

Assessment of Statistical Methods for Water Quality Monitoring in Maryland's Tidal Waterways

REU Site: Interdisciplinary Program in High Performance Computing

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Abstract

The Chesapeake Bay and its surrounding tributaries are home to over 3,600 species of plants and animals. In order to assess the health of the region, the Maryland Department of Natural Resources (DNR) monitors various parameters, such as dissolved oxygen, with monitoring stations located throughout the tidal waterways. Utilizing data provided by DNR, we assessed the waterways for areas of water quality concern. We analyzed the percentage of the readings taken for each parameter that failed to meet the threshold values and used the Wilcoxon Signed-Rank Test to determine the statuses of the stations. In order to assess the applicability of the Wilcoxon Test given the positive skew in the data, a simulation was performed. This simulation demonstrated that log-transforming the data prior to performing the Wilcoxon Test was not enough to reduce the Type I Error to reasonable levels. Thus our team developed a relative ranking using a set of multiple comparison methods: a version of the Tukey Test on variance-transformed proportions, the Bonferroni adjustment method, a Bayesian method, and the Benjamini-Hochberg rejection method. From the ranking results we identified when each ranking technique is most applicable to our data.

Key Words: Chesapeake Bay, Water quality monitoring, Wilcoxon signed-rank test, True Type I Error estimation, Benjamini-Hochberg method

1 Introduction

The Chesapeake Bay and its surrounding waterways provide a habitat for over 3,600 species of plants and animals. It is a valuable resource, both recreationally and commercially, to those who live in the basin. DNR operates 35 continuous monitoring stations. Three of these stations take readings at multiple depths for a total of 38 stations of interest. All of the stations take readings every 10 to 20 minutes, with the majority taking reading every 15 minutes. Various parameters such as water clarity, dissolved oxygen, pH, and chlorophyll, are used to help determine the health of the water [3]. Past analyses of this data have been used to determine trends in different regions of the tidal waterways as well as aid in assessing the success of DNR funded projects.

Four parameters are of particular importance in water quality monitoring: dissolved oxygen, chlorophyll, pH, and turbidity. Chlorophyll is the measure by which algae levels are evaluated. Density of chlorophyll in the water is measured in $\mu\text{g/L}$. An increase in algae



Figure 1.1: The Chesapeake Bay and its tributaries are located on the East Coast, spanning six states (MD, VA, WV, PA, NY, DE) and Washington DC and emptying into the Atlantic Ocean. Image used with permission.

levels corresponds to a decrease in water clarity and has a negative impact on dissolved oxygen levels as the algae decomposes. Turbidity is an important measurement of water clarity, the latter being necessary for light to reach submerged aquatic vegetation (SAV) and promote growth. Turbidity is measured in NTUs. When analyzing the measurements of chlorophyll and turbidity, larger numbers correspond to less healthy water. Conversely, when considering dissolved oxygen concentrations, higher readings are preferable. While a reading of 5mg/L is widely considered a failure threshold, a threshold of 3mg/L is often used to test for waters that are severely oxygen-deficient [6].

Summer is a time of particular interest for analyzing these parameters. When evaluating the concentration of dissolved oxygen, the time frame considered is June through September. Deficient oxygen levels are most prevalent during these warm summer months because warmer water holds less dissolved oxygen. Chlorophyll and turbidity are evaluated from April through September, the growing season for aquatic vegetation. High levels of chlorophyll and turbidity can have the most detrimental effect on the ecosystem during these months. Lastly, pH, a measure of the acidity of water, is also monitored during these months. This is due to the fact that extreme pH levels are detrimental to aquatic wildlife.

Our project focused on the four parameters listed above. The failure threshold(s) of each parameter and its time frame of greatest interest are summarized in Table 1.1.

Table 1.1: Failure threshold and time frame of greatest interest.

Parameter	Failure Threshold	Time Frame
Dissolved Oxygen (severe)	< 3mg/L	June to September
Dissolved Oxygen	< 5mg/L	June to September
Turbidity	> 7 NTU	April to September
Chlorophyll	> 30 μ g/L	April to September
pH	< 5.5 or > 8.3	April to September

In this paper, we assessed the station performances using the Wilcoxon Signed-Rank Test. This non-parametric test assumes that the distribution from which the data is taken is symmetric. However, the data for some parameters displayed significant asymmetry. To assess the impact of these violations, we simulated the performance of the Wilcoxon Signed-Rank Test on a skewed distribution. This simulation showed that even the standard techniques of log-transforming were not enough to bring the Type I Error rate of the Wilcoxon Signed-Rank test on such skewed data to reasonable levels. Thus we investigated relative ranking system which ranked stations in regard to their performance using multiple ranking techniques: a version of the Tukey Test on variance-transformed proportions, the Bonferroni adjustment method, a Bayesian method, and the Benjamini-Hochberg rejection method. From the ranking results we identified when each ranking technique is most applicable to our data.

This paper is organized as follows. Section 2 outlines the statistical methodologies used in the assessment along with a brief description of each method. Since the assessment involves numerous pairwise comparisons, special care is needed to conserve the overall Type I Error (False Positive). Improved methods are also illustrated in Section 2. Section 3 shows the results of the methods. This is proposed as a way of augmenting the assessment approach and may be helpful in the planning of Bay-wide restoration efforts. We conclude the report with final remarks in Section 4.

2 Methodologies

2.1 Percent Failure and Classification of Stations

We define percent failure as the number of readings that did not meet the DNR-provided threshold divided by the total number of readings. For each station, we determine the percent failure in the appropriate timespan for each parameter: dissolved oxygen, chlorophyll, turbidity and pH. We then proceed to use statistical tests that provide more complex insights that allow us to make more statistically sound conclusions.

One such test was the Wilcoxon’s Signed-Rank Test. The Wilcoxon Signed-Rank Test is a non-parametric test that we used to compare the median of the station’s data for a given parameter against the threshold value. The test assumes that the probability distribution from which the data is taken is symmetric. Let $\mathbf{x} = (x_1, x_2, \dots, x_m)$ be the data vector. The test statistic is given by

$$S = \left| \sum_{i=1}^m [R_i \cdot \text{sign}(x_i - \text{thresh})] \right|,$$

where R_i is the rank of $|x_i - \text{thresh}|$ in ascending order and the sign function is defined as

$$\text{sign}(x) = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{if } x = 0, \\ -1 & \text{if } x < 0. \end{cases}$$

The test rejects the null hypothesis that the median of the data is equal to the threshold if S is sufficiently large. For the purposes of our study, the Wilcoxon Signed-Rank test was used to test the hypothesis:

$$H_0 : \text{median}(\mathbf{x}) = \text{thresh}$$

$$H_A : \text{median}(\mathbf{x}) \neq \text{thresh}.$$

The station statuses were defined as “Good” or “Bad” if the Wilcoxon Test rejected the null hypothesis, depending upon if the station’s median for that parameter fell on the “unhealthy” or “healthy” side of the threshold value. The station was assigned “Borderline” if the null hypothesis could not be rejected. Our tests were conducted with $\alpha = 0.01$. In order to ensure the familywise Type I Error was α , we utilized the Benjamini-Hochberg rejection method. This method ranks the p-values in ascending order and rejects the null hypotheses corresponding to the lowest p-values until the cumulative sum of the rejected p-values is equal to α . Intuitively, this method rejects the null hypothesis of the tests where the null hypothesis is most likely to be false, that is the tests that have the lowest p-values, until the cumulative p-value is the chosen α .

2.2 Simulation

The Wilcoxon Signed-Rank Test is implemented under the assumption that the distribution from which the samples are drawn is symmetric. However, Figure 2.1 shows the distribution for turbidity at the Flats station which clearly displays a positive skew. Thus we

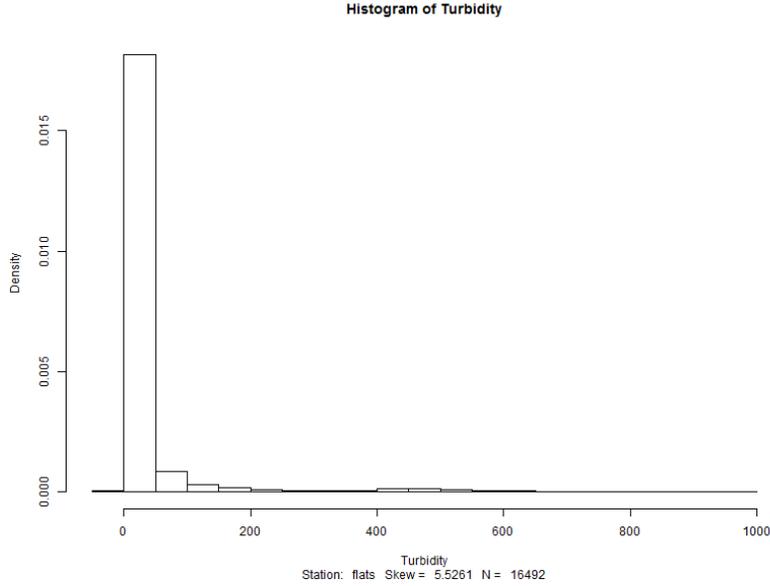


Figure 2.1: Turbidity readings from Flats station is an example of skewness in the data.

decided to run a simulation to assess the impact of the data's skew on the Type I Error rate for the Wilcoxon Signed-Rank Test. For the purposes of our study, we used sample data $(x_1, \dots, x_n) \sim \Gamma(\alpha_{shape}, \beta)$, where the gamma distribution is defined as having the probability density function

$$f(\alpha_{shape}, \beta) = \frac{\beta^{\alpha_{shape}}}{\Gamma(\alpha_{shape})} x^{\alpha_{shape}-1} e^{-\beta x},$$

where Γ is the gamma function. This simulation is performed with sample size $n = 1000$ on each gamma distribution with various values for shape and rate. For accuracy, the mean of the Type I Errors from 10,000 simulations was taken as the estimate for the Type I Error.

The simulation was performed using the **R** statistical programming environment. We generated random samples from the gamma distribution in **R** using the `rgamma()` function and calculated its median using the `qgamma()` function. On each sample, the Wilcoxon Signed-Rank Test was used to test the sample median against the true median of the gamma distribution. We tested the null hypothesis that the sample median does not differ from the true median with the alternative hypothesis that the medians were different. To summarize,

$$H_0 : \text{median}(\mathbf{x}) = \text{median}(\Gamma(\alpha_{shape}, \beta))$$

and

$$H_A : \text{median}(\mathbf{x}) \neq \text{median}(\Gamma(\alpha_{shape}, \beta)).$$

The Type I Error rate was the percentage of tests which rejected the null hypothesis. We performed the simulation using a claimed significance level $\alpha = 0.01$. However, with a skewed distribution, such as that of the gamma function, we suspected that the Wilcoxon test will

have a true Type I Error rate greater than the claimed 0.01. If the calculated Type I Error rate is less than our claimed $\alpha = 0.01$, then this would imply that the Wilcoxon Signed-Rank test is conservative for positively skewed data and therefore inefficient when applied to skewed distributions. If the Type I Error rate is greater than the claimed $\alpha = 0.01$, this would imply that the outcome of the Wilcoxon test is adversely affected by the lack of symmetry. In this case, we may find that the Wilcoxon Signed-Rank test may not be the most appropriate test to use to assess the status of the stations

2.3 Rankings of Salinity Regimes

In addition to classifying the stations' performance, we used the Tukey Honestly Significant Difference (HSD) test to see whether the salinity content of the station affects the station's performance. The Tukey HSD test is implemented by dividing all stations into salinity regimes and calculating the mean value of the percent fail (in terms of a particular parameter) within each salinity regime. The mean percent fail of each regime is then compared to all the others to test for significant differences between regimes. This comparison is based on the studentized range distribution. Significant differences between regimes indicate that salinity content significantly impacts stations' performance. The Tukey HSD test is conducted under the assumption that all observations are independent with equal variance. Classifications of each station's salinity regime was provided by DNR.

To implement the Tukey HSD, we paired each station's percent failure with its corresponding salinity regime. Once these two were paired, we fit them to the ANOVA model using the `aov` function in **R** with the percent failures as the response variables and salinity regimes as the explanatory variables. After fitting the ANOVA model, we used the `TukeyHSD` function in **R**.

2.4 Rankings of Stations

In addition to analyzing the statuses of the stations, we ranked the stations' performance. The result of the ranking techniques presents a set of stations that are tied in performance, thus giving groupings of stations that are linked by their similarities. To rank the stations, one must compare each station to all other stations and thus perform $\binom{n}{2}$ tests, where n is the number of stations. In order to bound the Type I Error rate, we utilized multiple comparison tests. We implemented four different ranking methods: the Tukey Test, the Bonferonni Test, and the Bayesian Ranking method using the stations' percent failures, and the Benjamini-Hochberg method using the stations' percent failures and means. Table 2.1 provides a brief summary of the ranking techniques, with more details in the following sub-sections.

2.4.1 Tukey Test

We implemented one of our ranking methods by performing a Tukey-Like Multiple Comparison Test among the stations. This testing involves a comparison of each possible pair of stations. In order to make the comparison, the percent fail first must be transformed by this

Table 2.1: A summary of ranking techniques used.

Method	Description
Tukey	Performs variance transform on the data; obtains the q statistic in the studentized range distribution
Bonferroni	Tests each of the $\binom{n}{2}$ hypotheses at a statistical significance level of $\frac{\alpha}{\binom{n}{2}}$, where n is the number of stations
Benjamini-Hochberg	For a given α , finds the largest k such that $P_{(k)} \leq \frac{k}{m}\alpha$, where m is the number of statistical tests
Bayesian Ranking	Transforms an assumed prior distribution to a posterior distribution; parameter estimates are then obtained from the posterior to rank the stations

function:

$$p' = \frac{1}{2} \left[\arcsin \sqrt{\frac{X}{n+1}} + \arcsin \sqrt{\frac{X+1}{n+1}} \right],$$

where X is the number of readings above the threshold and n is the number of observations in the sample (station). Once the percent failures are transformed, we rank them based on their numerical value from smallest to largest. Then, the differences are computed between all pairs of stations: the largest and smallest, the largest and second smallest, etc. A standard error (SE) is then computed for each of the pairs using the following formula,

$$SE = \sqrt{\frac{410.35}{n_A + 0.5} + \frac{410.35}{n_B + 0.5}},$$

where n_A is the sample size of one of the stations and n_B is the sample size of the other station in the pair being compared. After the difference and SE are computed for the pair, they are used to compute the test statistic:

$$q = \frac{p_A - p_B}{SE}.$$

This ‘q’ statistic is then compared to a $q_{0.01}$ critical value, which comes from the q-distribution attributed to the well-known statistician John Tukey, with $\alpha = 0.01$, degrees of freedom, v , equal to ∞ and $k =$ the number of groups (in this case $\binom{38}{2}$) given in Table B.5 of [7]. If the ‘q’ statistic $> q_{0.01}$ critical value, then we reject H_0 , which assumed the stations were not equal, and conclude the stations are the same; otherwise, we fail to reject H_0 and look to the original percent failure values to see which station is faring worse. A ranking is determined by ordering the stations by the number of pairwise comparisons a given station was declared as fairing better/worse.

2.4.2 Benjamini-Hochberg Method

A more recently developed Benjamini-Hochberg method of multiple comparisons is the Benjamini-Hochberg method. It is based on the idea of controlling the overall False Discovery Rate (FDR) as it is directly related to the expected proportion of falsely rejected hypotheses [2]. In this task, we have $\binom{38}{2}$ stations. Thus, letting $\alpha = 0.01$, the target is to reject no more than 10% of all tested hypotheses under the assumption that all compared pairs are indeed equal.

The implementation of Benjamini-Hochberg method is as follows [5]:

1. Sort the P-values $P_{(1)} \dots P_{(m)}$ where m is the number of tests
2. Finding the largest k such that $P_{(k)} \leq \frac{k}{m}\alpha$
3. Reject $P_{(1)} \dots P_{(k)}$.

For pairs of stations that are found significant, the winner for the test is the station with the lower percent fail, or the mean corresponding to the ‘healthier’ value. This process is repeated, keeping track of each station’s number of wins. The stations are then listed from most to least wins, creating a rank from best to worst condition.

2.4.3 Bayesian Method

Our analysis also included ranking the stations using a Bayesian ranking method. It estimates the ranks of certain unknown distribution parameters by ranking corresponding sample estimates. We model the distribution of the sample estimates for each station, x_i , by

$$x_i | \boldsymbol{\theta} \stackrel{\text{ind}}{\sim} N(\theta_i, \sigma_i^2), \quad i = 1, \dots, k,$$

where $\boldsymbol{\theta} = (\theta_1, \dots, \theta_k)$ is the vector of unknown quantities (the true percent failures). Let $\boldsymbol{\sigma} = (\sigma_1^2, \dots, \sigma_k^2)$ denote the vector of variances of the estimates $\boldsymbol{x} = (x_1, \dots, x_k)$. We can assume normality since the percent of failures are usually computed from a large sample. The Bayesian ranking procedure is based on the computation of posterior probabilities for all possible rankings. It determines the rank of θ_i by ordering the sample estimates and associating each x_i with a corresponding rank \hat{r}_i .

To implement this method, let \boldsymbol{S} be an $n \times 38$ matrix where columns correspond to 38 stations. Let $r = 1$. For each θ_i in \boldsymbol{S} , where $i \in \{1, \dots, 38\}$, compute the probability that θ_i is rank r . This implementation is based on simulating the posterior distribution and n corresponds to the simulation sample size which is set to a large number such as 1000. To compute the posterior probability, we find the maximum element in each row and set it equal to 1 while all other elements in the row are set to 0, and then compute the column sums. Once the column sums are obtained, we divide each sum by the total number of rows in \boldsymbol{S} . Let θ_j represent the station with the highest probability and associate the rank, r , with θ_j . Then eliminate that column from matrix \boldsymbol{S} so that \boldsymbol{S} is now an $n \times (38 - r)$ matrix. Continue this process, incrementing r , until all the stations are ranked.

For more on this method and other Bayesian ranking systems, please see [1].

3 Results

3.1 Stations' Statuses

A table of our results from the Wilcoxon Signed-Rank Test can be found in Table 3.1. Each row represents a station. They are listed in alphabetical order. The columns display the stations' statuses as assigned by the Wilcoxon Test for dissolved oxygen concentration at both the 3mg/l and 5mg/l threshold, turbidity, chlorophyll concentration, and pH.

According to the results of the Wilcoxon Signed-Rank Tests ran on the continuous monitoring stations' data for the summer of 2011, most stations are classified as exhibiting turbidity levels significantly above the threshold level, and all but the Bishopville station, Little Monie Creek station and the stations located at the bottom of Goose and Mansonville received a 'good' status for dissolved oxygen using the 5mg/l threshold. When the more critical dissolved oxygen concentration threshold of 3mg/l is used, only the Bishopville station's and Little Monie Creek station's status raised from 'bad' to 'good.' This suggests that the Bishopville station and Little Monie Creek station, while not as healthy as the rest of the bay, may not be in as critical condition as the two bottom stations whose statuses remain 'bad' regardless of which dissolved oxygen threshold is used. DNR mentions that low dissolved oxygen readings during the summer months are most prominent in measurements taken from the bottom stations due to decomposing algae that has sunken and the lack of mixing between surface and bottom waters [3]. These explanations support our conclusions that two of the bottom stations are faring the worst.

The statuses for turbidity are largely 'bad' in that many stations fell above the benchmark level. On the otherhand, the station statuses for chlorophyll faired quite well. This indicates that the algee blooms may have been a less of a concern during the summer of 2011 than dissolved oxygen or turbidity. Unlike turbidity, the statuses in pH are mostly 'good.' The exceptions are the Mataponi and Little Monie Creek stations which both received classifications of 'bad.' This shows that most of the stations fell within the benchmark range for pH. Some possible reasons for the 'bad' status could be because of an algae bloom or low salinity. Overall, in terms of pH the stations performed well. As our future research investigates into the appropriateness of the current methodology, namely the use of the Wilcoxon Signed-Rank to determine stations' statuses, we are hesitant to draw strong conclusions regarding the heath of the bay based on current statuses. The results of the methodology exploration is discussed further in Section 3.2.

3.2 Simulation Results

In this section we provide the results of the simualtion we ran to investigate the sensitivity of the Wilcoxon Signed-Rank to the non-symmetry in the data. This simualtion study is described in Section 2.2. Tables 3.2 and 3.4 show the results of the simulation. In both of the tables, the columns represent the values used for the β_{rate} and the rows represent a set of the values used for the α_{shape} in the simulation. Tables 3.3 shows the average skew of each parameter (listed in the columns) at each station (listed in the rows alphabetically).

Table 3.1: Wilcoxon Signed-Ranked test assignment of station statuses as “Good,” “Borderline,” or “Bad” of Dissolved oxygen (3mg), Dissolved oxygen (5mg), Chlorophyll, Turbidity, and pH.

Station Name	DO5	DO3	Turbidity	Chlorophyll	pH
Annapolis	Good	Good	Bad	Good	Good
Betterton	Good	Good	Bad	Good	Good
Big Annemessex	Good	Good	Good	Good	Good
Bishopville	Bad	Good	Bad	Bad	Good
Budds Landing	Good	Good	Bad	Bad	Good
Chesapeake Y. Club	Good	Good	Bad	Good	Good
Corisca River	Good	Good	Bad	Border	Good
Downs Park	Good	Good	Bad	Good	Good
Flats	Good	Good	Bad	Good	Good
Fort Armistead	Good	Good	Bad	Good	Good
Fort Howard	Good	Good	Bad	Good	Good
Fort Smallwood	Good	Good	Bad	Good	Good
Goose (bottom)	Bad	Bad	Good	Good	Good
Goose (surface)	Good	Good	Good	Good	Good
Gratitude Marina	Good	Good	Bad	Good	Good
Greys Creek	Good	Good	Bad	Good	Good
Harness Creek (down)	Good	Good	Bad	Good	Good
Havre de Grace	Good	Good	Bad	Good	Good
Indian Head	Good	Good	Bad	Good	Good
Iron Plot Landing	Good	Good	Bad	Good	Good
Little Monie	Bad	Good	Good	Good	Good
Love Point	Good	Good	Bad	Good	Good
Manokin	Good	Good	Good	Good	Good
Masonville (bottom)	Bad	Bad	Bad	Good	Good
Masonville Cove	Good	Good	Bad	Good	Good
Mataponi	Good	Good	Bad	Good	Good
Mattawoman	Good	Good	Bad	Good	Good
McHenry	Good	Good	Good	Good	Good
Newport Creek	Good	Good	Bad	Good	Good
Otter Point Creek	Good	Good	Bad	Good	Good
Possum (bottom)	Good	Good	Bad	Good	Good
Possum Point	Good	Good	Bad	Good	Good
Public Landing	Good	Good	Bad	Good	Good
Railroad Bridge	Good	Good	Bad	Good	Good
Sandy Point South	Good	Good	Bad	Good	Good
Sill	Good	Good	Bad	Good	Good
Sill (bottom)	Good	Good	Bad	Good	Good
St. George’s Creek	Good	Good	Good	Good	Good

Table 3.2: Type I Error of the Wilcoxon Test applied to samples drawn from the Gamma distribution with parameters α and β using a significance level of 0.01.

	β_{rate}	
	1	10
$\alpha_{shape} = 2$	0.8737	0.8692
$\alpha_{shape} = 4$	0.5054	0.5042
$\alpha_{shape} = 10$	0.1716	0.1701
$\alpha_{shape} = 50$	0.0304	0.0297
$\alpha_{shape} = 100$	0.0204	0.0205

Table 3.5 shows the skew of the $\Gamma(\alpha_{shape}, \beta_{rate})$ distribution with varying β_{rate} (listed in the columns) and α_{shape} (listed in the rows) values. From Table 3.5 we can see that as the α_{shape} parameter increases, the skew decreases. By looking at Table 3.2 and Table 3.4 we see a similar trend with the Type I Error, that is it decreases as α_{shape} and β_{rate} increase. We conclude that as the skew decreases, the Type I Error rate obtained with the Wilcoxon Signed-Rank Test also decreases. Although we see similar behavior using data with and without the log-transformation, the inflation of the Type I Error is less severe with the transformation.

This gives strong evidence that the use of the Wilcoxon Signed-Rank test on the log-transformed data over the raw data is more appropriate for the highly-skewed distributions we see in the data. Comparing the skew values for the gamma distributions found in Table 3.5 to the skew values of our station data found in Table 3.3 we see that dissolved oxygen's and pH's positive skew values are in the range of the gamma distributions' skews. For turbidity and chlorophyll, we see even larger skew values than those covered with our gamma distributions. Even with the lower skew values, we see in Table 3.2 that the Wilcoxon Test still yields a Type I Error rate much larger than our claimed significance level of 0.01 with the trend indicating the Type I Error will continue to inflate with larger skew values like those of turbidity and chlorophyll. Notice the simulation results indicate that the log-transformation does not significantly help reduce the Type I Error for the Wilcoxon Signed-Rank Test for data with this larger amount of skew.

Table 3.3: Average skew values for each station's data for dissolved oxygen, turbidity, chlorophyll and pH. Stations are listed alphabetically.

Station Name	DO	Turbidity	Chlorophyll	pH
Annapolis	0.8600	3.5520	5.1955	0.1123
Betterton	0.7221	3.9284	4.0422	0.6824
Big Annemessex	-0.0837	4.7184	2.4859	0.2181
Bishopville	0.6247	1.5330	3.5572	0.5359
Budds Landing	0.0033	22.896	0.8347	-0.8338
Chesapeake Y. Club	0.7433	2.0293	2.6650	0.5736
Corisca River	0.6647	3.6492	1.8978	-0.0347
Downs Park	-0.4362	3.7650	1.6356	0.4968
Flats	0.4660	5.5261	1.7057	-0.1534
Fort Armistead	0.4411	3.2652	2.6704	0.2692
Fort Howard	0.0265	3.5911	1.6136	0.3341
Fort Smallwood	-0.0350	7.0761	2.1032	0.3495
Goose (bottom)	0.9488	23.037	2.0776	0.5552
Goose (surface)	0.9363	3.6947	2.5232	-0.1927
Gratitude Marina	0.3992	3.6118	1.7231	0.7324
Greys Creek	1.0849	5.4151	1.7777	-0.1125
Harness Creek (down)	0.3694	3.3329	1.6014	-0.0118
Havre de Grace	0.5920	5.3488	1.6738	0.9268
Indian Head	0.0680	4.1753	1.1078	0.0945
Iron Plot Landing	0.6860	3.7273	8.3336	0.01653
Little Monie	0.2843	2.2775	13.122	0.5784
Love Point	0.0870	3.6370	0.9614	0.1996
Manokin	0.0360	5.1246	4.1258	-0.3016
Masonville (bottom)	0.8714	10.021	4.9516	1.4185
Masonville Cove	0.6011	5.5490	1.4686	0.3826
Mataponi	0.3116	9.2984	5.4602	0.5960
Mattawoman	0.3659	9.5283	0.3484	-0.4789
McHenry	0.4703	5.2170	3.2518	0.5557
Newport Creek	0.1511	4.5250	0.5743	-0.5666
Otter Point Creek	0.0653	4.4148	0.7785	0.1441
Possum (bottom)	0.4976	3.7311	1.9859	0.5159
Possum Point	0.8535	9.6415	1.5337	0.1973
Public Landing	0.0054	4.9307	0.5873	0.0193
Railroad Bridge	0.8822	7.9900	12.955	0.6746
Sandy Point South	0.2204	4.7515	1.1912	0.7569
Sill	0.8892	1.7235	2.3680	0.5063
Sill (bottom)	0.4632	1.1784	2.2671	0.8180
St. George's Creek	-0.2798	3.4172	2.8722	0.0064

Table 3.4: Type I Error of the Wilcoxon Test applied to the log-transform of samples drawn from the Gamma distribution with parameters α and β using a significance of 0.01.

	β_{rate}	
	1	10
$\alpha_{shape} = 2$	0.2183	0.2207
$\alpha_{shape} = 4$	0.1003	0.0977
$\alpha_{shape} = 10$	0.0407	0.0335
$\alpha_{shape} = 50$	0.0131	0.0145
$\alpha_{shape} = 100$	0.0116	0.0128

Table 3.5: Skew values for a Gamma distributions with parameters α_{shape} and β_{rate} .

	β_{rate}	
	1	10
$\alpha_{shape} = 2$	1.4431	1.2808
$\alpha_{shape} = 4$	0.8033	0.8793
$\alpha_{shape} = 10$	0.6059	0.6356
$\alpha_{shape} = 50$	0.2725	0.3453
$\alpha_{shape} = 100$	0.0633	0.2306

3.3 Salinity Regimes

The Tukey Test results in Table 3.6 display the p-value for each of the comparisons between the regimes. Noticing that all the p-values are greater than 0.01, we fail to reject H_0 and conclude that none of the regimes are significantly different from each other. Because of this fact, the ranking methods are not applicable or significant to the salinity regimes.

Table 3.6: Comparing stations by salinity regime TF = Tidal Fresh (0-0.5 ppt), OH = Oligohaline (0.5-5 ppt), MH = Mesohaline (5-18 ppt) and PH = Polyhaline (18-30 ppt).

p-value	Oxygen 3	Oxygen 5	Turbidity	Chlorophyll	pH
OH-MH	0.8528	0.6444	0.4824	0.9561	0.1745
TF-MH	0.7859	0.7499	0.9999	0.3261	0.7562
TF-OH	0.9992	0.9742	0.6079	0.4327	0.0818

3.4 Ranking

Tables 3.7–3.11 show the results of our ranking methodologies. The first column shows the percent failure of the given station. The rows are ordered by ascending percent failures. Each column shows the results of the respective ranking methodology.

The Tukey Test using the transformed percent failures was conservative, meaning it assigned ties even when station performances were dissimilar. The Bonferoni ranking method produced even more conservative groupings than the Tukey Test. The Bayesian Ranking method is the only method that does not readily lend itself to groupings and, as is shown in the ranking tables, it is not reliable. The problem stems from the simplicity of the method in where the posterior probability distribution comes from a normal distribution using the percent failure as the mean and the standard error from the sample, but since our data is skewed, the assumption is violated. Therefore, the Bayesian Ranking method was not suitable to accurately rank the data. Lastly, we can see that the Benjamini-Hochberg method using the two-proportions Z-test gives a better ranking for the percent fail values than the Benjamini-Hochberg method using the mean values. In fact, the Benjamini-Hochberg method using the mean values seems to be just as unreliable as the Bayesian method; this is because the means are impacted by the skews. In some of the tables, namely chlorophyll, turbidity, and pH, we see a similar pattern between the Benjamini-Hochberg using the means.

The meaning of these results depend on the use of the data. The Benjamini-Hochberg method is the ranking methodology most applicable when one wants the least conservative measurements. However, if one wishes to see larger groupings to have a more general representation of the bay, then either the Tukey or Bonferoni methods would be the most applicable.

Table 3.7: **Oxygen (5mg)** — Ranking of continuous monitoring stations (with 1 being the best) with respect to its percent failure, the Tukey Test, the Bonferroni Test, Benjamini Hochberg Method, and the Bayesian Simple Ranking Method, respectively.

Station Name	% Fail	Tukey	Bonferroni	Benjamini		Bayesian
				% Fail	Mean	
Betterton	0%	1	1	1	4	16
Havre de Grace	0%	1	1	1	5	17
Flats	0.01%	1	1	3	2	18
Goose (surface)	0.04%	1	1	3	3	15
Big Annemessex	1.34%	5	5	5	16	19
AnnapolisCIBS	2.23%	6	6	6	6	12
Manokin	2.62%	6	6	6	9	14
Sill	4.24%	8	8	8	9	10
Budds Landing	5.06%	8	8	8	1	7
Love Point	5.79%	10	10	10	8	2
Iron Plot Head	5.91%	10	10	10	23	13
Fort Howard	6.31%	10	10	10	7	11
St. George's Creek	7.62%	13	13	13	20	9
Sandy Point South	8.14%	14	13	14	15	4
Possum Point	11.98%	15	15	15	9	20
Gratitude Marina	12.60%	15	15	15	22	21
Fort Smallwood	15.09%	17	17	17	9	8
Fort Armistead	16.67%	17	17	17	14	6
Public Landing	18.58%	19	19	19	27	24
Railroad Bridge Crossing	19.50%	20	20	20	18	23
Downs Park	19.83%	20	20	20	16	1
Mattawoman	21.74%	22	22	22	25	5
Otter Point Creek	22.61%	23	23	23	9	22
Indian Head	23.73%	23	23	23	18	3
Masonville Cove	28.07%	25	25	25	20	27
Harness Creek	29.66%	25	25	25	23	26
Sill (bottom)	30.51%	25	25	25	29	25
Corisca River	37.03%	28	28	28	26	29
Mataponi	41.50%	29	29	29	33	30
Newport Creek	41.77%	29	29	29	32	32
Chesapeake Yacht Club	42.37%	29	29	29	29	28
McHenry	43.05%	39	29	29	29	31
Greys Creek	44.95%	33	33	33	27	33
Possum (bottom)	51.50%	34	34	34	34	34
Bishopville	55.78%	35	35	35	35	35
Little Monie	80.21%	36	36	36	36	37
Masonville (bottom)	80.41%	36	36	36	37	36
Goose (bottom)	89.75%	38	38	38	38	38

Table 3.8: **Oxygen (3mg)** — Ranking of continuous monitoring stations (with 1 being the best) with respect to its percent failure, the Tukey Test, the Bonferroni Test, Benjamini Hochberg Method, and the Bayesian Simple Ranking Method, respectively.

Station Name	% Fail	Tukey	Bonferroni	Benjamini		Bayesian
				% Fail	Mean	
Betterton	0%	1	1	1	4	19
Big Annemessex	0%	1	1	1	16	21
Flats	0%	1	1	1	2	22
Goose (surface)	0%	1	1	1	3	23
Havre de Grace	0%	1	1	1	5	24
Iron Plot Head	0%	1	1	1	23	25
Manokin	0%	1	1	1	9	26
Possum Point	0%	1	1	1	9	27
Railroad Bridge Crossing	0%	1	1	1	18	28
Sill	0.06%	1	1	10	9	29
AnnapolisCIBS	0.10%	1	1	10	6	20
Budds Landing	0.14%	12	1	10	1	17
Gratitude Marina	0.17%	12	1	10	22	30
St. George's Creek	0.31%	14	1	14	20	18
Public Landing	0.37%	14	1	15	27	16
Fort Armistead	0.58%	16	16	16	14	15
Indian Head	0.78%	17	17	17	18	13
Fort Howard	0.86%	18	17	17	7	14
Sandy Point South	1.05%	18	19	19	15	8
Love Point	1.09%	18	19	19	8	7
Fort Smallwood	2.10%	21	21	21	9	9
Mattawoman	3.09%	21	21	22	25	4
Sill (bottom)	3.30%	22	23	22	29	5
Possum (bottom)	5.26%	24	24	24	34	1
Otter Point Creek	5.54%	24	24	24	9	12
Corisca River	8.09%	26	26	26	26	3
Masonville Cove	8.52%	26	26	26	20	31
Harness Creek	8.61%	26	26	26	23	11
Downs Park	8.91%	26	26	26	16	2
Newport Creek	10.22%	30	30	30	32	32
Chesapeake Yacht Club	10.73%	31	31	30	29	10
McHenry	13.33%	32	32	32	29	6
Greys Creek	18.04%	33	33	33	27	34
Mataponi	19.06%	33	33	33	33	33
Bishopville	26.41%	35	35	35	35	35
Little Monie	27.86%	35	35	35	36	36
Masonville (bottom)	54.87%	37	37	37	37	37
Goose (bottom)	72.00%	38	38	38	38	38

Table 3.9: **Turbidity** — Ranking of continuous monitoring stations (with 1 being the best) with respect to its percent failure, the Tukey Test, the Bonferroni Test, Benjamini Hochberg Method, and the Bayesian Simple Ranking Method, respectively.

Station Name	% Fail	Tukey	Bonferroni	Benjamini		Bayesian
				% Fail	Mean	
Gooses (surface)	20.31%	1	1	1	3	3
Gooses (bottom)	23.65%	2	2	2	1	24
McHenry	27.01%	3	3	3	3	14
St George's Creek	32.30%	4	4	4	6	1
Manokin	35.40%	5	5	5	5	32
Little Monie	38.85%	6	6	6	1	31
Big Annemessex	42.28%	7	7	7	8	29
Flats	44.55%	7	7	8	30	37
AnnapolisCBIBS	45.45%	9	9	8	13	20
Harness Creek	52.95%	10	10	10	8	23
Fort Smallwood	53.29%	10	10	10	12	15
Havre de Grace	54.94%	10	10	10	30	5
Bishopville	56.94%	13	13	13	6	19
Love Point	57.14%	13	13	13	19	17
Indian Head	58.40%	15	15	15	10	18
Mattawoman	58.65%	15	15	15	11	26
Masonville Cove	59.77%	17	17	17	14	13
Newport Creek	68.24%	18	18	18	18	33
Grey's Creek	69.34%	18	18	18	16	35
Mataponi	73.33%	20	20	20	34	28
Downs Park	79.47%	21	21	21	22	4
Betterton	80.07%	21	21	21	30	36
Public Landing	80.67%	21	21	21	30	34
Fort Armistead	81.22%	21	21	21	16	22
Iron Pot Landing	82.06%	25	25	25	38	30
Otter Point Creek	86.52%	26	26	26	36	12
Sill	86.73%	26	26	26	14	8
Sandy Point South	87.85%	26	26	26	22	2
Possum Point	93.74%	29	29	29	21	10
Gratitude Marina	93.90%	29	29	29	27	21
Fort Howard	94.38%	29	29	29	28	16
Chesapeake Yacht Club	94.51%	29	29	29	20	9
Masonville (bottom)	98.22%	33	33	33	37	25
Corsica River	98.46%	33	33	33	24	11
Railroad Bridge Crossing	98.56%	33	33	33	34	27
Budds Landing	98.65%	33	33	33	29	38
Sill (bottom)	98.76%	33	33	33	25	7
Possum (bottom)	99.89%	38	38	38	26	6

Table 3.10: **Chlorophyll** — Ranking of continuous monitoring stations (with 1 being the best) with respect to its percent failure, the Tukey Test, the Bonferroni Test, Benjamini Hochberg Method, and the Bayesian Simple Ranking Method, respectively.

Station Name	% Fail	Tukey	Bonferroni	Benjamini		Bayesian
				% Fail	Mean	
Big Annemessex	0%	1	1	1	2	9
Mattawoman	0%	1	1	2	9	13
Public Landing	00.01%	1	1	2	11	23
Havre de Grace	00.03%	1	1	2	5	10
Betterton	00.08%	5	1	5	7	21
Manokin	00.10%	6	1	5	3	7
Flats	00.15%	7	1	7	4	3
Indian Head	00.15%	7	1	7	9	12
Love Point	00.22%	9	1	9	12	16
Otter Point Creek	00.24%	9	10	9	15	26
Iron Pot Landing	00.59%	11	11	11	1	25
Little Monie	00.72%	11	12	11	12	8
Mataponi	01.37%	13	13	13	6	20
Sandy Point South	01.44%	13	13	13	19	27
Gooses (surface)	01.62%	13	13	13	14	2
Gooses (bottom)	02.12%	13	13	13	7	1
Railroad Bridge Crossing	02.33%	17	17	17	17	33
Gratitude Marina	03.97%	18	18	18	17	31
AnnapolisCBIBS	04.15%	18	18	18	19	6
McHenry	06.41%	20	20	20	15	4
Fort Howard	08.20%	21	21	21	23	32
St. George's Creek	08.28%	21	21	21	21	5
Downs Park	09.48%	21	21	21	22	22
Masonville (bottom)	10.28%	24	24	24	26	34
Masonville Cove	10.66%	24	24	24	24	18
Fort Smallwood	13.28%	26	26	26	24	15
Sill (bottom)	17.92%	27	27	27	27	36
Fort Armistead	18.48%	27	27	27	27	24
Chesapeake Yacht Club	20.63%	29	29	29	30	30
Harness Creek	20.96%	30	30	29	29	14
Sill	22.61%	30	30	29	30	28
Possum (bottom)	26.45%	32	32	32	30	38
Possum Point	38.02%	33	33	33	33	29
Grey's Creek	40.99%	34	34	34	35	19
Corsica River	42.99%	34	34	34	37	35
Newport Creek	46.31%	36	36	36	33	17
Budds Landing	50.19%	37	37	37	36	37
Bishopville	77.97%	38	38	38	38	11

Table 3.11: **pH** — Ranking of continuous monitoring stations (with 1 being the best) with respect to its percent failure, the Tukey Test, the Bonferroni Test, Benjamini Hochberg Method, and the Bayesian Simple Ranking Method, respectively.

Station Name	% Fail	Tukey	Bonferroni	Benjamini		Bayesian
				% Fail	Mean	
Big Annemessex	0%	1	1	1	18	29
Iron Pot Landing	0%	1	1	1	4	30
Little Monie	0%	1	1	1	2	31
Manokin	0%	1	1	1	14	32
Mataponi	0%	1	1	1	1	28
Newport Creek	0%	1	1	1	7	33
Public Landing	0%	1	1	1	12	34
Railroad Bridge Crossing	00.05%	1	1	8	3	27
Gooses (bottom)	00.41%	9	9	9	9	24
Grey's Creek	00.43%	9	10	9	8	35
Masonville (bottom)	02.63%	11	11	11	6	25
Bishopville	04.62%	12	12	12	10	19
Havre de Grace	05.84%	13	13	13	11	5
Indian Head	07.99%	14	14	14	5	18
Betterton	08.15%	14	14	14	12	36
Gratitude Marina	08.39%	14	14	14	20	21
Love Point	09.44%	17	17	17	26	17
Sandy Point South	09.64%	18	17	18	21	2
Mattawoman	13.53%	19	19	19	14	26
McHenry	13.88%	19	19	19	17	14
AnnapolisCBIBS	14.45%	19	19	19	28	20
Possum (bottom)	17.49%	22	22	22	19	6
Sill (bottom)	18.39%	22	22	22	24	7
Downs Park	19.40%	22	22	24	26	4
Fort Armistead	20.26%	25	25	24	29	22
Masonville Cove	20.61%	26	26	26	21	13
Corsica River	20.92%	26	26	26	24	11
Sill	24.92%	28	28	28	30	8
Fort Smallwood	25.13%	28	28	28	32	15
Chesapeake Yacht Club	26.21%	28	28	28	21	9
Harness Creek	26.36%	28	28	28	30	23
Gooses (surface)	26.82%	28	28	28	35	3
St George's Creek	28.26%	33	33	33	35	1
Fort Howard	28.37%	33	33	33	32	16
Possum Point	30.89%	35	35	35	32	10
Otter Point Creek	31.07%	35	36	35	16	12
Flats	48.94%	37	37	37	38	37
Budds Landing	59.38%	38	38	38	37	38

4 Limitations of Project

Our results only apply for the tests we conducted on shallow-water stations and may not apply to the Chesapeake Bay as a whole. For further inquiry, www.eyesonthebay.net.

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