4.4.69

$$\int z^n \arctan z\,dz=\frac{z^{n+1}}{n+1}\arctan z-\frac{1}{n+1}\int\frac{z^{n+1}}{1+z^2}dz \qquad (n\neq-1)$$

4.4.70

$$\int z \operatorname{arccot} z\,dz=\frac{1}{2}(1+z^2)\operatorname{arccot} z+\frac{z}{2}$$

4.4.71

$$\int z^n \operatorname{arccot} z\,dz=\frac{z^{n+1}}{n+1}\operatorname{arccot} z+\frac{1}{n+1}\int\frac{z^{n+1}}{1+z^2}dz \qquad (n\neq-1)$$

4.5. Hyperbolic Functions

Definitions

4.5.1 $\sinh z=\dfrac{e^z-e^{-z}}{2} \qquad (z=x+iy)$

4.5.2 $\cosh z=\dfrac{e^z+e^{-z}}{2}$

4.5.3 $\tanh z=\sinh z/\cosh z$

4.5.4 $\operatorname{csch} z=1/\sinh z$

4.5.5 $\operatorname{sech} z=1/\cosh z$

4.5.6 $\coth z=1/\tanh z$

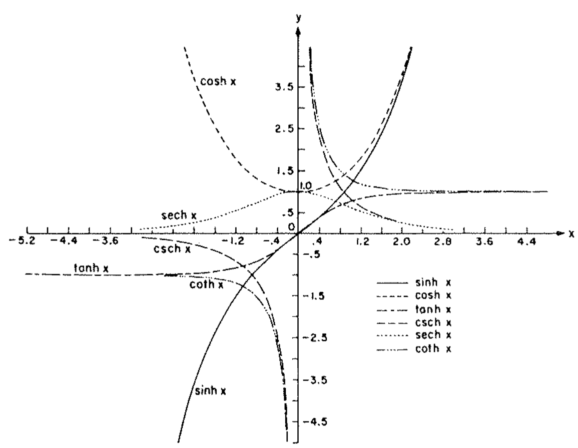

FIGURE 4.6. *Hyperbolic functions.*

Relation to Circular Functions (see 4.3.49 to 4.3.54)

Hyperbolic formulas can be derived from trigonometric identities by replacing z by iz

4.5.7 $\sinh z=-i\sin iz$

4.5.8 $\cosh z=\cos iz$

4.5.9 $\tanh z=-i\tan iz$

4.5.10 $\operatorname{csch} z=i\csc iz$

4.5.11 $\operatorname{sech} z=\sec iz$

4.5.12 $\coth z=i\cot iz$

Periodic Properties

4.5.13 $\sinh(z+2k\pi i)=\sinh z \qquad (k \text{ any integer})$

4.5.14 $\cosh(z+2k\pi i)=\cosh z$

4.5.15 $\tanh(z+k\pi i)=\tanh z$

Relations Between Hyperbolic Functions

4.5.16 $\cosh^2 z-\sinh^2 z=1$

4.5.17 $\tanh^2 z+\operatorname{sech}^2 z=1$

4.5.18 $\coth^2 z-\operatorname{csch}^2 z=1$

4.5.19 $\cosh z+\sinh z=e^z$

4.5.20 $\cosh z-\sinh z=e^{-z}$

Negative Angle Formulas

4.5.21 $\sinh(-z)=-\sinh z$

4.5.22 $\cosh(-z)=\cosh z$

4.5.23 $\tanh(-z)=-\tanh z$

Addition Formulas

4.5.24 $\sinh(z_1+z_2)=\sinh z_1\cosh z_2+\cosh z_1\sinh z_2$

4.5.25 $\cosh(z_1+z_2)=\cosh z_1\cosh z_2+\sinh z_1\sinh z_2$

4.5.26 $\tanh(z_1+z_2)=(\tanh z_1+\tanh z_2)/(1+\tanh z_1\tanh z_2)$

4.5.27 $\coth(z_1+z_2)=(\coth z_1\coth z_2+1)/(\coth z_2+\coth z_1)$

Half-Angle Formulas

4.5.28

$$\sinh\frac{z}{2}=\left(\frac{\cosh z-1}{2}\right)^{\frac{1}{2}}$$

4.5.29

$$\cosh \frac{z}{2}=\left(\frac{\cosh z+1}{2}\right)^{\frac{1}{2}}$$

4.5.30

$$\tanh \frac{z}{2}=\left(\frac{\cosh z-1}{\cosh z+1}\right)^{\frac{1}{2}}=\frac{\cosh z-1}{\sinh z}=\frac{\sinh z}{\cosh z+1}$$

Multiple-Angle Formulas

4.5.31 $$\sinh 2z=2 \sinh z \cosh z=\frac{2 \tanh z}{1-\tanh^2 z}$$

4.5.32 $$\cosh 2z=2 \cosh^2 z-1=2 \sinh^2 z+1=\cosh^2 z+\sinh^2 z$$

4.5.33 $$\tanh 2z=\frac{2 \tanh z}{1+\tanh^2 z}$$

4.5.34 $$\sinh 3z=3 \sinh z+4 \sinh^3 z$$

4.5.35 $$\cosh 3z=-3 \cosh z+4 \cosh^3 z$$

4.5.36 $$\sinh 4z=4 \sinh^3 z \cosh z+4 \cosh^3 z \sinh z$$

4.5.37 $$\cosh 4z=\cosh^4 z+6 \sinh^2 z \cosh^2 z+\sinh^4 z$$

Products of Hyperbolic Sines and Cosines

4.5.38 $$2 \sinh z_1 \sinh z_2=\cosh (z_1+z_2)-\cosh (z_1-z_2)$$

4.5.39 $$2 \cosh z_1 \cosh z_2=\cosh (z_1+z_2)+\cosh (z_1-z_2)$$

4.5.40 $$2 \sinh z_1 \cosh z_2=\sinh (z_1+z_2)+\sinh (z_1-z_2)$$

Addition and Subtraction of Two Hyperbolic Functions

4.5.41

$$\sinh z_1+\sinh z_2=2 \sinh \left(\frac{z_1+z_2}{2}\right) \cosh \left(\frac{z_1-z_2}{2}\right)$$

4.5.42

$$\sinh z_1-\sinh z_2=2 \cosh \left(\frac{z_1+z_2}{2}\right) \sinh \left(\frac{z_1-z_2}{2}\right)$$

4.5.43

$$\cosh z_1+\cosh z_2=2 \cosh \left(\frac{z_1+z_2}{2}\right) \cosh \left(\frac{z_1-z_2}{2}\right)$$

4.5.44

$$\cosh z_1-\cosh z_2=2 \sinh \left(\frac{z_1+z_2}{2}\right) \sinh \left(\frac{z_1-z_2}{2}\right)$$

4.5.45

$$\tanh z_1+\tanh z_2=\frac{\sinh (z_1+z_2)}{\cosh z_1 \cosh z_2}$$

4.5.46

$$\coth z_1+\coth z_2=\frac{\sinh (z_1+z_2)}{\sinh z_1 \sinh z_2}$$

Relations Between Squares of Hyperbolic Sines and Cosines

4.5.47

$$\sinh^2 z_1-\sinh^2 z_2=\sinh (z_1+z_2) \sinh (z_1-z_2)=\cosh^2 z_1-\cosh^2 z_2$$

4.5.48

$$\sinh^2 z_1+\cosh^2 z_2=\cosh (z_1+z_2) \cosh (z_1-z_2)=\cosh^2 z_1+\sinh^2 z_2$$

Hyperbolic Functions in Terms of Real and Imaginary Parts

$$(z=x+iy)$$

4.5.49 $$\sinh z=\sinh x \cos y+i \cosh x \sin y$$

4.5.50 $$\cosh z=\cosh x \cos y+i \sinh x \sin y$$

4.5.51 $$\tanh z=\frac{\sinh 2x+i \sin 2y}{\cosh 2x+\cos 2y}$$

4.5.52 $$\coth z=\frac{\sinh 2x-i \sin 2y}{\cosh 2x-\cos 2y}$$

De Moivre's Theorem

4.5.53 $$(\cosh z+\sinh z)^n=\cosh nz+\sinh nz$$

Modulus and Phase (Argument) of Hyperbolic Functions

4.5.54 $$|\sinh z|=(\sinh^2 x+\sin^2 y)^{\frac{1}{2}}=[\tfrac{1}{2}(\cosh 2x-\cos 2y)]^{\frac{1}{2}}$$

4.5.55 $$\arg \sinh z=\arctan (\coth x \tan y)$$

4.5.56 $$|\cosh z|=(\sinh^2 x+\cos^2 y)^{\frac{1}{2}}=[\tfrac{1}{2}(\cosh 2x+\cos 2y)]^{\frac{1}{2}}$$

4.5.57 $$\arg \cosh z=\arctan (\tanh x \tan y)$$

4.5.58 $$|\tanh z|=\left(\frac{\cosh 2x-\cos 2y}{\cosh 2x+\cos 2y}\right)^{\frac{1}{2}}$$

4.5.59 $$\arg \tanh z=\arctan \left(\frac{\sin 2y}{\sinh 2x}\right)$$

4.5.60 Relations Between Hyperbolic (or Inverse Hyperbolic) Functions

	$\sinh x=a$	$\cosh x=a$	$\tanh x=a$	$\operatorname{csch} x=a$	$\operatorname{sech} x=a$	$\coth x=a$
$\sinh x$	a	$(a^2-1)^{\frac{1}{2}}$	$a(1-a^2)^{-\frac{1}{2}}$	a^{-1}	$a^{-1}(1-a^2)^{\frac{1}{2}}$	$(a^2-1)^{-\frac{1}{2}}$
$\cosh x$	$(1+a^2)^{\frac{1}{2}}$	a	$(1-a^2)^{-\frac{1}{2}}$	$a^{-1}(1+a^2)^{\frac{1}{2}}$	a^{-1}	$a(a^2-1)^{-\frac{1}{2}}$
$\tanh x$	$a(1+a^2)^{-\frac{1}{2}}$	$a^{-1}(a^2-1)^{\frac{1}{2}}$	a	$(1+a^2)^{-\frac{1}{2}}$	$(1-a^2)^{\frac{1}{2}}$	a^{-1}
$\operatorname{csch} x$	a^{-1}	$(a^2-1)^{-\frac{1}{2}}$	$a^{-1}(1-a^2)^{\frac{1}{2}}$	a	$a(1-a^2)^{-\frac{1}{2}}$	$(a^2-1)^{\frac{1}{2}}$
$\operatorname{sech} x$	$(1+a^2)^{-\frac{1}{2}}$	a^{-1}	$(1-a^2)^{\frac{1}{2}}$	$a(1+a^2)^{-\frac{1}{2}}$	a	$a^{-1}(a^2-1)^{\frac{1}{2}}$
$\coth x$	$a^{-1}(a^2+1)^{\frac{1}{2}}$	$a(a^2-1)^{-\frac{1}{2}}$	a^{-1}	$(1+a^2)^{\frac{1}{2}}$	$(1-a^2)^{-\frac{1}{2}}$	a

Illustration: If $\sinh x=a$, $\coth x=a^{-1}(a^2+1)^{\frac{1}{2}}$

$\operatorname{arcsech} a=\operatorname{arccoth}(1-a^2)^{-\frac{1}{2}}$

4.5.61 Special Values of the Hyperbolic Functions

z	0	$\frac{\pi}{2}i$	πi	$\frac{3\pi}{2}i$	∞
$\sinh z$	0	i	0	$-i$	∞
$\cosh z$	1	0	-1	0	∞
$\tanh z$	0	∞i	0	$-\infty i$	1
$\operatorname{csch} z$	∞	$-i$	∞	i	0
$\operatorname{sech} z$	1	∞	-1	∞	0
$\coth z$	∞	0	∞	0	1

Series Expansions

4.5.62 $$\sinh z=z+\frac{z^3}{3!}+\frac{z^5}{5!}+\frac{z^7}{7!}+\ldots \qquad (|z|<\infty)$$

4.5.63 $$\cosh z=1+\frac{z^2}{2!}+\frac{z^4}{4!}+\frac{z^6}{6!}+\ldots \qquad (|z|<\infty)$$

4.5.64 $$\tanh z=z-\frac{z^3}{3}+\frac{2}{15}z^5-\frac{17}{315}z^7$$

* $$+\ldots+\frac{2^{2n}(2^{2n}-1)B_{2n}}{(2n)!}z^{2n-1}+\ldots \qquad \left(|z|<\frac{\pi}{2}\right)$$

4.5.65

$$\operatorname{csch} z=\frac{1}{z}-\frac{z}{6}+\frac{7}{360}z^3-\frac{31}{15120}z^5+\ldots$$

$$-\frac{2(2^{2n-1}-1)B_{2n}}{(2n)!}z^{2n-1}+\ldots \qquad (|z|<\pi)$$

4.5.66

$$\operatorname{sech} z=1-\frac{z^2}{2}+\frac{5}{24}z^4-\frac{61}{720}z^6+\ldots+\frac{E_{2n}}{(2n)!}z^{2n}+\ldots \qquad \left(|z|<\frac{\pi}{2}\right)$$

4.5.67

$$\coth z=\frac{1}{z}+\frac{z}{3}-\frac{z^3}{45}+\frac{2}{945}z^5-\ldots+\frac{2^{2n}B_{2n}}{(2n)!}z^{2n-1}+\ldots \qquad (|z|<\pi)$$

where B_n and E_n are the nth Bernoulli and Euler numbers, see chapter **23**.

Infinite Products

4.5.68 $$\sinh z=z\prod_{k=1}^{\infty}\left(1+\frac{z^2}{k^2\pi^2}\right)$$

4.5.69 $$\cosh z=\prod_{k=1}^{\infty}\left[1+\frac{4z^2}{(2k-1)^2\pi^2}\right]$$

Continued Fraction

4.5.70 $$\tanh z=\frac{z}{1+}\,\frac{z^2}{3+}\,\frac{z^2}{5+}\,\frac{z^2}{7+}\cdots \qquad \left(z\neq\frac{\pi}{2}i\pm n\pi i\right)$$

Differentiation Formulas

4.5.71 $$\frac{d}{dz}\sinh z=\cosh z$$

4.5.72 $$\frac{d}{dz}\cosh z=\sinh z$$

4.5.73 $$\frac{d}{dz}\tanh z=\operatorname{sech}^2 z$$

4.5.74 $$\frac{d}{dz}\operatorname{csch} z=-\operatorname{csch} z\coth z$$

*See page II.

4.5.75 $\frac{d}{dz}\operatorname{sech} z = -\operatorname{sech} z \tanh z$

4.5.76 $\frac{d}{dz}\coth z = -\operatorname{csch}^2 z$

Integration Formulas

4.5.77 $\int \sinh z\, dz = \cosh z$

4.5.78 $\int \cosh z\, dz = \sinh z$

4.5.79 $\int \tanh z\, dz = \ln \cosh z$

4.5.80 $\int \operatorname{csch} z\, dz = \ln \tanh \frac{z}{2}$

4.5.81 $\int \operatorname{sech} z\, dz = \arctan (\sinh z)$

4.5.82 $\int \coth z\, dz = \ln \sinh z$

4.5.83

$$\int z^n \sinh z\, dz = z^n \cosh z - n\int z^{n-1}\cosh z\, dz$$

4.5.84

$$\int z^n \cosh z\, dz = z^n \sinh z - n\int z^{n-1}\sinh z\, dz$$

4.5.85

$$\int \sinh^m z \cosh^n z\, dz = \frac{1}{m+n}\sinh^{m+1} z \cosh^{n-1} z + \frac{n-1}{m+n}\int \sinh^m z \cosh^{n-2} z\, dz$$

$$= \frac{1}{m+n}\sinh^{m-1} z \cosh^{n+1} z - \frac{m-1}{m+n}\int \sinh^{m-2} z \cosh^n z\, dz \qquad (m+n\neq 0)$$

4.5.86

$$\int \frac{dz}{\sinh^m z \cosh^n z} = \frac{-1}{m-1}\,\frac{1}{\sinh^{m-1} z \cosh^{n-1} z} - \frac{m+n-2}{m-1}\int \frac{dz}{\sinh^{m-2} z \cosh^n z} \qquad (m\neq 1)$$

$$= \frac{1}{n-1}\,\frac{1}{\sinh^{m-1} z \cosh^{n-1} z} + \frac{m+n-2}{n-1}\int \frac{dz}{\sinh^m z \cosh^{n-2} z} \qquad (n\neq 1)$$

4.5.87

$$\int \tanh^n z\, dz = -\frac{\tanh^{n-1} z}{n-1} + \int \tanh^{n-2} z\, dz \qquad (n\neq 1)$$

4.5.88

$$\int \coth^n z\, dz = -\frac{\coth^{n-1} z}{n-1} + \int \coth^{n-2} z\, dz \qquad (n\neq 1)$$

(See chapters **5** and **7** for other integrals involving hyperbolic functions.)

4.6. Inverse Hyperbolic Functions

Definitions

4.6.1 $\operatorname{arcsinh} z = \int_0^z \frac{dt}{(1+t^2)^{\frac{1}{2}}} \qquad (z=x+iy)$

4.6.2 $\operatorname{arccosh} z = \int_1^z \frac{dt}{(t^2-1)^{\frac{1}{2}}}$

4.6.3 $\operatorname{arctanh} z = \int_0^z \frac{dt}{1-t^2}$

The paths of integration must not cross the following cuts.

4.6.1 imaginary axis from $-i\infty$ to $-i$ and i to $i\infty$

4.6.2 real axis from $-\infty$ to $+1$

4.6.3 real axis from $-\infty$ to -1 and $+1$ to $+\infty$

Inverse hyperbolic functions are also written $\sinh^{-1} z$, arsinh z, $\mathscr{A}r \sinh z$, etc.

4.6.4 $\operatorname{arccsch} z = \operatorname{arcsinh} 1/z$

4.6.5 $\operatorname{arcsech} z = \operatorname{arccosh} 1/z$

4.6.6 $\operatorname{arccoth} z = \operatorname{arctanh} 1/z$

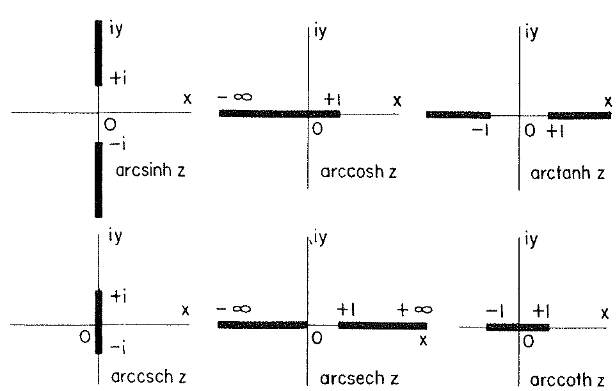

FIGURE 4.7. *Branch cuts for inverse hyperbolic functions.*

4.6.7 $\operatorname{arctanh} z = \operatorname{arccoth} z \pm \frac{1}{2}\pi i$

(see **4.5.60**) (according as $\mathscr{I}z \gtrless 0$)

Fundamental Property

The general solutions of the equations

$$z = \sinh t$$
$$z = \cosh t$$
$$z = \tanh t$$

are respectively

4.6.8 $t = \operatorname{Arcsinh} z = (-1)^k \operatorname{arcsinh} z + k\pi i$

4.6.9 $t = \operatorname{Arccosh} z = \pm \operatorname{arccosh} z + 2k\pi i$

4.6.10 $t = \operatorname{Arctanh} z = \operatorname{arctanh} z + k\pi i$

(k, integer)

Functions of Negative Arguments

4.6.11 $\operatorname{arcsinh}(-z) = -\operatorname{arcsinh} z$

***4.6.12** $\operatorname{arccosh}(-z) = \pi i - \operatorname{arccosh} z$

4.6.13 $\operatorname{arctanh}(-z) = -\operatorname{arctanh} z$

Relation to Inverse Circular Functions (see **4.4.20** to **4.4.25**)

Hyperbolic identities can be derived from trigonometric identities by replacing z by iz.

4.6.14 $\operatorname{Arcsinh} z = -i \operatorname{Arcsin} iz$

4.6.15 $\operatorname{Arccosh} z = \pm i \operatorname{Arccos} z$

4.6.16 $\operatorname{Arctanh} z = -i \operatorname{Arctan} iz$

4.6.17 $\operatorname{Arccsch} z = i \operatorname{Arccsc} iz$

4.6.18 $\operatorname{Arcsech} z = \pm i \operatorname{Arcsec} z$

4.6.19 $\operatorname{Arccoth} z = i \operatorname{Arccot} iz$

Logarithmic Representations

4.6.20 $\operatorname{arcsinh} x = \ln [x + (x^2+1)^{\frac{1}{2}}]$

4.6.21 $\operatorname{arccosh} x = \ln [x + (x^2-1)^{\frac{1}{2}}]$ $(x \geq 1)$

4.6.22 $\operatorname{arctanh} x = \frac{1}{2} \ln \frac{1+x}{1-x}$ $(0 \leq x^2 < 1)$

4.6.23 $\operatorname{arccsch} x = \ln \left[\frac{1}{x} + \left(\frac{1}{x^2}+1\right)^{\frac{1}{2}}\right]$ $(x \neq 0)$

4.6.24 $\operatorname{arcsech} x = \ln \left[\frac{1}{x} + \left(\frac{1}{x^2}-1\right)^{\frac{1}{2}}\right]$ $(0 < x \leq 1)$

4.6.25 $\operatorname{arccoth} x = \frac{1}{2} \ln \frac{x+1}{x-1}$ $(x^2 > 1)$

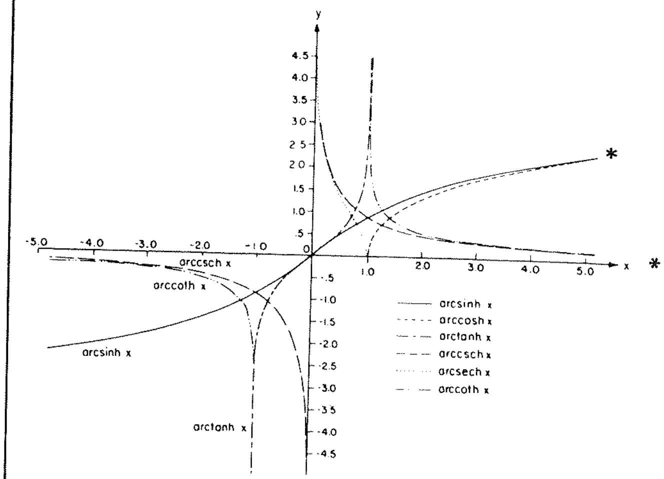

FIGURE 4.8. *Inverse hyperbolic functions.*

Addition and Subtraction of Two Inverse Hyperbolic Functions

4.6.26

$$\operatorname{Arcsinh} z_1 \pm \operatorname{Arcsinh} z_2 = \operatorname{Arcsinh} [z_1(1+z_2^2)^{\frac{1}{2}} \pm z_2(1+z_1^2)^{\frac{1}{2}}]$$

4.6.27

$$\operatorname{Arccosh} z_1 \pm \operatorname{Arccosh} z_2 = \operatorname{Arccosh} \{z_1 z_2 \pm [(z_1^2-1)(z_2^2-1)]^{\frac{1}{2}}\}$$

4.6.28

$$\operatorname{Arctanh} z_1 \pm \operatorname{Arctanh} z_2 = \operatorname{Arctanh} \left(\frac{z_1 \pm z_2}{1 \pm z_1 z_2}\right)$$

4.6.29

$$\operatorname{Arcsinh} z_1 \pm \operatorname{Arccosh} z_2 = \operatorname{Arcsinh} \{z_1 z_2 \pm [(1+z_1^2)(z_2^2-1)]^{\frac{1}{2}}\}$$
$$= \operatorname{Arccosh} [z_2(1+z_1^2)^{\frac{1}{2}} \pm z_1(z_2^2-1)^{\frac{1}{2}}]$$

4.6.30

$$\operatorname{Arctanh} z_1 \pm \operatorname{Arccoth} z_2 = \operatorname{Arctanh} \left(\frac{z_1 z_2 \pm 1}{z_2 \pm z_1}\right)$$
$$= \operatorname{Arccoth} \left(\frac{z_2 \pm z_1}{z_1 z_2 \pm 1}\right)$$

*See page II.

Series Expansions

4.6.31

$$\operatorname{arcsinh} z = z - \frac{1}{2\cdot 3} z^3 + \frac{1\cdot 3}{2\cdot 4\cdot 5} z^5 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7} z^7 + \ldots \qquad (|z|<1)$$

$$= \ln 2z + \frac{1}{2\cdot 2z^2} - \frac{1\cdot 3}{2\cdot 4\cdot 4z^4} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 6z^6} - \ldots \qquad (|z|>1)$$

4.6.32

$$\operatorname{arccosh} z = \ln 2z - \frac{1}{2\cdot 2z^2} - \frac{1\cdot 3}{2\cdot 4\cdot 4z^4} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 6z^6} - \ldots \qquad (|z|>1)$$

4.6.33 $$\operatorname{arctanh} z = z + \frac{z^3}{3} + \frac{z^5}{5} + \frac{z^7}{7} + \ldots \qquad (|z|<1)$$

4.6.34 $$\operatorname{arccoth} z = \frac{1}{z} + \frac{1}{3z^3} + \frac{1}{5z^5} + \frac{1}{7z^7} + \ldots \qquad (|z|>1)$$

Continued Fractions

4.6.35 $$\operatorname{arctanh} z = \frac{z}{1-}\,\frac{z^2}{3-}\,\frac{4z^2}{5-}\,\frac{9z^2}{7-}\ldots$$

(z in the cut plane of Figure 4.7.)

4.6.36

$$\frac{\operatorname{arcsinh} z}{\sqrt{1+z^2}} = \frac{z}{1+}\,\frac{1\cdot 2z^2}{3+}\,\frac{1\cdot 2z^2}{5+}\,\frac{3\cdot 4z^2}{7+}\,\frac{3\cdot 4z^2}{9+}\ldots$$

Differentiation Formulas

4.6.37 $$\frac{d}{dz}\operatorname{arcsinh} z = (1+z^2)^{-\frac{1}{2}}$$

4.6.38 $$\frac{d}{dz}\operatorname{arccosh} z = (z^2-1)^{-\frac{1}{2}}$$

4.6.39 $$\frac{d}{dz}\operatorname{arctanh} z = (1-z^2)^{-1}$$

4.6.40 $$\frac{d}{dz}\operatorname{arccsch} z = \mp\frac{1}{z(1+z^2)^{\frac{1}{2}}}$$

(according as $\mathcal{R}z \gtrless 0$)

4.6.41 $$\frac{d}{dz}\operatorname{arcsech} z = \mp\frac{1}{z(1-z^2)^{\frac{1}{2}}}$$

4.6.42 $$\frac{d}{dz}\operatorname{arccoth} z = (1-z^2)^{-1}$$

Integration Formulas

4.6.43 $$\int \operatorname{arcsinh} z\, dz = z \operatorname{arcsinh} z - (1+z^2)^{\frac{1}{2}}$$

4.6.44 $$\int \operatorname{arccosh} z\, dz = z \operatorname{arccosh} z - (z^2-1)^{\frac{1}{2}}$$

4.6.45 $$\int \operatorname{arctanh} z\, dz = z \operatorname{arctanh} z + \tfrac{1}{2}\ln(1-z^2)$$

4.6.46 $$\int \operatorname{arccsch} z\, dz = z \operatorname{arccsch} z \pm \operatorname{arcsinh} z \qquad *$$

(according as $\mathcal{R}z \gtrless 0$)

4.6.47 $$\int \operatorname{arcsech} z\, dz = z \operatorname{arcsech} z \pm \arcsin z \qquad *$$

4.6.48 $$\int \operatorname{arccoth} z\, dz = z \operatorname{arccoth} z + \tfrac{1}{2}\ln(z^2-1)$$

4.6.49

$$\int z \operatorname{arcsinh} z\, dz = \frac{2z^2+1}{4}\operatorname{arcsinh} z - \frac{z}{4}(z^2+1)^{\frac{1}{2}}$$

4.6.50

$$\int z^n \operatorname{arcsinh} z\, dz = \frac{z^{n+1}}{n+1}\operatorname{arcsinh} z - \frac{1}{n+1}\int \frac{z^{n+1}}{(1+z^2)^{\frac{1}{2}}}\, dz \qquad (n\neq -1)$$

4.6.51

$$\int z \operatorname{arccosh} z\, dz = \frac{2z^2-1}{4}\operatorname{arccosh} z - \frac{z}{4}(z^2-1)^{\frac{1}{2}}$$

4.6.52

$$\int z^n \operatorname{arccosh} z\, dz = \frac{z^{n+1}}{n+1}\operatorname{arccosh} z - \frac{1}{n+1}\int \frac{z^{n+1}}{(z^2-1)^{\frac{1}{2}}}\, dz \qquad (n\neq -1)$$

4.6.53

$$\int z \operatorname{arctanh} z\, dz = \frac{z^2-1}{2}\operatorname{arctanh} z + \frac{z}{2}$$

4.6.54

$$\int z^n \operatorname{arctanh} z\, dz = \frac{z^{n+1}}{n+1}\operatorname{arctanh} z - \frac{1}{n+1}\int \frac{z^{n+1}}{1-z^2}\, dz \qquad (n\neq -1)$$

4.6.55

$$\int z \operatorname{arccsch} z\, dz = \frac{z^2}{2}\operatorname{arccsh} z \pm \frac{1}{2}(1+z^2)^{\frac{1}{2}} \qquad *$$

(according as $\mathcal{R}z \gtrless 0$)

4.6.56

$$\int z^n \operatorname{arccsch} z\, dz = \frac{z^{n+1}}{n+1}\operatorname{arccsch} z \pm \frac{1}{n+1}\int \frac{z^n}{(z^2+1)^{\frac{1}{2}}}\, dz \qquad *$$

$(n\neq -1)$

*See page II.