# Contents

1 Introduction 2

2 Background 2

3 High Performance Computing at UMBC with tara 3
  3.1 Basics ................................................. 3
  3.2 IMPORTANT: Setting up your environment to use pbdR ................................................. 3
  3.3 Serial Hello World with R ........................................... 4

4 Getting Started with pbdR MPI 5
  4.1 Parallel Hello World ........................................... 5
  4.1.1 Example: Simple Parallel Hello ........................................... 5
  4.1.2 Example: Report Process Information ........................................... 6
  4.1.3 Example: Global cat command ........................................... 6
  4.2 Point-to-point communication ........................................... 8
  4.2.1 Example: Send Matrices ........................................... 8
  4.2.2 Example: Summing Matrices ........................................... 9
  4.3 Collective communication ........................................... 9
  4.3.1 Example: Summing Matrices with Reduce ........................................... 10
  4.3.2 Example: Scatter / Gather ........................................... 10
  4.4 What other MPI communications are available? ........................................... 13

5 Getting started with Distributed Matrices 13
  5.1 Example: Creating a distributed diagonal matrix ........................................... 13
  5.2 Example: SVD of a distributed matrix ........................................... 15
  5.3 Example: Building a distributed matrix in parallel ........................................... 16

6 Statistics Applications 17
  6.1 Monte Carlo Integration ........................................... 17
  6.2 Reading Datasets ........................................... 19
    6.2.1 Reading on all processes ........................................... 19
    6.2.2 Broadcasting from one process ........................................... 21
    6.2.3 Reading partitioned data ........................................... 21
  6.3 Distributed Regression ........................................... 23
  6.4 Distributed Bootstrap ........................................... 26
1 Introduction

pbdR (2012, "Programming with Big Data in R") is a recent package for high performance computing in R offered by the Remote Data Analysis and Visualization Center (RDAV) at the National Institute for Computational Sciences (NICS). pbdR, like its predecessor Rmpi, presents the appealing possibility of high level Message Passing Interface (MPI) programming in the open source math/statistics environment R. pbdR also supports higher level functionality for distributed dense matrix operations and scalable linear algebra. We have recently installed pbdR to the cluster tara at the High Performance Computing Facility (www.umbc.edu/hpcf). In this tutorial, we will demonstrate running pbdR programs through the scheduler on tara and some of the basic capabilities of the software.

This tutorial will assume knowledge of a typical Windows R user — that is, a basic knowledge of the programming language and being able to issue commands through the GUI. We will go from this to job submission on the cluster tara, to parallel jobs using pbdR.

The tutorial also assumes the reader has access to the cluster tara. However, most of the material is not specific to tara. Job submission is shown through the SLURM scheduler (computing.llnl.gov/linux/slurm), but this is open source software and can be obtained for free. pbdR also requires an MPI installation which can also be obtained freely.

All programs and datasets discussed in this tutorial are available in a companion tarball named pbdRtara2013.tar.gz. To extract the contents, use a command such as:

```bash
[araim10tara ~]$ tar xvzf pbdRtara2013.tar.gz
```

2 Background

I learned about pbdR on a trip to Knoxville, TN in March 2013. Its authors were giving a workshop at Univerity of Tennessee, Knoxville. Later in June 2013 I installed the package to tara. I found the installation to be straightforward, and that the package itself was a very friendly way to write MPI programs in R.

The R Project for Statistical Computing (R Core Team, 2013) is arguably the most popular software package for statistical computing. It is open source, and has a large user community. Users may extend the software by preparing contributed packages. The Comprehensive R Archive Network (CRAN, cran.r-project.org) is a large archive of contributed R packages, and currently contains over 4600 submissions ranging from LASSO regression to MCMC simulation. Programming in R is through a simple and intuitive (in my opinion) high level language, adapted from the S programming language, with rough similarity to Matlab programming.

MPI (Message Passing Interface) is one of the most popular standards for general purpose distributed computing; that is, computing which is split and synchronized over multiple computers. MPI programs are traditionally written in lower level languages like C, C++, or FORTRAN. To write an MPI program, one should think from an SPMD (single program multiple data) perspective. There are some number, say np, of MPI processes we control, which are assigned IDs from 0, . . . , np − 1. Otherwise they are all considered equals and all run the same program. The program is written from the perspective of a single MPI process and tells the process “what is my job?” An MPI program might specify that certain IDs do something different at certain times. The program also tells the processes when to communicate with one another. Many problems in statistics are considered “embarrassingly parallel”, so that minimal communication is needed between processes, and perhaps only at the very end of the computation to produce (say) a sample average. However, many problems which are interesting from a parallel computing viewpoint exist in statistics, and cannot be solved independently in parallel.

A predecessor to pbdR called Rmpi allows MPI programs to be written in R as well. The documentation in Rmpi seemed to promote a master-slave computing perspective, where one process is responsible for controlling the workflow of the others. This seems to go against the spirit of MPI. We found that, on tara, the performance of master-slave computation was inadequate so that a single process Rmpi program was orders of magnitude slower than a serial R program. It is however possible to run Rmpi in SPMD mode. More information about this can be found on www.umbc.edu/hpcf/resources-tara/how-to-run-R.html. The author of Rmpi appears to be involved in the development of pbdR. The documentation for
Another popular R package for parallel computing is called SNOW (Simple Network Of Workstations). SNOW uses a master-slave perspective; the programmer works from the perspective of the master and distributes computations to available workers. SNOW is probably easier to learn than MPI, but is also less expressive. An example of a SNOW command is `parSapply`, which applies a function to each element of a vector, and where all such computations are farmed out to available workers. For more information on using SNOW on tara, see www.umbc.edu/hpcf/resources-tara/how-to-run-R.html. An exhaustive list of R HPC packages, besides the few noted in this tutorial, is available at cran.r-project.org/web/views/HighPerformanceComputing.html.

As mentioned earlier, the documentation for the pbdR project is fairly extensive. For more information, good starting places are:

- rdav.nics.tennessee.edu/2012/09/pbdr
- thirteen-01.stat.iastate.edu/snoweye/pbdr
- r-pbd.org

3 High Performance Computing at UMBC with tara

3.1 Basics

UMBC High Performance Computing Facility (HPCF) is the community-based, interdisciplinary core facility for scientific computing and research on parallel algorithms. See the center’s web page at www.umbc.edu/hpcf for more information, including:

- Current projects
- Publications based on work carried out on the HPCF’s computing clusters
- Obtaining an account
- How to get started on the computing cluster

The current computing cluster tara is an 86 node machine. Each node features two quad core Intel Nehalem X5550 processors (2.66 GHz, 8192 kB cache) and 24 GB of memory. All nodes run the Redhat Enterprise Linux operating system. Attached to the cluster is 160 TB of central storage. For more information, see:

- www.umbc.edu/hpcf/resources-tara/system-description.html

Running programs on an HPC cluster is different than running them on a standard workstation. On a cluster, the user typically works on a “head node”. This is where we develop and compile our program. When the program is ready to be run, we submit it to the set of available compute nodes. We don’t specify directly which compute nodes to use, but instead submit a special script to an entity called the scheduler. In the case of tara, the scheduling software is SLURM. The script specifies important information such as how many compute nodes our job will need, how much time it will need to run, how much memory, and of course the instructions to run the job itself. The scheduler will check whether the requested resources are available, as there may be many other jobs running on the system. If resources are not available, the scheduler will place our job into a queue and allow it to wait. In this tutorial, we will see some basics of interacting with the scheduler. More information is available at:

- www.umbc.edu/hpcf/resources-tara/how-to-run.html
- www.umbc.edu/hpcf/resources-tara/scheduling-policy.html
- www.umbc.edu/hpcf/resources-tara/how-to-run-R.html

3.2 IMPORTANT: Setting up your environment to use pbdR

Before we begin programming, we need to prepare our R environment. See the section “Using contributed libraries” of the web page www.umbc.edu/hpcf/resources-tara/how-to-run-R.html. The pbdR library won’t load without taking these preliminary steps.
3.3 Serial Hello World with R

There are several ways to run R programs. Typical R users (especially when using Windows) will start the R GUI and work interactively. Our goal is to run programs on an HPC cluster which are distributed across multiple compute nodes. Our first step will be to run R as a batch job through the scheduler. We will practice with a Hello World program.

- Although there are GUIs available for R in Linux, typically the R command line is used. Let’s run a program interactively using R command line

```
[araim1@tara-fe1 interactive]$ R
R version x.xx.x (xxxx-xx-xx)
Copyright (C) xxxx The R Foundation for Statistical Computing
...>
> cat("Hello World!\n")
Hello World!
```

- Next we’ll run the program from the Linux command line. We can issue R code, but this gets messy with more than a few commands.

```
[araim1@tara-fe1 batch-cmd]$ Rscript -e 'cat("Hello World!\n")'
Hello World!
[araim1@tara-fe1 batch-cmd]$
```

Or we can run a script.

```
[araim1@tara-fe1 batch-cmd]$ cat driver.R
cat("Hello World!\n")
[araim1@tara-fe1 batch-cmd]$ Rscript driver.R
Hello World!
[araim1@tara-fe1 batch-cmd]$
```

- Now let’s try running our program in batch on a compute node using a submission script

```
#!/bin/bash
#SBATCH --job-name=helloR
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
# Run a command
Rscript -e 'cat("Hello World!\n")'
```

```
#!/bin/bash
#SBATCH --job-name=helloR
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
# Run a script
Rscript driver.R
```

```
[araim1@tara-fe1 batch-scheduler]$ sbatch run1.slurm
Submitted batch job 1327727
[araim1@tara-fe1 batch-scheduler]$ cat slurm.err
[araim1@tara-fe1 batch-scheduler]$ cat slurm.out
Hello World!
[araim1@tara-fe1 batch-scheduler]$
```

```
[araim1@tara-fe1 batch-scheduler]$ sbatch run2.slurm
Submitted batch job 1327728
```

4
4 Getting Started with pbdR MPI

Now that we’re comfortable running batch R jobs through the scheduler, let’s write some pbdR programs.

4.1 Parallel Hello World

4.1.1 Example: Simple Parallel Hello

Let’s start with a simple parallel version of our serial Hello World script

```r
library(pbdMPI, quiet = TRUE)
init()
cat("Hello World!\n")
finalize()
```

Notice that we have now included the pbdMPI library, with some initialization at the beginning and cleanup at the end. The option `quiet = TRUE` prevents some extra output to stderr if dependencies are auto-loaded. In the submission script, we have added the options `--nodes=2` and `--ntasks-per-node=3` so that the code will run on 6 MPI processes across two compute nodes. Notice also that when we invoke `Rscript`, we use `mpirun` to make sure it happens in parallel via the MPI framework.

Not bad, but how do we know that this is really coming from 6 MPI processes in parallel? Let’s modify the program a bit.
4.1.2 Example: Report Process Information

```r
library(pbdMPI, quiet = TRUE)
init()
.comm.size <- comm.size()
.comm.rank <- comm.rank()
.hostname <- Sys.info()["nodename"]
cat("Hello World from ID", .comm.rank, "on host", .hostname, "of", 
 .comm.size, "processes\n")
finalizing()
```

Now we have accessed two very useful MPI variables.

- `.comm.size` is the number of MPI processes available to our program
- `.comm.rank` is the ID of the current MPI process

In addition, we have also requested the name of the machine which is hosting each process. We have modified our `cat` command to print these variables as part of the “hello” greeting. We may use the same `run.slurm` script as above for submission, and will continue to use the same script throughout the tutorial unless otherwise noted.

```
#!/bin/bash
#SBATCH --job-name=pbdRJob
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
#SBATCH --nodes=2
#SBATCH --ntasks-per-node=3
mpirun Rscript driver.R
```

As an alternative to the `cat` function, you may prefer `sprintf` which is useful for formatted printing

```
> msg <- sprintf("Hello from ID %d on host %s of %d processes!\n", .comm.rank, .hostname, .comm.size)
> cat(msg)
```

4.1.3 Example: Global cat command

Let’s try one more variation of parallel Hello World using `pbdR`’s `comm.cat` function. This is a convenient way to write out text from our group of MPI processes.
library(pbdMPI, quiet = TRUE)
init()
.comm.size <- comm.size()
.comm.rank <- comm.rank()
.hostname <- Sys.info()$"nodename"

# Comm cat messages
msg <- sprintf("Hello from ID %d on host %s of %d processes!\n", 
                .comm.rank, .hostname, .comm.size)

# Comm cat with all ranks
comm.cat(msg, all.rank = TRUE)

# Comm cat with rank 0
comm.cat(msg, rank.print = 0)

# Comm cat with rank 1
comm.cat(msg, rank.print = 1)

finalize()

Notice that we can control which process prints in `comm.cat`; we can choose a rank or ask all ranks to print.

```
[arain1@tara-fe1 comm-cat]$ sbatch run.slurm
Submitted batch job 1327730
[arain1@tara-fe1 comm-cat]$ cat slurm.err
COMM.RANK = 0
About to say hello from all processes... COMM.RANK = 0
Hello from ID 0 on host n2 of 6 processes!
COMM.RANK = 0
Hello from ID 1 on host n2 of 6 processes!
COMM.RANK = 1
Hello from ID 2 on host n2 of 6 processes!
COMM.RANK = 2
Hello from ID 3 on host n3 of 6 processes!
COMM.RANK = 3
Hello from ID 4 on host n3 of 6 processes!
COMM.RANK = 4
Hello from ID 5 on host n3 of 6 processes!
COMM.RANK = 5

[arain1@tara-fe1 comm-cat]$ cat slurm.out
COMM.RANK = 0
About to say hello from some specific processes...
COMM.RANK = 0
Hello from ID 0 on host n2 of 6 processes!
COMM.RANK = 1
Hello from ID 1 on host n2 of 6 processes!
COMM.RANK = 1
Hello from ID 1 on host n2 of 6 processes!
COMM.RANK = 1
Hello from ID 0 on host n2 of 6 processes!
```

Having the `COMM.RANK` heading can be useful, but here it is redundant. We can get rid of it with the option `quiet = TRUE`

```
> comm.cat(msg, rank.print = 0, quiet = TRUE)
Hello from ID 0 of 6 processes!
>
```
4.2 Point-to-point communication

Having a bunch of MPI processes running independently is okay, but they’ll need to communicate to do anything really useful. Here we’ll demonstrate point-to-point communication, where processes send messages directly to each other.

4.2.1 Example: Send Matrices

```r
library(pbdMPI, quiet = TRUE)
init()
.comm.size <- comm.size()
.comm.rank <- comm.rank()

A <- matrix(.comm.rank, 3, 3)

if(.comm.rank == 0) {
  cat("At ID 0, my matrix is:
")
  print(A)

  for (id in seq(1, .comm.size-1))
    y <- recv(rank.source = id)
    cat("At ID 0, recv'ed matrix from ID", id, ":
")
    print(y)
} else {
  send(A, rank.dest = 0)
}
finalize()
```

```bash
#!/bin/bash
#SBATCH --job-name=pbdRJob
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
#SBATCH --nodes=1
#SBATCH --ntasks-per-node=4

mpirun Rscript driver.R
```

In this program, each MPI process creates a $3 \times 3$ matrix $A$ containing its process ID repeated 9 times. Each process sends its copy of $A$ to process 0, who receives them and prints them in order (by process ID). Notice that the `send` and `recv` commands are used, and an if statement controls the program flow: process 0 does one thing, and all other processes do something else.

Readers familiar with MPI may notice that the `send` and `recv` commands have very simple argument lists. We don’t need to specify the number of elements in $A$ being sent, or their type. This is handled by pbdR. Some options like the MPI communicator have been set to reasonable defaults (i.e. `MPI_COMM_WORLD`) but may be changed by the user.

```
[araim0taras-fel send-matrices]$ sbatch run.slurm
Submitted batch job 1327736
[araim0taras-fel send-matrices]$ cat slurm.err
[araim0taras-fel send-matrices]$ cat slurm.out
At ID 0, my matrix is:
[,1] [,2] [,3]
[1,] 0 0 0
[2,] 0 0 0
[3,] 0 0 0
At ID 0, recv'ed matrix from ID 1 :
[,1] [,2] [,3]
[1,] 1 1 1
[2,] 1 1 1
[3,] 1 1 1
At ID 0, recv'ed matrix from ID 2 :
[,1] [,2] [,3]
```

8
4.2.2 Example: Summing Matrices

We can easily extend the previous example to compute the sum of the local matrices

```r
library(pbdMPI, quiet = TRUE)
init()
.comm.size <- comm.size()
.comm.rank <- comm.rank()
A <- matrix(.comm.rank, 3, 3)
if(.comm.rank == 0) {
  X <- A
  for (id in seq(1, .comm.size-1))
    y <- recv(rank.source = id)
    X <- X + y
} else {
  send(A, rank.dest = 0)
}
comm.cat("Sum of all matrices X =\n", quiet = TRUE)
comm.print(X, quiet = TRUE)
finalize()
```

```
#SBATCH --job-name=pbdRJob
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
#SBATCH --nodes=1
#SBATCH --ntasks-per-node=4

mpirun Rscript driver.R
```

```
Sum of all matrices X =
[,1] [,2] [,3]
[1,] 6  6  6
[2,] 6  6  6
[3,] 6  6  6
```

4.3 Collective communication

Collective MPI communications are more convenient than point-to-point for many operations. We will demonstrate a few.
4.3.1 Example: Summing Matrices with Reduce

Here is a more convenient version of the example in section 4.2:

```r
library(pbdMPI, quiet = TRUE)
init()
A <- matrix(.comm.rank, 3, 3)
X <- reduce(A, op = "sum")
comm.print(A, all.rank = TRUE)
comm.print(X)
finalize()
```

```bash
#!/bin/bash
#SBATCH --job-name=pbdRJob
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
#SBATCH --nodes=1
#SBATCH --ntasks-per-node=4
mpirun Rscript driver.R
```

4.3.2 Example: Scatter / Gather

In the following example, we’ll compute an iid sample of 1000 observations from \(N(\mu, \sigma^2)\) for each \(\mu \in \{-10, -9.5, \ldots, 9.5, 10\}\), and return the mean of each sample. To do this, we’ll split the vector \((-10, -9.5, \ldots, 9.5, 10)\) into a list of vectors, so that the first vector will be processed by ID 0, the second vector will be processed by ID 1, and so forth. Here is the splitting code in serial, assuming that .comm.size has been set to 8.
> all.mu.levels <- seq(-10, 10, 0.5)
> idx <- 1:length(all.mu.levels) %% .comm.size
> x <- split(all.mu.levels, idx)
> print(idx)
[1] 1 2 3 4 5 6 7 0 1 2 3 4 5 6 7 0 1 2 3 4 5 6
[39] 7 0 1
> print(x)
$'0'
[1] -6.5 -2.5 1.5 5.5 9.5
$'1'
[1] -10 -6 -2 2 6 10
$'2'
[1] -9.5 -5.5 -1.5 2.5 6.5
$'3'
[1] -9 -5 -1 3 7
$'4'
[1] -8.5 -4.5 -0.5 3.5 7.5
$'5'
[1] -8 -4 0 4 8
$'6'
[1] -7.5 -3.5 0.5 4.5 8.5
$'7'
[1] -7 -3 1 5 9
>
Notice that the $j$th element of all.mu.levels is assigned to the process with ID $\equiv j \mod 8$. From here, we can use the \texttt{scatter} command so the vectors will be distributed as desired.

```r
library(pbdMPI, quiet = TRUE)
init()
.comm.size <- comm.size()
.comm.rank <- comm.rank()

all.mu.levels <- seq(-10, 10, 0.5)
idx <- 1:length(all.mu.levels) %% .comm.size
x <- split(all.mu.levels, idx)
mu.levels <- scatter(split(all.mu.levels, idx))

comm.cat("\nAfter scatter, mu.levels:\n", quiet = TRUE)
comm.print(mu.levels, all.rank = TRUE)

m <- numeric( length(mu.levels) )
for (i in 1:length(mu.levels))
{
  mu <- mu.levels[i]
  x <- rnorm(n = 1000, mean = mu, sd = 1)
  m[i] <- mean(x)
}

means <- gather(m, rank.dest = 0)

comm.cat("\nOur result on process 0:\n", quiet = TRUE)
comm.print(means, quiet = TRUE)
comm.print(unlist(means), quiet = TRUE)

finalize()
```

---

File: code/collective-comm/gather-vector/driver.R

---
#!/bin/bash
#SBATCH --job-name=pbdRJob
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
#SBATCH --nodes=1
#SBATCH --ntasks-per-node=8

mpirun Rscript driver.R

[araim1@tara-fe1 gather-vector]$ sbatch run.slurm
Submitted batch job 46
[araim1@tara-fe1 gather-vector]$ cat slurm.err
[araim1@tara-fe1 gather-vector]$ cat slurm.out

After scatter, mu.levels:
COMM.RANK = 0
[[1]] -6.5 -2.5 1.5 5.5 9.5
COMM.RANK = 1
[[1]] -10 -6 -2 2 6
COMM.RANK = 2
[[1]] -9.5 -5.5 -1.5 2.5 6.5
COMM.RANK = 3
[[1]] -9 -5 -1 3 7
COMM.RANK = 4
[[1]] -8.5 -4.5 -0.5 3.5 7.5
COMM.RANK = 5
[[1]] -8 -4 0 4 8
COMM.RANK = 6
[[1]] -7.5 -3.5 0.5 4.5 8.5
COMM.RANK = 7
[[1]] -7 -3 1 5 9

Our result on process 0:
[[1]]
[[2]]
[[3]]
[[4]]
[[5]]
[[6]]
[[7]]
[[8]]

Our result on process 0 after unlist:
[6] -10.0265972 -5.985492 -1.971049 1.990389 5.960975
[16] 6.4903893 -0.02659720 -4.98549176 -0.97104886 2.99038933
[26] 7.46097501 -8.02659720 -3.98549176 0.02895114 3.49038933
[31] 7.96097501 -7.52659720 -3.48549176 0.52895114 4.49038933
[41] 8.96097501
4.4 What other MPI communications are available?

pbdR does not implement the entire MPI specification, therefore some commands that are available in C and FORTRAN MPI implementations may not be available. However, many MPI commands and options are available in pbdR that we have not covered. For more information, see:

- pbdMPI reference manual at: cran.r-project.org/web/packages/pbdMPI
- Full MPI specification: www.mpi-forum.org/docs
- Quick list of all MPI commands: www.mcs.anl.gov/research/projects/mpi/www

5 Getting started with Distributed Matrices

pbdR supports higher level programming for distributed dense matrices using the sub-package pbdDMAT. A matrix is distributed among a group of processes using a block-cyclical distribution as in ScaLAPACK. The details will not be discussed here, but the reader can refer to acts.nersc.gov/scalapack/hands-on/datadist.html and the package vignette for pbdDMAT at cran.r-project.org/web/packages/pbdDMAT.

5.1 Example: Creating a distributed diagonal matrix

```r
library(pbdDMAT, quiet = TRUE)
init.grid()
Y <- diag(1:6, type = "ddmatrix")
comm.cat("class of Y on each process:
", quiet = TRUE)
comm.print(class(Y), all.rank = TRUE)
comm.cat("print:
", quiet = TRUE)
print(Y)
comm.cat("print with all = TRUE:
", quiet = TRUE)
print(Y, all = TRUE)
X <- as.matrix(Y, proc.dest = 0)
comm.cat("Converted to regular matrix:
", quiet = TRUE)
comm.print(X, proc.rank = 0, quiet = TRUE)
finalize()
```

```bash
#!/bin/bash
#SBATCH --job-name=pbdRJob
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
#SBATCH --nodes=1
#SBATCH --ntasks-per-node=3
mpirun Rscript driver.R
```

Submitted batch job 61

Using 1x3 for the default grid size

class of Y on each process:
COMM_RANK = 0
[1] "ddmatrix"
attr(,"package")
DENSE DISTRIBUTED MATRIX

@Data: 1e+00, 0e+00, 0e+00, 0e+00, ...
Process grid: 1x3
Global dimension: 6x6
(max) Local dimension: 6x4
Blocking: 4x4
BLACS ICTXT:
0

print with all = TRUE:

dx@Data[ 1, 1]= 1.000000000000000000
dx@Data[ 2, 1]= 0.000000000000000000
dx@Data[ 3, 1]= 0.000000000000000000
dx@Data[ 4, 1]= 0.000000000000000000
dx@Data[ 5, 1]= 0.000000000000000000
dx@Data[ 6, 1]= 0.000000000000000000
dx@Data[ 1, 2]= 0.000000000000000000
dx@Data[ 2, 2]= 2.000000000000000000
dx@Data[ 3, 2]= 0.000000000000000000
...
dx@Data[ 5, 6]= 0.000000000000000000
dx@Data[ 6, 6]= 6.000000000000000000

Converted to regular matrix:

[1,] 1 0 0 0 0 0
[2,] 0 2 0 0 0 0
[3,] 0 0 3 0 0 0
[4,] 0 0 0 4 0 0
[5,] 0 0 0 0 5 0
[6,] 0 0 0 0 0 6
[arain10tara-fe1 diag]$
5.2 Example: SVD of a distributed matrix

```r
library(pbdDMAT, quiet = TRUE)
init.grid()

X <- ddmatrix(data=0, nrow=6, ncol=6)
for (i in 1:6) {
  X[i,i] <- 4
}
for (i in 1:5) {
  X[i,i+1] <- 1
  X[i+1,i] <- 1
}
X.local <- as.matrix(X, proc.dest = 0)
comm.cat("X.local:
", quiet = TRUE)
comm.print(X.local, proc.rank = 0, quiet = TRUE)

S <- svd(X)
comm.cat("SVD, d =
", quiet = TRUE)
comm.print(S$d, proc.rank = 0, quiet = TRUE)

U.local <- as.matrix(S$u, proc.dest = 0)
comm.cat("SVD, u =
", quiet = TRUE)
comm.print(U.local, proc.dest = 0, quiet = TRUE)

V.local <- as.matrix(S$v, proc.dest = 0)
comm.cat("SVD, v =
", quiet = TRUE)
comm.print(V.local, proc.dest = 0, quiet = TRUE)

D <- as.ddmatrix(diag(S$d, nrow=6, ncol=6))
Y <- S$u %*% D %*% t(S$v)
Y.local <- round( as.matrix(Y, proc.dest = 0), 4)
comm.cat("Y.local:
", quiet = TRUE)
comm.print(Y.local, proc.rank = 0, quiet = TRUE)

finalize()
```

```bash
#!/bin/bash
#SBATCH --job-name=pbdRJob
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
#SBATCH --nodes=1
#SBATCH --ntasks-per-node=3

mpirun Rscript driver.R
```

```
[araim1@tara-fe1 svd]$ sbatch run.slurm
Submitted batch job 73
[araim1@tara-fe1 svd]$ less slurm.out
[araim1@tara-fe1 svd]$ cat slurm.err
[araim1@tara-fe1 svd]$ cat slurm.out
Using 1x3 for the default grid size

X.local:
[1,] 4 1 0 0 0 0
[2,] 1 4 1 0 0 0
[3,] 0 1 4 1 0 0
[4,] 0 0 1 4 1 0

15
```
5.3 Example: Building a distributed matrix in parallel

For a big data application such as analysis of a large distributed database, the data may not fit on a single process. Then a natural way to build a distributed matrix will be to have each process contribute their local piece. Again, more detail is given in the package vignette for pbdDMAT at cran.r-project.org/web/packages/pbdDMAT.

```R
library(pbdDMAT, quiet=TRUE)
init.grid()
.grid.rank <- comm.rank()
.grid.size <- comm.size()

eclntxt <- minctxt()
init.grid(NPROW = 1, NPCOL = .comm.size, ICTXT = newctxt)
.grid <- blacs(ICTXT = newctxt)
.grid.row <- grid$MYROW
.grid.col <- grid$MYCOL

msg <- sprintf("My row is %d and my col is %d\n", .grid.row, .grid.col)
comm.cat(msg, all.rank = TRUE)

x <- matrix(.grid.col, 2, 3)
comm.print(x, all.rank = TRUE)

A <- ddmatrix(data = x, nrow = 2, ncol = 9, bldim = c(2,3), ICTXT = newctxt)
print(A)
global.A <- as.matrix(A)
comm.print(global.A)

finalize()
```
We’ve constructed a $1 \times \text{np}$ grid of processes, corresponding to a matrix with $1 \times \text{np}$ blocks. Some special syntax is needed to create a grid with this configuration (indexed internally by `newictxt`), and to retrieve information on the grid we just created. Each process retrieves its row and column position in the grid, and subsequently creates the matrix that will be placed in that position. The distributed matrix is created by the `ddmatrix` function. There are some small complications when not all blocks are the same size; this case is shown in section 6.3.

```plaintext
[araim1@tara-fe1 build-parallel]$ sbatch run.slurm
Submitted batch job 1329212
[araim1@tara-fe1 build-parallel]$ cat slurm.err
Using 1x3 for the default grid size
Grid ICTXT=3 of size 1x3 successfully created
COMM.RANK = 0
My row is 0 and my col is 0
COMM.RANK = 1
My row is 0 and my col is 1
COMM.RANK = 2
My row is 0 and my col is 2
COMM.RANK = 0
[,1] [,2] [,3]
[1,] 0 0 0
[2,] 0 0 0
COMM.RANK = 1
[,1] [,2] [,3]
[1,] 1 1 1
[2,] 1 1 1
COMM.RANK = 2
[,1] [,2] [,3]
[1,] 2 2 2
[2,] 2 2 2

DENSE DISTRIBUTED MATRIX
-------------------------------
@Data: 0e+00, 0e+00, 0e+00, 0e+00, ...
Process grid: 1x3
Global dimension: 2x9
(max) Local dimension: 2x3
Blocking: 2x3
BLACS ICTXT: 3
COMM.RANK = 0
[1,] 0 0 0 1 1 1 2 2 2
[2,] 0 0 0 1 1 1 2 2 2
```

6 Statistics Applications

6.1 Monte Carlo Integration

Consider the probability simplex

$$S^J = \left\{ p \in [0,1]^J : \sum_{j=1}^{J} p_j = 1 \right\},$$
and let \( p = (p_1, \ldots, p_J) \) follow a Dirichlet\(_J(\alpha)\) distribution on the simplex. Suppose we would like to compute \( E(p_1 \cdots p_J) \). A simple monte carlo estimator for this quantity is

\[
\frac{1}{R} \sum_{r=1}^{R} \{ p_1^{(r)} \cdots p_J^{(r)} \}, \quad \text{where } p^{(1)}(1), \ldots, p^{(R)}(J) \text{ iid } \sim \text{Dirichlet}_J(\alpha).
\]

The following \texttt{pbdR} codes compute the monte carlo estimator in serial and parallel with \texttt{pbdR} using \( R = 2^{22} \) repetitions. The parallel program splits the \( R \) repetitions among \( np \) available processes. We have set \( \alpha = (1, \ldots, 1) \). Table 1 shows a small timing study carried out with this program on tara. We can see that varying \( J \) from 2 to 10 does not have a large impact on the run time, but for a fixed \( J \), the serial code performs similarly to the \texttt{pbdR} code with one process, and that doubling \( np \) halves the run time. This near-ideal behavior is expected given the embarassingly parallel nature of this problem.

Note that before running this program, you may need to install the \texttt{MCMCpack} library to your account. This can be done by running the following command.

\[
\texttt{> install.packages("MCMCpack")}
\]

---

File: code/monte-carlo/serial.R

```r
library(MCMCpack)

start.time <- Sys.time()

# Read J and R from the command line, e.g.
# Rscript driver.R 2 1048576
cmd <- commandArgs(TRUE)
J <- as.integer(cmd[1])
R <- as.integer(cmd[2])

alpha <- rep(1,J)

n <- R

# Don’t sample them all at once, or we might run out of memory
r <- 0
while (r < n)
{
    X <- rdirichlet( min(1000, n-r), alpha)
    total <- total + sum( apply(X, 1, prod) )
    r <- r + nrow(X)
}

prob.mc <- total / R

cat("Monte carlo probability:\n")
print(prob.mc, digits = 20)

elapsed <- as.numeric(Sys.time() - start.time, units = "secs")
cat("Elapsed secs: ", elapsed, "\n")
```

---

18
library(pbdMPI, quiet = TRUE)
library(MCMCpack)
init()

start.time <- Sys.time()

.comm.size <- comm.size()
.comm.rank <- comm.rank()
.hostname <- Sys.info()

msg <- sprintf("Process ID %d out of %d processes running on %s\n",
  .comm.rank, .comm.size, .hostname)
comm.cat(msg, all.rank = TRUE, quiet = TRUE)

# Read J and R from the command line, e.g.
# Rscript driver.R 2 1048576
cmd <- commandArgs(TRUE)
J <- as.integer(cmd[1])
R <- as.integer(cmd[2])

alpha <- rep(1, J)

n <- R / .comm.size

# Don’t sample them all at once, or we might run out of memory
r <- 0

while (r < n)
{
  X <- rdirichlet(min(1000, n-r), alpha)
  total <- total + sum(apply(X, 1, prod))
  r <- r + nrow(X)
}

prob.global <- reduce(total / R, op = "sum", rank.dest = 0)

comm.cat("Monte carlo probability:\n", quiet = TRUE)
comm.print(prob.global, digits = 20, quiet = TRUE)

elapsed <- as.numeric(Sys.time() - start.time, units = "secs")
comm.cat("Elapsed secs: ", elapsed, "\n", quiet = TRUE)

finalize()

Here is an example of the submission script; of course the settings for J, --nodes, and --ntasks-per-node must be changed for each entry of the performance study.

!bin/bash
#SBATCH --job-name=pbdRJob
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
#SBATCH --nodes=1
#SBATCH --ntasks-per-node=4
mpirun Rscript driver.R 4 4194304

6.2 Reading Datasets

Real statistical applications involve data, and therefore it’s worth considering some different ways to read a data file into a parallel R program.

6.2.1 Reading on all processes

The easiest and most obvious thing to do is read the data on all processes. If there are many processes and a large amount of data, this can be very wasteful and processes may contend with each other for access to I/O devices.
Table 1: Monte Carlo timing study. Entries are elapsed seconds used to compute the approximation. The serial column represents the serial code, while the np columns represent the pbdr code with np processes. All runs with 8 processes used a single node, while those with 16 processes used two nodes, and those with 32 processes used four nodes.

<table>
<thead>
<tr>
<th>J</th>
<th>Serial</th>
<th>np=1</th>
<th>np=2</th>
<th>np=4</th>
<th>np=8</th>
<th>np=16</th>
<th>np=32</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>14.0007</td>
<td>13.9731</td>
<td>7.8291</td>
<td>4.0788</td>
<td>2.2214</td>
<td>1.6099</td>
<td>0.7294</td>
</tr>
<tr>
<td>3</td>
<td>14.9866</td>
<td>14.7969</td>
<td>7.7737</td>
<td>4.3116</td>
<td>2.3701</td>
<td>1.3460</td>
<td>0.9655</td>
</tr>
<tr>
<td>4</td>
<td>16.2859</td>
<td>15.6357</td>
<td>8.3377</td>
<td>4.7176</td>
<td>2.4998</td>
<td>1.3869</td>
<td>0.9151</td>
</tr>
<tr>
<td>5</td>
<td>17.3330</td>
<td>16.6487</td>
<td>8.9406</td>
<td>4.8157</td>
<td>2.5994</td>
<td>1.4578</td>
<td>0.8544</td>
</tr>
<tr>
<td>6</td>
<td>18.7816</td>
<td>18.1040</td>
<td>9.4614</td>
<td>4.9742</td>
<td>2.7160</td>
<td>1.5420</td>
<td>0.9212</td>
</tr>
<tr>
<td>7</td>
<td>18.7773</td>
<td>19.1563</td>
<td>10.0340</td>
<td>4.9283</td>
<td>2.8270</td>
<td>1.7059</td>
<td>1.1007</td>
</tr>
<tr>
<td>8</td>
<td>20.1023</td>
<td>19.6903</td>
<td>10.2211</td>
<td>5.4114</td>
<td>2.9414</td>
<td>1.6890</td>
<td>0.9308</td>
</tr>
<tr>
<td>9</td>
<td>21.6230</td>
<td>20.8582</td>
<td>10.9944</td>
<td>5.6989</td>
<td>3.1924</td>
<td>1.8046</td>
<td>0.9473</td>
</tr>
<tr>
<td>10</td>
<td>22.6599</td>
<td>21.8145</td>
<td>11.4868</td>
<td>5.9748</td>
<td>3.2320</td>
<td>1.7895</td>
<td>1.0265</td>
</tr>
</tbody>
</table>

---

File: code/read-data/all-read/driver.R

```r
library(pbdtMPI, quiet = TRUE)
init()
.comm.size <- comm.size()
.comm.rank <- comm.rank()

dat <- read.table("../../../data/HondaOdysseyData.csv", sep = ",", head = TRUE)
comm.print(dat, all.rank = TRUE)
finalize()
```

---

File: code/read-data/all-read/run.slurm

```bash
#!/bin/bash
#SBATCH --job-name=pbdRJob
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
#SBATCH --nodes=1
#SBATCH --ntasks-per-node=3

mpirun Rscript driver.R
```

---

[araim1@tara-fe1 all-read]$ sbatch run.slurm
 Submitted batch job 1329245
[araim1@tara-fe1 all-read]$ cat slurm.err
[araim1@tara-fe1 all-read]$ cat slurm.out

COMM.RANK = 0

<table>
<thead>
<tr>
<th>Model</th>
<th>Price</th>
<th>Color</th>
<th>Mileage</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honda</td>
<td>20890</td>
<td>Dark Cherry Pearl</td>
<td>41922</td>
<td>2008</td>
</tr>
<tr>
<td>Odyssey</td>
<td>EX</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

COMM.RANK = 1

<table>
<thead>
<tr>
<th>Model</th>
<th>Price</th>
<th>Color</th>
<th>Mileage</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honda</td>
<td>20890</td>
<td>Dark Cherry Pearl</td>
<td>41922</td>
<td>2008</td>
</tr>
<tr>
<td>Odyssey</td>
<td>EX</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

COMM.RANK = 2

<table>
<thead>
<tr>
<th>Model</th>
<th>Price</th>
<th>Color</th>
<th>Mileage</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honda</td>
<td>20890</td>
<td>Dark Cherry Pearl</td>
<td>41922</td>
<td>2008</td>
</tr>
<tr>
<td>Odyssey</td>
<td>EX</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

[araim1@tara-fe1 all-read]$
6.2.2 Broadcasting from one process

A better idea is to read the file on one process and broadcast the data to all other processes. Again, this assumes the data isn’t large, and having \texttt{np} copies won’t be too expensive.

```r
library(pbdMPI, quiet = TRUE)
init()
.comm.size <- comm.size()
.comm.rank <- comm.rank()
dat0 <- NULL
if (.comm.rank == 0)
{
  filename <- "../../../data/HondaOdysseyData.csv"
  dat0 <- read.table(filename, sep = "", head = TRUE)
}
dat <- bcast(dat0, rank.source = 0)
comm.print(dat, all.rank = TRUE)
finalize()
```

6.2.3 Reading partitioned data

The data may be too large to fit on a single process. Suppose our data file has been partitioned into three pieces, with 34 rows in the first piece, 34 in the second piece, and 32 in the third piece. These partitioned files are named \texttt{HondaOdysseyData\_part0.csv}, \texttt{HondaOdysseyData\_part1.csv}, and \texttt{HondaOdysseyData\_part2.csv} respectively. We can use three processes to build a distributed matrix as follows, without a single process ever having to store the combined data.

```
#!/bin/bash
#SBATCH --job-name=pbdRJob
#SBATCH --output=slurm.out
#SBATCH --error=slurm.err
#SBATCH --partition=develop
#SBATCH --nodes=1
#SBATCH --ntasks-per-node=3
mpirun Rscript driver.R
```

```
araim1@tara-fe1 process0-read]$ sbatch run.slurm
Submitted batch job 1329250
[araim1@tara-fe1 process0-read]$ cat slurm.err
[araim1@tara-fe1 process0-read]$ cat slurm.out
COMM.RANK = 0
<table>
<thead>
<tr>
<th>Model</th>
<th>Price</th>
<th>Color</th>
<th>Mileage</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honda Odyssey</td>
<td>20890</td>
<td>Dark Cherry</td>
<td>41922</td>
<td>2008</td>
</tr>
<tr>
<td>EX</td>
<td></td>
<td>Pearl</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Honda Odyssey</td>
<td>26810</td>
<td>Green</td>
<td>42533</td>
<td>2008</td>
</tr>
<tr>
<td>EX-L</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
100 Honda Odyssey EX-L 23714 | Light Blue | 29293 | 2008 |
COMM.RANK = 1
<table>
<thead>
<tr>
<th>Model</th>
<th>Price</th>
<th>Color</th>
<th>Mileage</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honda Odyssey</td>
<td>20890</td>
<td>Dark Cherry</td>
<td>41922</td>
<td>2008</td>
</tr>
<tr>
<td>EX</td>
<td></td>
<td>Pearl</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Honda Odyssey</td>
<td>26810</td>
<td>Green</td>
<td>42533</td>
<td>2008</td>
</tr>
<tr>
<td>EX-L</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
100 Honda Odyssey EX-L 23714 | Light Blue | 29293 | 2008 |
COMM.RANK = 2
<table>
<thead>
<tr>
<th>Model</th>
<th>Price</th>
<th>Color</th>
<th>Mileage</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honda Odyssey</td>
<td>20890</td>
<td>Dark Cherry</td>
<td>41922</td>
<td>2008</td>
</tr>
<tr>
<td>EX</td>
<td></td>
<td>Pearl</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Honda Odyssey</td>
<td>26810</td>
<td>Green</td>
<td>42533</td>
<td>2008</td>
</tr>
<tr>
<td>EX-L</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
100 Honda Odyssey EX-L 23714 | Light Blue | 29293 | 2008 |
```
library(pbdDMAT, quiet=TRUE)
init.grid()
.comm.rank <- comm.rank()
.comm.size <- comm.size()

# Create a np x 1 grid of processes where we'll distribute our matrix
newctxt <- minctxt()
init.grid(NPROW = .comm.size, NPCOL = 1, ICTXT = newctxt)
.grid <- blacs(ICTXT = newctxt)
.grid.row <- grid$MYROW
.grid.col <- grid$MYCOL

msg <- sprintf("ID %d: My grid position is (%d,%d)\n", .comm.rank, grid$MYROW, grid$MYCOL)
comm.cat(msg, all.rank = TRUE, quiet = TRUE)

dd <- "../../../data/"
filename <- sprintf("%s/HondaOdysseyData_part%d.csv", dd, .grid.row)
dat <- read.table(filename, sep = ",", head = TRUE)
X.local <- cbind(dat$Price, dat$Mileage, dat$Year)

msg <- sprintf("ID %d: Read data file %s\n", .comm.rank, filename)
comm.cat(msg, all.rank = TRUE, quiet = TRUE)
comm.print(X.local, all.rank = TRUE)

k <- ncol(X.local)
l.n <- nrow(X.local)
blrows <- allreduce(l.n, op = "max")

X <- new("ddmatrix", Data=X.local, dim=c(n,k), ldim=dim(X.local),
         bldim=c(blrows,k), ICTXT = newctxt)
X <- redistribute(X, bldim=c(4,4))

print(X)
X.global <- as.matrix(X)
comm.print(X.global)
finalize()

This code is a bit more complicated than before. We are again working with a process grid as in section 5.3. One (realistic) complication here is that the number of rows is not equal across all pieces of the data. Notice that the variable `blrows`, which represents the number of rows in each block, is taken to be the maximum of the number of rows of the data pieces. The partitioning of the data into 34, 34, and 32 rows (where all blocks are of equal size except for the last block which has the leftovers) has been selected on purpose to be compatible with the matrix blocking format. The distributed matrix is initially created in a natural way, so that process 0 holds the first set of rows, process 1 holds the second set, and process 2 holds the third set. This is not an ideal distribution for parallel matrix operations, so we use `redistribute` to rearrange the blocking scheme.

One issue that we have avoided is that some of our columns are not numeric. As of right now, `pbdR` appears not to support distributed data frames or matrices of strings. We have ignored the non-numeric columns in this example. We may code them into numeric variables, but the coding should be the same across all processes, and therefore would require some communication. We show one way to handle this in section 6.3.
Submitted batch job 1329259
[araim1@tara-fe1 partitioned-read]$ cat slurm.err
[araim1@tara-fe1 partitioned-read]$ cat slurm.out
Using 1x3 for the default grid size

Grid ICTXT=3 of size 3x1 successfully created
ID 0: My grid position is (0,0)
ID 1: My grid position is (1,0)
ID 2: My grid position is (2,0)
ID 0: Read data file ../../../data//HondaOdysseyData_part0.csv
ID 1: Read data file ../../../data//HondaOdysseyData_part1.csv
ID 2: Read data file ../../../data//HondaOdysseyData_part2.csv
COMM.RANK = 0

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20890</td>
<td>41922</td>
</tr>
<tr>
<td>2</td>
<td>26810</td>
<td>42533</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

COMM.RANK = 1

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21900</td>
<td>51938</td>
</tr>
<tr>
<td>2</td>
<td>29491</td>
<td>20895</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

COMM.RANK = 2

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27988</td>
<td>46975</td>
</tr>
<tr>
<td>2</td>
<td>25900</td>
<td>48479</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

COMM.RANK = 0

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20890</td>
<td>41922</td>
</tr>
<tr>
<td>2</td>
<td>26810</td>
<td>42533</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.3 Distributed Regression

Now we will extend the example from section 6.2.3 into a distributed regression. Since the dataset isn’t large, we can first run the regression in serial and then compare the result with our parallel version.
filename <- sprintf("../../data/HondaOdysseyData.csv")
dat <- read.table(filename, sep = ",", head = TRUE)
Model.levels <- levels(dat$Model)
Year.levels <- levels(as.ordered(dat$Year))
Price <- dat$Price
Model <- factor(dat$Model, levels = Model.levels)
Year <- factor(dat$Year, levels = Year.levels)
Mileage <- dat$Mileage
X <- model.matrix(~ Model + Mileage + Year)
y <- Price
beta.hat <- solve(t(X) %*% X) %*% t(X) %*% y
cat("beta.hat = \\
"
print(beta.hat)

Next is the parallel version. Notice that when we code the non-numeric variables into numbers, we need to do some communication so that the coding is the same on all processes.
library(pbdDMAT, quiet=TRUE)
init.grid()
.comm.rank <- comm.rank()
.comm.size <- comm.size()

# Create a np x 1 grid of processes where we'll distribute our matrix
newctxt <- minctxt()
init.grid(NPROW = .comm.size, NPCOL = 1, ICTXT = newctxt)
.grid.row <- grid$MYROW
.grid.col <- grid$MYCOL

msg <- sprintf("ID %d: My grid position is (%d,%d)\n", .comm.rank, .grid.row, .grid.col)
comm.cat(msg, all.rank = TRUE, quiet = TRUE)

# ------------ Read data for regression ------------
dd <- "../../data"
filename <- sprintf("%s/HondaOdysseyData_part%d.csv", dd, .grid.row)
dat <- read.table(filename, sep = ",", head = TRUE)

msg <- sprintf("ID %d: Read data file %s\n", .comm.rank, filename)
comm.cat(msg, all.rank = TRUE, quiet = TRUE)

# We need a global coding for factor and ordered variables
Model.levels <- unique(unlist(allgather(levels(dat$Model))))
Year.levels <- unique(unlist(allgather(levels(as.ordered(dat$Year)))))

Price <- dat$Price
Model <- factor(dat$Model, levels = Model.levels)
Year <- factor(dat$Year, levels = Year.levels)
Mileage <- dat$Mileage

# Create a model matrix for the local regressors
X.local <- model.matrix(~ Model + Mileage + Year)
y.local <- as.matrix(Price)

# If we don't clear the dimension names, we get an error later
# from redistribute
attributes(X.local)$dimnames <- NULL

# ------------ Create distributed matrices for X and y ------------
k <- ncol(X.local)
l.n <- nrow(X.local)
blrows <- allreduce(l.n, op = "sum")
bldim <- allreduce(1.n, op = "max")
X <- new("ddmatrix", Data=X.local, dim=c(n,k), ldim=dim(X.local),
bldim=c(blrows,k), ICTXT = newctxt)
y <- new("ddmatrix", Data=y.local, dim=c(n,1), ldim=dim(y.local),
bldim=c(blrows,1), ICTXT = newctxt)
X <- redistribute(X, bldim=c(4,4))
y <- redistribute(y, bldim=c(4,1))

# ------------ Fit regression in parallel ------------
beta.hat <- solve(t(X) %*% X) %*% t(X) %*% y

comm.cat("beta.hat =\n", quiet = TRUE)
comm.print(global.beta.hat, quiet = TRUE)
finalize()
6.4 Distributed Bootstrap

Distributed computing could also be helpful for the bootstrap algorithm in a large data setting. Suppose again that our dataset is too large to fit on a single process, but is split into four pieces. Four MPI processes will be used to compute the bootstrap in a distributed manner, where each is responsible for one piece of the data. We demonstrate on the `law82` dataset which can be found in the `bootstrap` package. The `bootstrap` package can be installed by the following command.

```r
> install.packages("bootstrap")
```

The goal is to compute the standard error of the sample correlation between the variables `LSAT` and `GPA`. This example was adapted from (Rizzo, 2008, Chapter 7). A serial version is given first.
library(bootstrap)
set.seed(1234)

n <- nrow(law82)
B <- 500

T.boot <- numeric(B)

for (b in 1:B)
  idx <- sample(1:n, replace = TRUE)
  LSAT.boot <- law82$LSAT[idx]
  GPA.boot <- law82$GPA[idx]
  T.boot[b] <- cor(LSAT.boot, GPA.boot)

print(T.boot)

se.boot <- sd(T.boot)
cat("se.boot =", se.boot, "\n")

For our parallel version, process 0 is responsible for drawing the indices which will be included in each
bootstrap sample. These indices are the broadcast to all other processes. Note that if the number of
observations \( n \) was very large, this might not be feasible, and we would have to reconsider doing it in
a distributed way. Each process checks bootstrap indices against the list of local observations. Using
those observations and a parallel inner product function, all processes cooperate to compute the sample
correlation. After \( B = 500 \) bootstrap iterations, all processes contain \( B \) sample correlations, which they
can then use to compute the standard error.
library(pbdMPI, quiet = TRUE)
init()

l.n <- nrow(dat)
l.idx <- as.integer(rownames(dat))
n <- allreduce(l.n, op = "sum")
B <- 500
T.boot <- numeric(B)

inner.prod.mpi <- function(x, y, idx)
{
  # idx are global indices
  idx <- idx[idx %in% l.idx]
  # jdx are local indices that index into the local data structures
  jdx <- idx - min(l.idx) + 1
  x.y <- sum(x[jdx] * y[jdx])
  # Just in case this process didn't receive any indices, use x.y = 0
  if (is.na(x.y)) { x.y <- 0 }
  x.y <- allreduce(x.y, op = "sum")
  return(x.y)
}

cor.mpi <- function(x, y, idx)
{
  # Compute the means for the current bootstrap sample given by idx
  # and subtract the means from the data
  ones <- matrix(1/n, nrow = n, ncol = 1)
  m.x <- inner.prod.mpi(y, ones, idx)
  m.y <- inner.prod.mpi(x, ones, idx)
  x <- x - m.x
  y <- y - m.y
  # Compute the correlation
  r <- x.y / ( sqrt(x.x) * sqrt(y.y) )
  return(r)
}

for (b in 1:B)
{
  idx0 <- NULL
  if (.comm.rank == 0)
  {
    idx0 <- as.matrix(sample(1:n, replace = TRUE))
  }
  idx <- bcast(idx0, rank.source = 0)
  T.boot[b] <- cor.mpi(dat$LSAT, dat$GPA, idx)
}

comm.cat("T.boot:
", quiet = TRUE)
comm.print(T.boot)
se.boot <- sd(T.boot)
comm.cat("se.boot =", se.boot, "\n", quiet = TRUE)

finalize()
Acknowledgements

Thanks to Dr. George Ostrouchov at Oak Ridge National Laboratory for his invitation, as well as for arranging financial support from the NICS Remote Data and Visualization Center (funded by NSF), allowing the author to attend the NICS training event in March 2013 and gain exposure to pbdR. Sections 5.3, 6.2, 6.3, and 6.4 of this document were suggested by Dr. Nagaraj Neerchal after a two hour workshop (“Introduction to pbdR with tara @ HPCF”) which was held on 6/16/2013. Thanks to all attendees of this workshop for their participation and helpful feedback.

The hardware used in this tutorial is part of the UMBC High Performance Computing Facility (HPCF). The facility is supported by the U.S. National Science Foundation through the MRI program (grant nos. CNS–0821258 and CNS–1228778) and the SCREMS program (grant no. DMS–0821311), with additional substantial support from the University of Maryland, Baltimore County (UMBC). See www.umbc.edu/hpcf for more information on HPCF and the projects using its resources. The author additionally acknowledges financial support as HPCF RA.

References

