TENSES, ANALYTICITY, AND TIME'S ETERNITY

STEPHEN E. BRAUDE

Although we may not know exactly what an analytic sentence is, we can conveniently and plausibly classify some tensed sentences, e.g., ‘Nobody has squared the circle’ and ‘It was either raining or not raining’ as analytic. Like paradigm analytic sentences, e.g., ‘All bachelors are unmarried’, and unlike most tensed sentences, the truth or falsity of these analytic tensed sentences is curiously independent of the circumstances of their production. In this paper I want to expose an interesting discrepancy between our ordinary understanding of some analytic (as well as some contingent) tensed sentences, and the treatment of these sentences in tense logic and standard predicate logic. In particular, I want to show that the truth-conditions of these sentences in ordinary discourse differ significantly from the truth-conditions of the translations of those sentences in the formal languages of either tense logic or standard predicate logic.

I

Many philosophers have maintained that the sense of an ordinary tensed sentence is unambiguously expressed only in certain kinds of quantified formulas. For example, the sentences

(1) Hugo is smiling
(2) Hugo smiled
(3) Hugo will smile

are supposed to be re-expressed perspicuously by, respectively,

(1') (∃t) (t is simultaneous with _____ & Hugo smiles at t)
(2') (∃t) (t is earlier than _____ & Hugo smiles at t)
(3') (∃t) (t is later than _____ & Hugo smiles at t)
The first conjunct in (1’)-(3’) serves to locate the time of the event or state of affairs which in the second conjunct is said to occur or obtain then, and both conjuncts are supposed to be tenseless. Philosophers have various prejudices concerning the content of the first conjunct. Some prefer to fill in the blank with the word ‘now’, or with expressions such as ‘this utterance’ and ‘the time of this utterance’, while others prefer to fill in the blank with dates (usually, the date of the sentence’s time of production).¹ All these formulations pose the rest of problems noted below.

In standard tense logics, the first three sentences are usually written

\[(1'') \ p \]
\[(2'') \ Pp \]
\[(3'') \ Fp \]

where ‘p’ is a schematic letter replaceable by present-tense sentences (in this case ‘Hugo is smiling’), and where ‘P’ and ‘F’ are the tense modalities for the past and future, respectively. (2’’), for example, would normally be read as ‘It was the case that p’. Since formulas of tense logic are assigned truth-values which may differ from moment to moment, valuations are relativized to moments of time. Thus, where \(A\) is a formula of tense logic, an expression ‘\(PA\)’ is said to be true at a moment \(M\) if and only if \(A\) is true at some moment before \(M\). Moreover, changing the position of a negation sign relative to a past or future-tense operator in a tense-logical formula significantly alters the truth-conditions of the formula. For example,

\( \lnot F\sim A\), is true at moment \(M\) iff \(\sim \lnot A\), is true at a time later than \(M\)

\( \sim FA\), is true at moment \(M\) iff \(\sim A\), is true at all times later than \(M\)

Thus ‘F~ (Hugo is smiling)’ would correspond to the informal ‘Hugo will not smile [i.e., at some time]’, whereas ‘~F(Hugo is smiling)’ corresponds to ‘Hugo will never smile’.

As a final prelimenary, I should state briefly what I understand a tensed sentence to be. First, I take a sentence to be an instance of a concatenation of morphemes, which has a truth-value, and whose visual or sound pattern may be replicated.² A sentence \(S\) is tensed, then, if and only if it is necessary that for any two distinct moments of time, replicas of \(S\) produced at those
times have different truth-conditions. I have defended and explained this position elsewhere, but what I have in mind is (very roughly) the following.

The truth-conditions for a classic, simple past tense sentence of ordinary English may be stated as follows.

A past-tense sentence ‘S was φ’ is true iff S is φ (or is φing) prior to its time of production.

The sentence to the right of this biconditional is tenseless – that is, its truth-conditions are not relativized to its time of production. Moreover, the minimal, but significant, respect in which the truth-conditions of a tensed sentence change is that the time relative to which, and of course the period of time in which, what the sentence reports must occur in order to be true, changes from moment to moment. Thus a replica of ‘S φd’, produced at M, will be true just in case S φs before M, and another replica of ‘S φd’, produced at a later time M', will be true just in case S φs before M’. Although the difference in truth-conditions between successive replicas of a tensed sentence may be very slight (for example, when two replicas are produced only a few seconds apart), often this difference in truth-conditions results in a difference in truth-values for replicas of a tensed sentence, as in the case where replicas of ‘S φd’ are produced before and after S’s only case of φing.

II

Consider

\[ p \lor \neg p \]

Sentences having this form are usually regarded as analytic, since these sentences would be instances of a logical truth. According to the canons of standard sentential logic, then, both

\[ \text{Either Hugo is smiling or Hugo is not smiling} \]

\[ \text{Either Hugo will smile or it is not the case that Hugo will smile} \]

are analytic. But are these sentences tensed? Tense logic answers this question affirmatively, but only indirectly, by providing different symbolizations for (5) and (6). (5) would be written as (4), and (6) is written as

\[ \neg Fv \lor \neg Fp \\
Fp \]
(5) and (6) also count as tensed according to the definition of ‘tensed sentence’ I supplied earlier. (5), for example, is true if and only if either Hugo is smiling at its time of production or Hugo is not smiling at its time of production, and this time must change with each successive replication of (5). Of course (5) will be true whenever produced, these changes in truth-conditions notwithstanding.4

While it is certainly intriguing that some instances of logical truths are tensed, the interesting cases I want to consider concern different kinds of tensed analytic sentences.

Consider again

(6) Either Hugo will smile or it is not the case that Hugo will smile

and its tense-logical formalization

(7) Fp v ∼Fp

Ordinarily, if we wanted to utter a sentence with the future-tense tautological form of (7), we would not utter a sentence with the awkward second disjunct of (6). Instead, we would say something more ambiguous than (6), for example,

(8) Hugo will either smile or not smile
(9) Either Hugo will smile or Hugo will not smile

But while we might frequently use (8) or (9) when – in the interest of precision – we should use (6), we might instead be trying to produce sentences having the tense-logical forms of

(10) F(p v ∼p)
(11) Fp v F∼p

respectively. One reason for this, of course, is that the second disjunct in (6) has the same sense as ‘Hugo will never smile’, and we often do not want to say that. We may instead want to say something having the force of ‘Hugo will not smile [i.e., at some time]’. In fact, I think we would ordinarily understand (8) as expressing what would be more perspiciously but less elegantly expressed by

(12) It will be the case that either Hugo smiles or Hugo does not smile

Similarly, we would ordinarily understand (9) as expressing what would be expressed by
(13) Either Hugo will smile or it will be the case that Hugo does not smile

This becomes clear once we understand what the second disjunct of (6) really says. Accordingly, I shall hereafter regard (8) and (9) as equivalent in sense to (12) and (13), respectively.

In some systems of tense logic, (10) and (11) — and thus (12) and (13) — are not equivalent. For example, if time is regarded as branching into the future (i.e., into possible futures), and if a future-tense expression $FA$ is true at moment $M$ if and only if $A$ is true at a later time in some possible future relative to $M$, then (10) can be true while (11) is false. But this interpretation of the future tense, called the ‘Peircean’ theory of the future by Prior, does not seem to correspond to the ordinary use of future-tense sentences. In this paper, however, I shall concern myself only with the more humdrum analysis of the future tense mentioned in the previous section, according to which (10) and (11) are equivalent, and to keep matters simple, I shall concentrate on sentences having the form of (10) and its past-tense counterpart $P(p \lor \sim p)$. As it turns out, though, some of the problems I raise below for the standard analysis of the future tense afflict more exotic analyses as well.

Notice that (10) is a straightforward instance of (3'''). In an ordinary tense logic, in which (5) would be a wff, and in which the result of prefixing a wff by ‘F’ or ‘P’ is a wff, (10) would be well-formed. Moreover (10) does not seem to be equivalent to (7). While (7) and its past-tense analogue should be theorems of any tense logic, (10) and its past-tense analogue will be theorems only if it is presupposed that time has neither a beginning nor an end. Otherwise, at the last moment of time, for example, (7) would be true but (10) would be false. Tense logic takes (7) to be true at moment $M$ provided that its second disjunct is true, and that disjunct is true at $M$ if and only if for all $N$, if $N$ is later than $M$, then $\sim p$ is true at $N$. Thus $\sim Fp$, and hence (7), will be vacuously satisfied at time’s last moment. But (10) is true at moment $M$ just in case $(p \lor \sim p)$ is true at some time later than $M$. Thus (10) will be false at time’s last moment, since there is then no later time at which $(p \lor \sim p)$ is true.

But instances of (10) in the natural language are true at any time, including the last moment of time. In ordinary language, (8), for example, is true if and only if Hugo either smiles or does not smile after its time of production. And since Hugo will not smile
after (8)’s production, if (8) is produced at the last moment of
time, (8) will be true at that moment.

Thus it appears that standard tense logics inadequately
formulate the truth-conditions for sentences of the natural
language having the form of (10) or its past-tense analogue, since
in ordinary discourse the analyticity of such sentences does not
hinge on the presupposition that time is eternal.

An analogous problem afflicts the predicate-logic translation
technique adumbrated above. According to that technique, (8)
should be written as

(14) \((\exists t) [t \text{ is later than } _____ \& (\text{Hugo smiles at } t 
\lor \sim \text{Hugo smiles at } t)]\)

while (6) would be re-expressed as

(15) \[ (\exists t) (t \text{ is later than } _____ \& \text{Hugo smiles at } t) 
\lor (\exists t)(t \text{ is later than } _____ \& \text{Hugo smiles at } t) \]

Like (7), (15) is true at any time, including time’s last moment.
But since the first conjunct of (14) locates time \(t\) as being later
than the time of production of (14), that conjunct, and hence
(14) itself, will be false at that time. Thus (14) does not seem to
re-express (8), which is true at time’s last moment.

(8) might, of course, be rewritten as the universally
quantified

(16) \((t) [t \text{ is later than } _____ \supset (\text{Hugo smiles at } t \lor 
\sim \text{Hugo smiles at } t)]\)

Even if time can have an end, (16) will be true at the last
moment, since its antecedent is false at that time. But (16) cannot
be regarded as a satisfactory translation of (8). It is more likely a
translation of

(17) Hugo will always either smile or not smile

The difference between (8) and (17) is presumably significant and
worth preserving when we translate tensed sentences into formulas
quantifying over times. But by translating (8) as (16), we lose the
distinction in predicate logic between (8) and (17).

The sorts of analytic tensed sentences we have considered
to far might plausibly be regarded as tense-logical tautologies.
But the failure of tense logic and predicate logic to reproduce the
truth-conditions of these analytic tensed sentences extends also to
tensed sentences which are analytic but not tautologous. Consider
(18) Hugo will not square the circle  
(19) Hugo has not been a married bachelor  

Ordinarily, these sentences would be regarded as true at any time, including time's first and last moments. (18), for example, is true if and only if Hugo does not square the circle after its time of production. And since Hugo clearly cannot square the circle at any time, much less after time's last moment, the truth of (18) at any time is guaranteed.  

In tense logic, however, the truth-conditions of (18) are stated differently, and apparently inaccurately. Tense logic takes (18) as true at moment $M$ just in case 'Hugo is not squaring the circle' is true at some moment after $M$. But since at the last moment of time there is no later moment, (18) would be false at that time. 

Similarly, by translating (18) as  

(20) $(\exists t)(t \text{ is later than } _____ \& \text{ Hugo is not squaring the circle at } t)$  

we again have a sentence that is false at the last moment of time, since the first conjunct locating $t$ after the sentence’s time of production, will be false. 

Some might protest that I am confusing sentences of the form ‘$\neg Fp$’ (or ‘$\neg Pp$’) with sentences of the form ‘$F\neg p$’ (or ‘$P\neg p$’), and that (18), for example, is really an instance of the former and hence vacuously true at time’s last moment. But I am not making this mistake. The ordinary language counterpart in this case to a sentence of the form ‘$\neg Fp$’ would be  

(21) Hugo will never square the circle  

Granted, there may be cases in ordinary discourse where a sentence like (18) might be understood as (21). But I am simply stipulating that we are not to understand (18) in this way. There is no reason why (18) cannot have the future-tense force of more mundane future-tense sentences like  

(22) Hugo will not attend John’s party  

where this last sentence is not to be understood as (the rather odd)  

(23) Hugo will never attend John’s party$^8$  

In fact, we can now see that even a *synthetic* tensed sentence of the form ‘$F\neg p$’ has truth-conditions in ordinary discourse distinct from those ascribed to its translations in standard
tense and predicate logics. (22), for example, will be true at time’s last moment, not because it has the sense of (23), but because at that time there is no later time at which John’s party will take place. But according to the tense-logical method of rendering truth-conditions for tensed sentences, (22) comes out false at time’s last moment, since at that time there is no later moment at which ‘Hugo is not attending John’s party’ is true. Similarly, at time’s last moment (22) will also turn out to be false under its predicate logic translation.

Notice that I am not claiming here that (22), for example, can be true at time’s last moment, while

\[(22')\] It will be the case that it is not the case that Hugo is attending John’s party

is false. I am only claiming that in tense logic the truth-conditions of (22') are stated in such a way that the two sentences can differ in truth-value, even though they are supposed to be logically equivalent. I don’t want to quarrel with those who consider (22) to have the form of (22'), although I would say that the standard predicate logic translation of (22) fails to capture the form of that sentence. Whether or not (22') is a perspicuous version of (22), the truth-conditions of tensed sentences in ordinary language differ from those of their tense-logical translations in a way that is an embarrassment for tense logic.9

III

It thus appears that both tense logic and standard predicate logic distort, in an interesting way, the truth-conditions of some tensed sentences in ordinary language, by making it a necessary condition for the satisfaction of those sentences that there be times in addition to their times of production. Probably the most interesting result of this requirement is that some sentences which would ordinarily be regarded as analytic turn out to be synthetic. Differences in truth-value assignments to tensed sentences (analytic or synthetic) would emerge only at time’s last or first moment. These discrepancies are perhaps more interesting in the case of tense logic, since we would expect the truth-conditions of tense-logical formulas to match those of the tensed sentences they represent. Moreover, it is ironic that at the last (first) moment of time, it is the very same fact about time — namely, that there is no next (previous) moment — which makes analytic tensed sen-
TENSES, ANALYTICITY AND TIME'S ETERNITY

tences like those considered above true in ordinary language but false under their tense-logical translations.

Of course as long as we presuppose the two-way eternity of time, neither tense logic nor predicate logic will diverge in these ways from the language they purport to symbolize. And not surprisingly, many philosophers are willing to make this presupposition. Still, since some tensed sentences are analytic even when time is regarded as having a first and last moment, and since some synthetic past and future-tense sentences would be regarded as true at the first and last moment, respectively, it is reasonable to expect that analyticity and truth under these conditions would be preserved in our formal languages.

UNIVERSITY OF MARYLAND BALTIMORE COUNTY
BALTIMORE, MARYLAND 21228
USA

NOTES

1 In my paper ‘Tensed Sentences and Free Repeatability’, Philosophical Review, vol. 82, no. 2 (April 1973), pp. 188-214, I indicated why certain of these substitutes cannot be used if (1')-(3') are to be tenseless.

2 Understood in this way, a sentence is a linguistic event, and is more like a sentence-token than a sentence-type. In any case, as far as the points I am making here are concerned, nothing serious hangs on this particular terminological convention.


4 It is interesting, of course, that some tensed sentences have invariant truth-values, since a popular view of tensed sentences is that all such sentences have variable truth-values. I discuss this point in detail in ‘Tensed Sentences and Free Repeatability’.


7 We can accomplish this in a tense logic with axioms for linearity simply by including as axioms both ‘¬F¬A ⊃ FA’ and ‘¬P¬A ⊃ PA’. At the last and first moments of time, respectively, the antecedents of these

Actually, (23) does have a colloquial use in which it is equivalent in sense to (22). But in this case the occurrence of 'never' serves as an emphatic device.

One final point merits our attention. In tense logic, sentences of the forms 'Gp' or 'Hp' (respectively, 'it will always be the case that p' and 'it has always been the case that p', and equivalent, respectively, to '~F~p' and '~P~p') are vacuously true at time's last and first moments, respectively. An expression 'GA', for example, is true at M iff for all N > M, A is true at N. The truth-conditions for the predicate logic analogue of 'it will always be the case that p' are stated similarly. But this seems highly counter-intuitive, since it turns analytically false sentences — for example,

(24) Hugo will always square the circle'

into truths. But presumably such sentences are false at any time. (I owe this point to Audrey McKinney.)