Homework #1
Sept 3, 2015

1 For the classical diffusion equation $u_t = \nabla \cdot (5\nabla u)$ (in 3 space dimensions)

find TWO changes of variables which changes the diffusion constant from 5 to $D = 1$ for the new coordinate system.

2 Verify that $y = \frac{1}{\mu}\int_0^t \sin(\mu(t-s))f(s)ds$ satisfies the ODE $y'' + \lambda y = f(t)$ where $\mu = \sqrt{\lambda}$. [Hint: Use the Lagrange formula (extension of the Fundamental Theorem of Calculus).]

$$\frac{d}{dt}\int_{a(t)}^{b(t)} F(t,s)ds = b'(t)F(t,b(t)) - a'(t)F(t,a(t)) + \int_{a(t)}^{b(t)} F_t(t,s)ds.$$

`Bell's 01_Introduction.pdf`

3 [#3 pg 11] Verify that $u(x,t) = \int_0^{\frac{x}{2\sqrt{t}}} e^{-s^2}ds$ is a solution to $u_t = u_{xx}$.

4 [#4 pg 11] Verify that $u(x,y) = f(x)g(y)$ is a solution to $uu_{xy} = u_x u_y$ for all pairs of differentiable functions f, g of a single variable in $\mathbb{R}$.

5 [#5 pg 11 - modified] Verify that the solution $u(x,t)$ of the transport equation

$$\frac{\partial u}{\partial t} + \phi(u)\frac{\partial u}{\partial x} = 0, \quad u(x,0) = f(x),$$

for sufficiently smooth ϕ is implicitly given by $u = f(x - \phi(u)t)$.