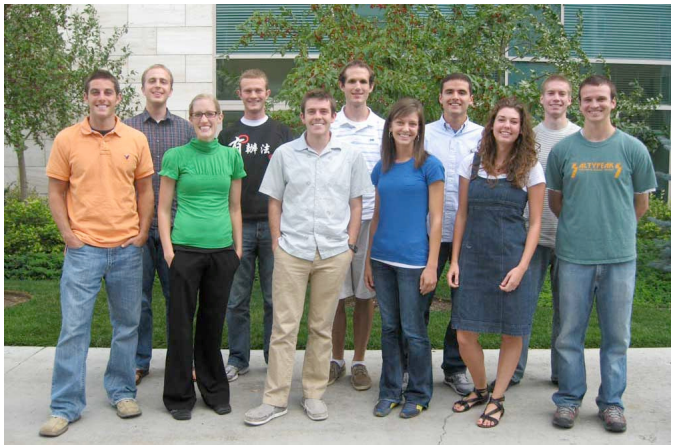


Mathematics needs to ***assert*** its ***leadership*** role in STEM.

NSF-CSUMS: Computational Science Training for Undergraduates in the Mathematical Sciences

- Seeking new models for research and education in math sciences with integrated strengths in computation.
- Cohort structure of 6 or more students
- Year long research experience
- Roughly 10k/student
- Cohort-level activities, seminars, etc.
- Strengthen research infrastructure, capacity

IMPACT: Interdisciplinary Mentoring Program in Analysis, Computation, & Theory



Attract and Retain Students into the Mathematical Sciences through:

- Interdisciplinary Research
  - Attracts students with diverse interests into math
  - Connects with faculty in different departments
- Modernized Curriculum
  - Cuts through jargon, eliminates barriers, reduces redundancy
  - Makes mathematics, statistics, and computation the common core in pure and applied science
- Socialization and Team Building
  - Vertically and horizontally integrated research groups
  - Socialization provides a safety net, reduces attrition
- Industrial Cooperation
  - Interesting problems, data, additional funding.
  - Provides opportunities for internships, employment, entrepreneurship

Attract and Retain Students into the Mathematical Sciences through:

- Interdisciplinary Research
  - Attracts students with diverse interests into math
  - Connects with faculty in different departments
- Modernized Curriculum
  - Cuts through jargon, eliminates barriers, reduces redundancy
  - Makes mathematics, statistics, and computation the common core in pure and applied science
- Socialization and Team Building
  - Vertically and horizontally integrated research groups
  - Socialization provides a safety net, reduces attrition
- Industrial Cooperation
  - Interesting problems, data, additional funding.
  - Provides opportunities for internships, employment, entrepreneurship

Attract and Retain Students into the Mathematical Sciences through:

- Interdisciplinary Research
  - Attracts students with diverse interests into math
  - Connects with faculty in different departments
- Modernized Curriculum
  - Cuts through jargon, eliminates barriers, reduces redundancy
  - Makes mathematics, statistics, and computation the common core in pure and applied science
- Socialization and Team Building
  - Vertically and horizontally integrated research groups
  - Socialization provides a safety net, reduces attrition
- Industrial Cooperation
  - Interesting problems, data, additional funding.
  - Provides opportunities for internships, employment, entrepreneurship

Attract and Retain Students into the Mathematical Sciences through:

- Interdisciplinary Research
  - Attracts students with diverse interests into math
  - Connects with faculty in different departments
- Modernized Curriculum
  - Cuts through jargon, eliminates barriers, reduces redundancy
  - Makes mathematics, statistics, and computation the common core in pure and applied science
- Socialization and Team Building
  - Vertically and horizontally integrated research groups
  - Socialization provides a safety net, reduces attrition
- Industrial Cooperation
  - Interesting problems, data, additional funding.
  - Provides opportunities for internships, employment, entrepreneurship

Program Outline

- Summer Bootcamp (8 weeks)
  - Learn advanced linear algebra, optimization, signal processing, dynamical systems, control theory, statistics…
  - Long days, ask students to put in a minimum of 60 hours/week
  - Computer labs integrate with instruction.
- School Year
  - Break out into research groups
  - Weekly meetings
  - Local MAA conference in March
- Spring Wrap-up
  - Work on finished product

Progress To Date

- 50 student participants (roughly 40 CSUMS, 10 other)
- 17 from under-represented groups (roughly 1/3)
- Several papers, conference talks, and posters
- Excellent graduate school placements
- Increase in Math-Science majors (math department up 50% in 5 years, hard to assign credit)
- 2 marriages, 1 serious relationship…

Lessons Learned

- Investigators are tired…comes at a high cost
- Large-scale undergraduate research is financially unsustainable
- Need a model that integrates research and mentoring into the curriculum
- Need a tightly coordinated curriculum so that we can teach more efficiently
- Need to completely rethink our approach to how we teach computation
- Need more courses in application areas

Lessons Learned

- Investigators are tired…comes at a high cost
- Large-scale undergraduate research is financially unsustainable
- Need a model that integrates research and mentoring into the curriculum
- Need a tightly coordinated curriculum so that we can teach more efficiently
- Need to completely rethink our approach to how we teach computation
- Need more courses in application areas

Lessons Learned

- Investigators are tired…comes at a high cost
- Large-scale undergraduate research is financially unsustainable
- Need a model that integrates research and mentoring into the curriculum
- Need a tightly coordinated curriculum so that we can teach more efficiently
- Need to completely rethink our approach to how we teach computation
- Need more courses in application areas

Lessons Learned

- Investigators are tired…comes at a high cost
- Large-scale undergraduate research is financially unsustainable
- Need a model that integrates research and mentoring into the curriculum
- Need a tightly coordinated curriculum so that we can teach more efficiently
- Need to completely rethink our approach to how we teach computation
- Need more courses in application areas

Lessons Learned

- Investigators are tired…comes at a high cost
- Large-scale undergraduate research is financially unsustainable
- Need a model that integrates research and mentoring into the curriculum
- Need a tightly coordinated curriculum so that we can teach more efficiently
- Need to completely rethink our approach to how we teach computation
- Need more courses in application areas

Lessons Learned

- Investigators are tired…comes at a high cost
- Large-scale undergraduate research is financially unsustainable
- Need a model that integrates research and mentoring into the curriculum
- Need a tightly coordinated curriculum so that we can teach more efficiently
- Need to completely rethink our approach to how we teach computation
- Need more courses in application areas

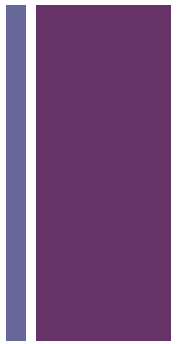
Let's Design an Applied and Computational Mathematics Major

Possibly an integrated MS program

A Few Big Ideas: Accessible to Undergrads

- Orthonormal Sets
- Spectral Theory
- Matrix Decompositions and Special Matrices
- Fixed-Point Theorems
- Convexity
- Central Limit Theorem
- Other?

Orthonormal Sets



- Orthogonal Projections, Least Squares
- Regression, Curve Fitting, Estimation (BLUE)
- Fourier Coefficients
- Fourier Series
- Fourier Transforms
- Gram Schmidt, QR
- Orthogonal Polynomials
- Wavelets

Spectral Theory

- Decomposition of Matrices & Operators
- Spectral Theorem for Self-Adjoint& Normal Matrices
- Jordan Form
- Cayley Hamilton Theorem
- Peron Frobenius Theorem
- Differential Equations
  - Linear Theory
  - Quantum Mechanics
  - Sturm-Liouville,

Matrix Decompositions and Special Matrices

- SVD
  - Polar Decompositions
  - Generalized Inverse
  - PCA and friends
  - Low rank approximations
- QR, LU, QZ
- Hessenberg
- Toeplitz
- Hankel

Fixed-Point Theorems

- Contraction Mapping Principle
  - Newton's Method(s)
    - Inverse and Implicit Function Theorems
    - Optimization: BFGS, Interior-Point, etc.
    - Continuation Methods
    - Kalman Filtering, State Estimation
    - Algorithms to Solve Many/Most Inverse Problems
  - Successive Approximations: Existence and Uniqueness of Solutions to Myriad Problems.
  - Small Gain Theorem
- Brower's Fixed Point Theorem
  - Perron's Theorem
  - Hartman-Grobman
  - Equilibrium proofs in Game Theory, Economics.

Convexity

- Jensen's Inequality
  - Most inequalities (Young, AGM, Holder, etc.)
  - Information Theory (Gibbs inequality)
  - Probability (Transforms, Expectation)
  - Statistics (Rao-Blackwell)
- Separation Theorems
  - Hahn-Banach Theorem
  - Important in Finance, Economics, etc.
- Convex Optimization
  - Important class of problems
  - Curve fitting, regression, estimation, etc.

Central Limit Theorem

- Obvious Importance in Probability & Statistics
  - The basis of sampling, inference
  - The core of filtering, noise canceling
  - Brownian motion, stochastic processes
- High-Dimensional Algorithms
  - Monte Carlo Methods
  - High-Dimensional Integration
  - Johnson Lindenstrauss Theorem
  - Compressed Sensing

Other Important Ideas

- Bayes Theorem
- Chinese Remainder Theorem
- Interpolation &Splines
- Cauchy Integral Formula
- Maximum Modulus Theorem
- Lie Groups & Algebras
- Fundamental Theorem of Algebra, Liouville's Theorem
- Stability
- Conditioning
- Variation of Constants Formula
- Euler-Lagrange Equation
- Numerical Range (Field of Values)
- Sylvester and Riccati Equations
- Euler Characteristic

Proposed New Major: Applied and Computational Mathematics

- Pre-Requisites
  - Math Minor
    - Calculus I & II
    - Multivariable Calculus
    - Introduction to Proof
    - Linear Algebra
    - Ordinary Differential Equations
  - First Semester Analysis
- Math Sciences Core
  - Linear & Nonlinear Analysis
  - Computation & Optimization
  - Probability & Statistics
  - Differential & Integral Equations
- Pick an Area of Specialty (Double Majors or Minors)
  - Actuarial Science
  - Biology/Bioinformatics
  - Business Analytics
  - Chemistry
  - Computer Science, Computer Graphics, Animation
  - Economics
  - Engineering (ME, EE, CiE, ChE)
  - Geo-science
  - Finance
  - Operations Research
  - Physics/Astronomy
  - Statistics/Bio-Statistics

Math Science Core: The Foundations of Applied Mathematics

- Linear&Nonlinear Analysis
- Computation & Optimization
- Probability & Statistics
- Differential & Integral Equations

Each core course has a computer lab where students learn computation and applications

Junior Core: Fall Term

Analysis I

- Abstract Vector Spaces
- Linear Transformations
- Inner Product Spaces
- Spectral Theory
- Metric Space Topology
- Differentiation
- Contraction Mappings & Applications
- Convex Analysis

Computation& Optimization I

- Combinatorics& Graphs
- Complexity & Data
- Approximation Theory
- Analysis of Algorithms
- Introduction to Optimization
- Linear Optimization
- Unconstrained Optimization
- Constrained Optimization

Junior Core: Winter Term

Analysis II

- Riemann-Darboux Integration
- Line & Surface Integration
- Complex Integration
- Exterior Algebra & Differential Forms
- Advanced Spectral Theory
- Generalized Inverses
- Perturbation of Linear Operators
- Matrix Groups and Permutations

Computation& Optimization II

- Dyn Opt (Finite Horizon)
- Dyn Opt (Infinite Horizion)
- Dyn Opt (Uncertain Stopping)
- Dyn Opt (Overlapping Generations)
- Discrete Transforms
- Advanced Algorithms
- Advanced Complexity
- Conditioning & Stability

Senior Core: Fall Term

Probability & Statistics I

- Random Spaces and Variables
- Distributions
- Expectation
- Limit Theorems
- Markov Processes
- Poisson, Queuing, Renewal
- Information Theory
- Martingales, Diffusion

Differential and Integral Equations I

- Linear Dynamical Systems
- Nonlinear Dynamical Systems
- Bifurcation Theory
- Control Theory
- Modeling PDE
- Hyperbolic PDE
- Parabolic PDE
- Elliptic PDE

Senior Core: Winter Term

Probability& Statistics II

- Estimation
- Likelihood
- Regression
- Hypothesis Testing
- Multivariate Statistics
- Bayesian Statistics
- State Estimation
- Time Series Analysis & System Identification

Differential and Integral Equations II

- Integral Equations
- Calculus of Variations
- Optimal Control I
- Optimal Control II
- Stochastic Calculus
- Stochastic Differential Equations
- Stochastic Optimal Control
- Applications

Final Talking Points

- Lock-step approach is efficient
  - Cohort approach—students in all the same classes.
  - Students apply to the program (e.g., Junior core, Senior core).
  - Classes would be full as a result.
- Comes at a cost of 2 FTES (8 courses/year),could graduate 40 majors/year.
- More opportunities for funding
- More opportunities for collaboration