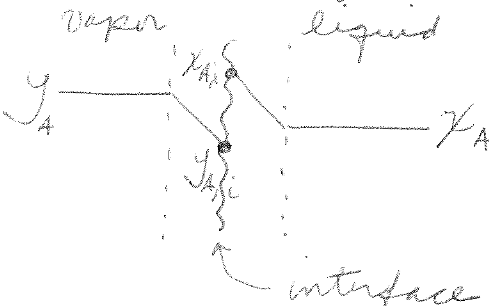
Design of a packed column, e.g., a "stripper" where solute "A" is transferred from a liquid to a vapor.

Combination of mass transfer resistances:

N_A taken as positive if flux is into vapor



Liquid phase: $N_A = k_x(x_A - x_{A,i})$

Vapor phase: $N_A = k_y(y_{A,i} - y_A)$

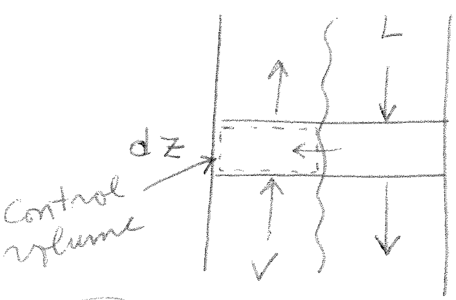
Assume equilibrium at interface: $y_{A,i} = K_A x_{A,i}$

Combine above 3 eqns. to eliminate $y_{A,i}$ and $x_{A,i}$:

$$N_A = \underbrace{\left(\frac{1}{k_y} + \frac{K_A}{k_x}\right)}_{K_{OY}} \left(\underset{\uparrow\; K_A x_A}{y_A^*} - y_A\right) = \underbrace{\left(\frac{1}{K_A k_y} + \frac{1}{k_x}\right)}_{K_{OX}} \left(x_A - \underset{\uparrow\; \frac{y_A}{K_A}}{x_A^*}\right)$$

"overall" driving forces and mass transfer coefficients

Material balance in column:



$$\underbrace{V\,dy_A}_{\text{outflow - inflow by convection of vapor}} = \underbrace{K_{OY}\left(y_A^* - y_A\right)}_{\text{inflow across interface per unit area}}\; a \; \underbrace{A_c\,dz}_{\text{volume of column under consideration}}$$

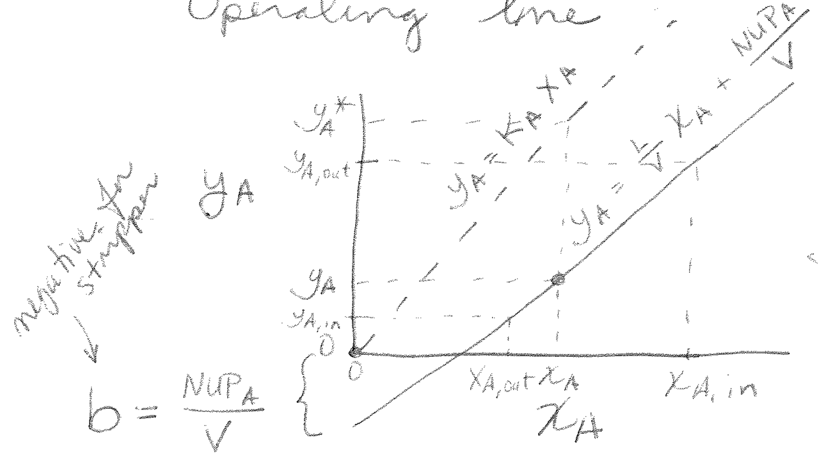
a: contact area per unit total volume (i.e., vapor plus liquid)

Rearrange:

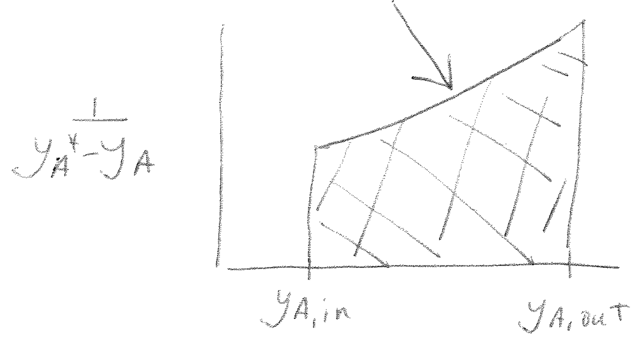
$$\underbrace{\int_{y_{A,in}}^{y_{A,out}} \frac{dy_A}{y_A^* - y_A}}_{NTU_{OY}} = \int_0^{h} \frac{dz}{\underbrace{\left(\frac{V}{K_{OY} A_c\, a}\right)}_{HTU_{OY}}} = \frac{h}{HTU_{OY}}$$

h: column height

Operating line



$$NTU_{OY} = \int_{y_{A,in}}^{y_{A,out}} \frac{dy_A}{y_A^* - y_A}$$



Analytical result for NTU_{OY} for straight equilibrium and operating lines

Equilibrium: $y_A^* = K_A x_A$

Operating line: $y_A = L/V\, x_A + b$ or $x_A = \dfrac{y_A - b}{L/V}$

Combining: $y_A^* = (y_A - b)\left(\dfrac{K_A V}{L}\right)$

Substitute to get:

$$NTU_{OY} = \int_{y_A^{in}}^{y_A^{out}} \frac{dy_A}{(y_A - b)\frac{K_A V}{L} - y_A}$$

$$= \frac{1}{1 - \frac{K_A V}{L}} \ln\left(\frac{y_{A,in}\left(\frac{K_A V}{L} - 1\right) - b\frac{K_A V}{L}}{y_{A,out}\left(\frac{K_A V}{L} - 1\right) - b\frac{K_A V}{L}}\right)$$

This form, however, is not very convenient for a stripper. Instead, it is more convenient to have NTU_{OY} a function of $x_{A,in}$, $x_{A,out}$, and $y_{A,in}$, where often $y_{A,in} = 0$. An overall balance on the process yields: $L(x_{A,in} - x_{A,out}) = V[y_{A,out} - y_{A,in}]$ or $y_{A,out} = \frac{L}{V}(x_{A,in} - x_{A,out}) + y_{A,in}$. Substituting yields:

$$NTU_{OY} = \frac{1}{1 - \frac{K_A V}{L}} \ln\left(\frac{y_{A,in}\left(\frac{K_A V}{L} - 1\right) - b\frac{K_A V}{L}}{\left(\frac{K_A V}{L} - 1\right)\left(\frac{L}{V}(x_{A,in} - x_{A,out}) + y_{A,in}\right) - b\frac{K_A V}{L}}\right)$$

$y_{A,in}$ often $= 0$

The preceding equations also apply to an "adsorber", although in that case they should for convenience be written in terms of $y_{A,in}$, $y_{A,out}$, and $x_{A,in}$.

There is an equivalent analysis using the overall liquid phase driving force $(x_A - x_A^*)$ and the corresponding mass transfer coefficient (K_{OX}):

$$\underbrace{\int_{x_{A,in}}^{x_{A,out}} \frac{dx_A}{x_A - x_A^*}}_{NTU_{OX}} = \int_0^h \frac{dz}{\underbrace{\frac{L}{K_{OX} A_c a}}_{HTU_{OX}}} = \frac{h}{HTU_{OX}}$$

NTU_{OX} and HTU_{OX} are related to NTU_{OY} and HTU_{OY} as follows:

$$HTU_{OY} = \frac{L}{K_A V} HTU_{OX} \; ; \; NTU_{OY} = \frac{K_A V}{L} NTU_{OX}$$

The Height Equivalent to a Theoretical Equilibrium Plate (HETP) can also be defined as $HETP = h/N_{eq}$, where N_{eq} is the number of equilibrium stages required for the separation, as calculated by a stage-to-stage calculation. HETP is related to HTU_{OY} (and HTU_{OX}) according to:

~1 foot for "structured" packings

~3 feet for "random" packings

$$HETP = \frac{HTU_{OY} \ln (K_A V/L)}{\frac{K_A V}{L} - 1}$$

applies to linear equilibrium and operating lines.

Mechanical Contactors for Liquid-Liquid Extraction

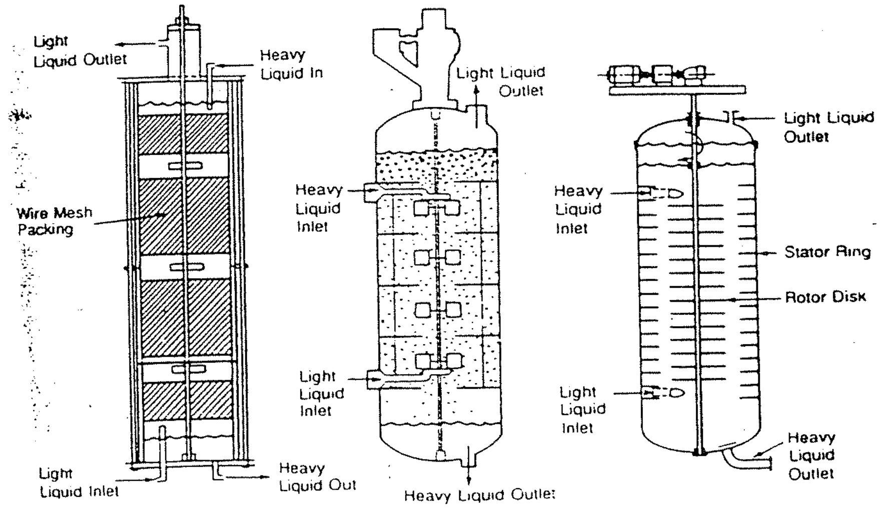


Figure 5.4-1. Differential extractors. The heavy and light liquids move countercurrently, as in mixer–settlers. The concentrations in these liquids are not near equilibrium, unlike those in mixer–settlers. (After R. E. Treybal, *Mass Transfer Operations*, McGraw-Hill, 1980.)

Example Problem:

1. A fermentation broth containing 0.1 mole/L penicillin must be fed to an extractor at 5 L/hr to recover 95 % of the penicillin. The solvent, which contains no penicillin, is fed to the extractor at a rate of 2 L/hr and offers a partition coefficient of 8.

(a) How many equilibrium stages are required to achieve this separation?

(b) How tall must a differential extractor be if it has a diameter of 12 cm and a *ka* of 1 hr^{-1}?

(c) For the differential extractor in part (b) plot how the aqueous and organic phase concentrations (y and x) change as a function of position (i.e. height).

2. You are trying to extract an aqueous fermentation fluid that contains two different types of penicillin, the desired penicillin with the proper structure and an undesired penicillin-analog that has an additional hydroxyl group. The pH and solvent have been chosen to enhance the difference in partition coefficients between these two penicillins. You need to design a column that can achieve a 98 % recovery of the desired penicillin and to calculate what percent of the undesired penicillin is also extracted.

(a) A pilot extraction study showed that a short column 10 cm in diameter and 1 m tall extracted 60 % of the desired penicillin. The aqueous flow rate in this pilot column was 6 L/hr while the solvent flow was 4 L/hr. Determine *ka* for this extractor. [You will use this *ka* to solve parts (b) and (c).]

(b) Using the same column conditions as the pilot system, determine how tall the column must be to achieve 98 % recovery of the desired penicillin. State all assumptions.

(c) For the column in part (b), calculate the % recovery of the undesired penicillin that will also be extracted into the solvent.

	Desired Penicillin	Undesired Analog
Partition Coefficient	10	5
y_{top} (mole/L)	5×10^{-2}	0.5×10^{-2}
x_{bottom}	0	0