ENCH 630: Transport Phenomena – Problem Set 3

Introduction to COMSOL Multiphysics

Consider the entrance region in a cylindrical tube which is 1 cm in diameter and 25 cm in length, and where liquid water is flowing inside the tube. At the entrance of the tube, the fluid is in uniform plug flow in the axial direction with a velocity of v_{inlet} . The outlet of the tube is at 1 atm absolute pressure. The no-slip condition applies at the tube walls, and the entire system can be assumed to be isothermal at 298 K and to be at steady state. Determine using numerical calculations performed by COMSOL Multiphysics the fluid velocity inside the tube as a function of position for a variety of values for v_{inlet}. Repeat your calculations using liquids other than liquid water by employing appropriate physical properties (*i.e.*, by changing the viscosity and density).

When performing your calculations make sure that v_{inlet} is selected so that laminar flow applies everywhere inside the tube. Show using graphs of your results the boundary region at the tube wall and the inviscid fluid core at the center of the tube for several different types of liquids and for particular choices of v_{inlet}. Also make a plot of pressure versus distance along the tube centerline for several values of v_{inlet} and verify that the Hagen Poiseuille law is approached asymptotically inside the tube.

A common dimensionless correlation for the length of tube needed to establish a fully developed parabolic velocity profile (denoted as L_e) when starting with plug flow at the tube inlet is given by the following empirical relation:

$$L_e / D = 0.035 \text{ Re} \qquad (1)$$

In eq. 1, D is the tube diameter and Re is the Reynolds number. However, it is unclear in the above relation what the criterion is for establishing fully developed flow, and it appears that different criterion will yield different numerical constants in that equation.. Consequently, use your "numerical experiments" performed as described above to develop a more general entrance length correlation of the following form:

$$z / D = f\,(\text{Re},\ v_{\text{CL}}/v_{\text{CL.max}}) \qquad (2)$$

In eq. 2, $v_{\text{CL}}/v_{\text{CL,max}}$ is the ratio of the centerline fluid velocity at the axial location z in the tube to the maximum centerline fluid velocity for an infinitely long tube, and f is a function that you will determine empirically. Note that eq. 2 includes eq. 1 as a subcase that corresponds to a particular choice of $v_{\text{CL}}/v_{\text{CL,max}}$ which is implied when using eq. 1. Note also that, according to the analytical solution for fully developed laminar flow in a tube of circular cross section the maximum centerline fluid velocity is given by $v_{\text{CL,max}} = 2 * v_{\text{inlet}}$.

One numerical artifact that you may need to consider in this problem is that, at the location $z = 0$, it is not possible to simultaneously enforce the conditions that $v = v_{\text{inlet}}$ and that there is no slip at the wall. Consequently, unless a very fine finite element mesh is employed, the observed value of $v_{\text{CL,max}}$ for a long column will be somewhat less that the theoretical value of $2 * v_{\text{inlet}}$. For this reason you may want to use the observed value of $v_{\text{CL,max}}$ rather the theoretical value when you develop your correlation.

A three part tutorial which provides details on how to solve this problem is available on the YouTube channel associated with the ENCH 630 course at the following link:

https://www.youtube.com/channel/UCXBBPN_Kypq5dcbwyQXiHQA

You may also find it useful to view the following YouTube video which contains some background information related to this problem.

https://www.youtube.com/watch?v=y0hvefFLWFM