Mathematics Handout

1. Derivatives

Let f(x) = x^n and g(x) = a + bx where n, a and b are constants.

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

$$\frac{dg(x)}{dx} = b$$

$$\frac{d[f(x)+g(x)]}{dx} = nx^{n-1}+b$$

$$\frac{d[f(g(x))]}{dx} = n(a+bx)^{n-1}b$$

2. Summation notation

Let a - over a variable indicate the arithmetic mean or average value of that variable and k be a constant.

$$a_1+a_2+a_3=\sum_{i=1}^{3} a_i$$

$$a_1+a_2+...+a_n=\sum_{i=1}^{n} a_i$$

$$a+a+a=\sum_{i=1}^{3} a=3a$$

$$\begin{aligned}\sum\left[(y_i-\bar{y})(x_i-\bar{x})\right] &= \sum(y_i x_i-\bar{y}x_i-\bar{x}y_i+\overline{xy}) \\ &= \sum(y_i x_i)-n\bar{x}\bar{y} \\ &= \sum(y_i-\bar{y})x_i\end{aligned}$$

$$\sum_{i}^{n}\sum_{j}^{n} x_i x_j = \sum_{1}^{n} x_i^2 + 2\sum_{i<j} x_i x_j = \left[\sum_{i}^{n} x_i\right]^2$$

$$\sum_{i}^{n} k a_i = k\sum_{i}^{n} a_i$$

3. Derivatives in Summation Notation

Let f(x) be any arbitrary function of x.

$$\frac{d(\sum_{i}^{n} f(x))}{dx} = \sum_{i}^{n} \frac{df(x)}{dx}$$

Suppose that x = a + by and f(x) = x^2.

$$\frac{d(\sum_{i}^{n} (a+by_i)^2)}{da} = 2\sum_{i}^{n} (a+by_i)$$

$$\frac{d(\sum_{i}^{n} (a+by_i)^2)}{db} = 2\sum_{i}^{n} [(a+by_i)y_i]$$

4. Expectations, Means, and Variances

Recall from statistics that E() is called the expectations operator and that the expected value of a random variable is its mean, or population average. Let x, y, z and , be random variables where z = " + $x + ,. If a bar appears over the variable it denotes the sample mean and s^2 is the sample variance. Let μ and F^2 be the population mean and variance respectively.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$s_x^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

$$E(x) = \mu_x$$

$$E[(x-\mu_x)(y-\mu_y)] = E(xy) - \mu_x\mu_y$$
$$= \mathrm{F}_{xy}$$

$$E(xz) = E(x(\text{"} + \$x + ,)) = E[\text{"}x + \$x^2 + ,x]$$
$$= \text{"}E(x) + \$E(x^2) + E(,x)$$
$$= \text{"}\mu_x + \$E(x^2) + E(,x)$$

$$E[(x-\mu_x)(y-\mu_y)] = E(xy) - \mu_x\mu_y$$
$$= \mathrm{F}_{xy}$$

$$E[x-E(x)]^2 = E[x-\mu_x]^2$$
$$= E(x^2) - \mu_x^2$$
$$= \mathrm{F}_x^2$$