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Power in Game Forms

by

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Abstract: This essay defines power in game forms in what I believe to be a plausible and natural manner. It distinguishes between the affirmative and preclusive "faces" of power and identifies general properties that characterize all power relations (and all game forms). It also identifies certain special properties that may characterize some power relations (and some game forms). It defines "effective" preference in a manner similar to "domination" in the sense of cooperative game theory. And finally it asks this question: what conditions on power relations assure that, even as preferences may change, effective preference has "nice" properties such that society can make collective decisions easily. The answer is this: if there are three or more possible outcomes, and if no actor has all the power, it is impossible to structure power in such a way that effective preference has "nice" properties.

1. Introduction

Power is often identified as the central concept in political science. Yet there is little scholarly consensus on how to define power, how to observe and measure it, or even how to think about it. Indeed, March [1966, p. 79] has concluded that "on the whole ... power is a disappointing concept".

Here we shall conceive of power as the capacity of an actor, alone or (more likely) in combination with others, to bring about or preclude certain outcomes. The subsequent technical sections of this essay are devoted to formalizing this notion and examining some of its properties and implications. But first, I will point out how this concept differs from some other looser notions of power, and also from the various "power indices", and I will try to provide some justification for this concept.

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2. Power and political analysis

Often (especially in more empirically-oriented studies) what is estimated and identified as power is, at least implicitly and roughly, the relative success of political actors in getting decisions they like. This simple notion - that the politically powerful are precisely those who are most successful in getting desired outcomes - is certainly straightforward and appeals, at least at first glance, to common sense. Moreover, it is methodologically convenient for empirical research, since we can usually ascertain the preference of actors in political situations, observe the decisions, and match the former with the latter.

However, this approach really does not bear theoretical scrutiny.[1] It suggests that all factors that make for political success are components of power and that measured power is simply a summary of these advantages. To say that "A got what he wanted, because he is powerful" then becomes an *a priori* (and vacuous) truth. It is more useful to have a concept of power that allows us to say that "A failed to get what he wanted, although he is powerful" or "A got what he wanted, although he is not powerful".

In a particular decision, or typically over a sequence of decisions, advantages that make for success may include, among others, the following. An actor may be successful because:

(i) he has preferences similar to those of one or a few other actors who are powerful;

(ii) he has preferences similar to those of most other actors;

(iii) he has preferences that are, in some sense, about "average";

(iv) he holds, in some sense, the "balance of power" between two opposing groups and he can tip the decision as he prefers; or

(v) in any other way his preferences are related to the overall distribution of preferences in such a way that he derives advantages.

Such circumstances are centainly advantageous to the actor in question, but they may not be enduring since they depend on fortuitous, and perhaps shortlived, configurations in the distribution of preferences. If his preferences were different, or if the preferences of (some) other actors were different, his advantage would likely evaporate. In any case, we certainly want to distinguish systematically between such *preference-based advantages* in political decision making (contingent upon particular preference distributions) and other *non-preference-based advantages* (invariant under changes in

[1]Indeed power has been explicitly distinguished from "success" or "satisfaction" in some recent formal analyses; see especially Brams and Lake [1978] and Straffin et al. [1982]; also see Barry [1974, pp. 195 ff.; 1980].

the distribution of preferences). I believe that most people, upon reflection, would agree that the concept of power should be confined to non-preference based advantages in decision making, and perhaps to a subset thereof. But it bears emphasis that the success of an actor in getting decisions he likes, or his expected satisfaction with political outcomes, will depend in large part - in fact primarily (especially in large systems) - on his preference-based advantages and only in part - perhaps only in small part and sometimes not at all - on other non-preference-based advantages, including power.[2]

What further (non-preference-based) advantages may an actor have? He may be successful because:

(vi) he is strategically skillful, while (some) others are not;

(vii) he has information that (some) others do not have;

(viii) he can cooperate freely with other actors, while (some) others are hindered from doing the same; or

(ix) he is active and exploits all opportunities available to him, while (some) others are relatively inert.

I am inclined to think it is redundant to say that a given decision was reached because actor *A* wanted it and is powerful, and *A* is powerful because (for example) he is more strategically skillful than others. It is more direct to say that the decision was reached because *A* wanted it and *A* is more skillful than others. In any case, I shall keep such non-preference-based advantages as (vi) - (ix) conceptually distinct from power.

Finally, an actor may be successful because:

(x) he is able to modify the preferences of other actors and thus also their actions;

(xi) he is able to modify the actions of other actors without changing their preferences (e.g., by means of threats or promises); or

(xii) he is able (perhaps in combination with others) to bring about outcomes he likes or preclude outcomes he dislikes, regardless of the preferences or actions of others (or those outside the combination).

In the event of (x), I say that an actor exercises *persuasive influence*; in the event of

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[2]Probably "pluralist" and "elitist" analyses of the political process really disagree about the characteristic distribution of preferences or interests in society more than about the distribution and structure of power.

(xi), I say that the actor exercies *coercive influence*.[3] But in either event, I distinguish *influence* - the ability to modify the actions of others (which is accordingly an *interpersonal relation*) - from *power* - the ability of an actor, or a combination of actors, to bring about or preclude outcomes (which is accordingly a *relation between actors and outcomes*) - as manifested in the event of (xii). (Coercive influence, however, is based in large part on power, viz. on the ability to bring about or preclude outcomes desired or undesired by other actors. But coercive influence is in part preference-based, for "to destroy it, nothing more is required than to be indifferent to its threats, and to prefer other goods to those which it promises" [Tawney, p. 176].)

It follows from this understanding that power is a *capacity* or *potential*. [Cf. Barry, 1976, 71, 96-97.] In any particular case, a powerful actor (or combination) may not exercise his (its) power - because of inattention, forebearance, because he will get what he wants anyway, because to do so would not accomplish what he wants (or, in the case of a combination, because of disunity, lack of coordination, etc.).

This view suggests that power may be conceived of as a generalization of "voting power". That is, a group in a voting body can succeed when it can outvote (or "overpower") its opposition - and this is true regardless of how preferences are distributed or how skillful the opposition is, whether or not the group exercises influence, and so forth. At the same time, a group that "has the power" to impose its will on a voting body may fail to do so - out of forebearance, because of internal disunity, lack of mobilization, etc. Conversely, groups that might be outvoted (or even those entirely lacking voting power, e.g., non-voting delegates) often succeed, i.e., get outcomes they like, but they succeed for reasons such as (i)-(xi) above; the fact remains that such actors or groups can always be outvoted - they can neither secure nor preclude any outcome.

Many voting bodies are *majoritarian* and are characterized by very simple *power relations*, viz. any group constituting a majority of the body is all-powerful and can secure or preclude any outcome (e.g., any bill or package of bills), while any other group (constituting less than a majority) is powerless and can neither secure nor preclude any outcome. But if we want to understand decision making in a voting body - or in a larger and less formally structured political system - it is not sufficient to look only at power relations. Power may be the first thing to look at to understand who wins and who loses, "who gets, what, when, how," but it certainly is not the last or only thing to look at. We need to look also at the ideological structure or distribution of preferences or interests in the body, the distribution of information and skills, communications patterns, opportunities for influence, and so forth. Indeed, power relations in

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[3]Perhaps this label really is appropriate only for influence based on threats, and we might dub influence based on promises "inducive" influence. But since our concern here is with power, not influence, there is no present need to introduce or maintain such a terminological distinction.

most voting bodies are so relatively simple (even if not strictly majoritarian) that analyses of the legislative process typically focus almost exclusively on other variables.

The various measures of voting power - i.e., the power indices due to Shapley and Shubik [1954], Banzhaf [1965], Coleman [1971], and Deegan and Packel [1979] - with which formal theorists are familiar are (when properly used and interpreted[4]) indeed measures of power (or, more precisely, of what we will call "power in the general sense") as we conceive it here. Precisely for this reason, they are rather uninteresting when applied to majoritarian or other unicameral "one man, one vote" bodies, since they tell us only that all members have an equal fraction of the power. (They become more interesting when applied to bicameral systems, weighted voting systems, etc.)

Furthermore, most of these indices are *normalized* - that is, defined in such a way that the power values of all actors always sum to unity. Accordingly, they focus entirely on the *distribution* of power and are insensitive to variations in the structure of power or the precise nature of power relations, which may significantly affect the nature of political decision making. For example, according to either the Shapley-Shubik or Banzhaf index, each of the members of a majoritarian voting body has a power index value of $1/n$. The same is true of each member in a "universal veto" or "pure bargaining" (*liberum veto*) system in which unanimous consent is required to bring about any change in the status quo. Yet clearly the structure of power, the nature of power relations, and the character of decision making are very different in the two cases.

The Coleman index and the non-normalized variant of the Banzhaf index [cf. Straffin] do distinguish between the two cases, because their power values do not automatically sum to unity (or any other constant) and do reflect the fact that power relations are (as we shall say) "stronger" in the first case. Yet the fact remains that two actors in two different voting bodies characterized by quite different power relations may nevertheless have the same power value (on any of these indices). Indeed, this defect inevitably characterizes any measure that attempts to summarize the power position of an actor by means of a single numerical value. Power relations may be sufficiently complex and varied that a single index value cannot adequately indicate the power position of an actor.

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[4]Some of the alleged "paradoxes of power" (e.g., "the paradox of quarrelling members") based on power indices really result from inappropriate use of them [cf. Barry, 1980, 192-194]. As conceived here, the power of members of a voting body cannot increase (or, for that matter, decrease) as a result of their "quarrelling" (i.e., always having opposite preferences), and the power of the non-quarrelling third member is likewise unaffected. The non-quarrelling third member will be highly successful, however, benefiting from preference-based advantage (iv) above.

And finally we should note that these power indices do not apply at all to more complex and varied situations. They apply only to those voting or "voting-like" situations that are formally characterizable as "simple games" [cf. Shapley, 1962]. Voting or "voting-like" situations - and simple games - are characterized by a kind of neutrality among outcomes - if an actor or coalition can secure (or preclude) one outcome x, it can also secure (or preclude) any other outcome y; only the status quo or "do nothing" outcome may be allowed a distinguished position. Thus, in simple games, every coalition belongs to one of just three categories: winning, losing, and blocking. A winning coalition is all-powerful; it can secure outcome x and also outcomes y, z, etc. A coalition that can secure (or preclude) outcomes x and y, but not outcomes z and w, takes us beyond the scope of simple games and "voting-like" situations and accordingly beyond the scope of the power indexes. But in general, we must take account of such more complex situations and be prepared to deal with them theoretically.

It might seem that we need only move from simple games to games in general. But games in general effectively incorporate preference information in a way unsuitable for our purposes. Thus the rules of the parlor games that provided the original models for game theory determine, in effect, the preferences of the players (e.g., simply by defining what constitutes "winning"). Likewise, the "rules" of the "economic games" for which game theory was originally developed [by von Neumann and Morgenstern, 1953] substantially determine players' preferences by imposing on them such constraints as egoism and individualism (more for me is better than less for me; otherwise I am different). In conventional mathematical game theory, and especially cooperative theory, games are for the most part, played for money (strictly, for something called "transferable utility"), thus also determining players preferences. For this reason, while the Shapley-Shubik index is a special case of the Shapley [1953] "value" applied to simple games, and while the Shapley value itself is defined for all games, the "value" index does not provide an appropriate measure of power in general - it presumes in effect that all collective decision making is a battle over fully divisible spoils and that all actors prefer more of the spoils to less. But much (or most) collective decision making concerns fixing public policies, which are "public goods" (at least partial and/or local public goods) in that they bear on all actors, but at the same time are differently evaluated by different actors [cf. Barry, 1980, 189-191]. Thus, political rules and institutions do not determine or particularly constrain the preferences of those participating under the rules. Suppose, for example, that a given set of political rules gives some actor a "veto"; nothing in the nature of the rules themselves suggests how valuable this "power" may be to that actor. Nor in general can we specify the "value" of political coalitions; it all depends on the preferences of the participants, which likely vary from case to case.

For these reasons, "game forms" [the term is due to Gibbard, 1973], rather than games as such, provide the appropriate framework for our analysis. A *game form* is, in effect, a game minus any specification of the preferences (interests, utilities, payoffs, etc.) that motivate the players in any particular instance. It should be clear that a constitution, electoral system, voting procedure, or any other strategic environment for political action is formally described, not by a game, but by a game form.

The analysis of power that follows is based on game forms. Power in game forms is defined in what I believe to be a plausible and natural manner. I distinguish between two "faces" of power and identify general properties that characterize all power relations (and all game forms). I identify also certain special properties that may characterize some power relations (and some game forms). I define "effective" preference in a manner similar to "domination" in the sense of cooperative game theory. And finally I ask this question: what conditions on power relations will assure that, even as preferences may change, effective preference has "nice" properties such that society can make collective decisions easily.

The answer to the question is this: if there are three or more possible outcomes, and if no one actor has all the power, it is impossible to structure power in such a way that effective preference has "nice" properties. This conclusion clearly is a variant of Arrow's [1963] General Impossibility Theorem. As an analysis of power, however, what follows is most closely related to work by March [1957], Rae [1971, 1975], and perhaps Goldman [1972].

3. Game forms

A game form is, in effect, a game without payoff functions. To quote Gibbard [1973, p. 589; notation has been modified]:

> Formally, then, a *game form* is a function g with a domain of the following sort. To each player 1 to n is assigned a non-empty set, $S_1,...,S_n$ respectively, of *strategies* .... The domain of the function g consists of all n-tuples $(s_1,...,s_n)$, where $s_1 \in S_1,...,s_n \in S_n$. The values of the function g are called *outcomes*.

Following Farquharson [1969, p. 21], I call strategy n-tuples *situations*. A game form, then, maps situations into outcomes, in a deterministic fashion.

A two-player game form may be represented by a matrix, one row of which corresponds to each strategy in S_1 and one column of which corresponds to each strategy in S_2. Each cell then corresponds to a situation and the entry in the cell (say, x) indicates the outcome to which the situation belongs. (In a two-player game - as opposed to game form - in normal form, the cell entries would be numbers indicating utility payoffs to the players.)

An n-player game form may be likewise represented by an n-dimensional matrix. But from the point of view of any focal player i, the game form may be "contracted" into a two-dimensional matrix, a row of which corresponds to each strategy in S_i and a column of which corresponds to each possible *contingency* for i (again I follow the usage of Farquharson [1969, p. 28]), i.e., each (n-1)-tuple of strategies, one for each player other than i. Likewise, from the point of view of any focal coalition (subset) T of players, the game form may be "contracted" into a two-dimensional matrix, the rows of which correspond to all possible combinations of strategies, one for each player in T, and the columns of which correspond to all possible combinations of strategies, one for each player not in T.

The general result on game forms due to Gibbard [1973] says this: only under very limited conditions can a game form be "straightforward" - that is, offer each player i, regardless of his preferences, a *dominant* strategy, i.e., a clearly best strategy that, in every contingency, gives an outcome that is at least as good (according to i's preferences) as the outcome given by any other of i's strategies. In particular, a game form can be straightforward only if it has fewer than three outcomes or if it makes one player all-powerful. Gibbard's perspective is non-cooperative and individualistic: each player in isolation asks, "Do I have a clearly best strategy, or do I have to try to anticipate what other players will do?" The perspective here, on the other hand, is cooperative: each possible coalition asks, "What can we accomplish on our own, what outcomes can we impose or preclude - generally, what constraints can we impose on the realization of outcomes - regardless of what the other players do?" And roughly, we say the more numerous and diversified these possible constraints, the more powerful the coalition.

4. *The power of players and coalitions*

We begin by considering the "potency" of a single strategy of a single player i. Let V designate the set of all outcomes in the game form. Then let $V'_i \subseteq V$ be the set of outcomes that can possibly be realized (which one depending on the strategy selections of the other players) given that player i selects his strategy $s'_i \in S_i$. In the contracted (from i's point of view) game form matrix, V'_i is simply the union of all outcomes in the row corresponding to s'_i. The complement $V - V'_i$ is, of course, the set of the set of outcomes that certainly cannot be realized (regardless of the strategy selections of the other players) given that player i selects s'_i. In other words, by selecting strategy s'_i, player i can preclude any outcome in $V - V'_i$, so if this set is non-empty, he has some power.

Clearly $\varnothing \subset V'_i \subseteq V$.[5] The inclusiveness of V'_i is a (negative) measure of the constraint that selection of s'_i imposes on the realization of outcomes; and thus it is a (negative) measure of the "potency" of the strategy s'_i [cf. March, 1957, p. 210]. If $V'_i = V$, s'_i is *impotent*; if $V'_i \subset V$, s'_i is (more or less) potent for all outcomes x in V'_i; and if $V'_i = \{x\}$ (i.e., if V'_i is a one-element set - in other words if x belongs to every cell in the row corresponding to s'_i), s'_i is maximally potent or, as we shall say, *decisive* for x.

We now consider the power of an individual player i. He has the strategy set S_i; corresponding to each strategy $s'_i \in S_i$ is the set of outcomes V'_i discussed just above. γ_i designates this family of sets of outcomes, one set for each of i's strategies. Some elements of γ_i may intersect; some may be identical; in the extreme, all elements of γ_i may be identical, in which case necessarily $V_i = V$ for all $s_i \in S_i$. If this extreme possibility holds, i.e., if $\cap V_i = V$ (where $\cap V_i$ designates the intersection of all elements of γ_i), every one of i's strategies is impotent and player i is *powerless*. Otherwise, i.e., if $(\cap V_i) \subset V$, i has at least one potent strategy and player i is (more or less) *powerful*. And if at least one element of γ_i is a one-element set, e.g., $V'_i = \{x\}$, i has at least one strategy that is decisive for x, and we say likewise that player i is *decisive* for x. A player who has at least one strategy decisive for each outcome $x \in V$ is all-powerful or a *dictator*.

A coalition T is simply a set of players. The *joint strategy set* S_T of coalition T is the set of all possible combinations of (individual) strategies, one for each player in T. Corresponding to each strategy combination s'_T is a set of outcomes V'_T that can possibly be realized given that the players in T select the strategies that define s'_T; in the contracted (from the point of view of the coalition T) game form, V'_T is the union of all the outcomes in the row corresponding to s'_T. Thus joint strategies may be characterized as impotent, (more or less) potent, and decisive (for a specified outcome) in just the same manner as individual strategies. And thus also coalitions may be characterized as powerless, (more or less) powerful, and decisive (for specified outcomes) in just the same manner as individual players.

From now on, I shall speak generally of the power of a coalition, it being understood that an individual player can be regarded as a one-player coalition.

5. Affirmative and preclusive power

The power of a coalition T is fully specified by the family of sets of outcomes γ_T, one

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[5] " $A \subset B$ " means A is a *proper* subset of B, i.e., everything in A is also in B and something in B is not in A. " $A \subseteq B$ " allows for improper set inclusion, i.e., A and B may be equal.

set for each of T's joint strategies. This specification may be complex, reflecting the possible complexity of power relations. Sometimes, therefore, it is convenient to look only at certain summary aspects of coalition power. And sometimes it is not necessary to look beyond these aspects - that is, sometimes power relations are not so complex.

Of particular interest as a summary aspect of a coalition T's power is the union of one element sets in γ_T; let $D(T)$ designate this set of outcomes. $D(T)$ specifies the outcomes for which T is decisive; T can impose any outcome in $D(T)$ as the realized outcome, i.e., if $x \in D(T)$, T has a joint strategy that, if used, assures that x is the realized outcome, regardless of the strategy selections of players not in T. Thus $D(T)$ specifies the *affirmative power*, or "power of impose", of the coalition T, which derives from decisive joint strategies.

An *all-powerful* coalition T has maximum (affirmative) power, i.e., $D(T) = V$ - in words, an all-powerful coalition can impose any outcome. The coalition of the whole is always all-powerful, i.e., always $D(N) = V$. In a majoritarian voting body, any majority coalition is all-powerful. A *minimal* all-powerful coalition is an all-powerful coalition that contains no proper subcoalition (subset) that is itself all-powerful. A dictator, of course, is a one-element all-powerful coalition, necessarily minimal.

Not all power is affirmative power; power has a "second face". For if a coalition T has any "power to impose", i.e., $D(T) \neq \varnothing$, it also has some "power to preclude". That is, to say that T can impose x as the realized outcome is also to say that that T can preclude any $y \in V - \{x\}$ as the realized outcome. But not all *preclusive power* is such a "complement" of affirmative power, since any potent strategy combination gives a coalition some preclusive power; i.e., given any s'_T such that $V'_T \subset V$, T can preclude any outcome in $V - V'_T \neq \varnothing$. In general, let $F(T)$ designate the set of outcomes any one of which T can preclude, i.e., $F(T) = [V - \cap V_T]$. Note that if $x, y \in D(T)$ and $x \neq y$, i.e., if T is decisive for at least two distinct outcomes, then

$$F(T) = (V - \{x\}) \cup (V - \{y\}) = V.$$

Several points should be emphasized. First, T can preclude *any one* of the outcomes in $F(T)$, not (necessarily) *any several* simultaneously. T can preclude x and y simultaneously only if there is some particular joint strategy s'_T such that $x, y \notin V'_T$ (which implies, but is not implied by, $x, y \in F(T)$).

Second, and related to the previous point, preclusive power is not equivalent to "veto power" as the term is ordinarily used (e.g., in connection with the U.N. Security Council). To say an actor has "veto power" ordinarily means that the actor can *simultaneously* preclude every outcome but one (usually the status quo or "doing nothing"). But this is only a slightly roundabout way of saying that the actor is decisive for that one outcome. Thus "veto power" in the ordinary sense actually entails some affirmative power. And henceforth we shall say formally that a coalition that is decisive for a single outcome has *veto power.*

Finally, it bears repeating that in general the power of a coalition T is fully specified by the family of sets γ_T. $D(T)$ and $F(T)$ can always be inferred from γ_T but not vice versa.

6. General properties of power relations

Thus far, we have considered the power of particular coalitions. We now turn to consider the general properties of power relations among coalitions.

A coalition T is *at least as powerful* as coalition S *with respect to outcome* x if, for every element V'_S of γ_S such that $x \in V'_S$, there is some element V'_T of γ_T such that $x \in V'_T \subseteq V'_S$. In words, for every joint strategy S has that precludes some outcome other than x, T has a joint strategy that precludes at least the same outcomes and perhaps others (but not x). T and S are *equally powerful with respect to* x if T is at least as powerful as S with respect to x and also vice versa. T is *more powerful* than S *with respect to* x if T is at least as powerful as S with respect to x but not vice versa.

More generally, T is *at least as powerful* as S if T is at least as powerful as S with respect to every outcome in V. And following the same pattern of definition as just above, we may say that T and S are *equally powerful*[6] or that T is more powerful than S.

In many cases, of course, the power of coalitions cannot be compared, because comparisons are based on the incomplete relation of set inclusion.

However, the power of two coalitions can always be compared when one is a subset of the other. Consider a coalition T, any outcome x, and any joint strategy s'_T such that $x \in V'_T$. Now consider any coalition S of which T is a subset. Let $Q = S - T$, i.e., Q is the coalition made up of the players in S but not in T. Now consider any strategy s'_Q such that $x \in V'_Q$. Thus $x \in V'_T \cap V'_Q$, and the coalition $S = T \cup Q$ has a joint strategy (the combination of s'_T and s'_Q - call it s'_S) that is at least as potent for x as s'_T (or s'_Q). (Necessarily $V'_S \subseteq (V'_T \cap V'_Q)$, so $V'_S \subseteq V'_T$.) Since the same argument can be made for any outcome, the coalition S is at least as powerful as T (or Q). Thus in general:

Theorem 1: (*Inclusiveness*) If $T \subset S$, S is at least as powerful as T.

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[6]It follows - as we should hope it follows - that, if T and S are equally powerful, $D(T) = D(S)$ and $F(T) = F(S)$. This has been formally demonstrated by Harmut Kliemt (1980, personal communication). The reverse, of course, is not true.

We now consider the relationship between the power of two *disjoint* coalitions T and S. Clearly this relationship must exhibit some kind of consistency - that is, the power of one coalition must be limited *at least* by the power of all disjoint coalitions (or, equivalently by virtue of Inclusiveness, by the power of its complement). Most clearly, the affirmative (preclusive) power of one coalition must be limited at least by the preclusive (affirmative) power of all disjoint coalitions.

T and S being two disjoint coalitions, consider, for any s'_T and s'_S, the intersection $V'_T \cap V'_S$. Since T and S are disjoint, all such strategy selections can happen simultaneously, and this intersection cannot be empty; in the limit, if T and S are complements $(T=-S)$, s'_T and s'_S together define a unique situation, which belongs to a given outcome, say x. Thus in any case we have $x \in V'_T \cap V'_S$. Formally:

Theorem 2: (*Consistency*) For every pair of disjoint coalitions T and S and for every pair of joint strategies s'_T and s'_S, $V'_T \cap V'_S \neq \varnothing$.

Once stated, this point is no doubt obvious and may not deserve to be labelled a "theorem". But it has some corollaries that are important and less obvious.

Suppose $V'_T = \{x\}$, i.e., $x \in D(T)$; then by the theorem x belongs to every element of γ_S; i.e., $x \in \cap V_S$. Generalizing:

Corollary 2.1: For every pair of disjoint coalitions T and S, $D(T) \subseteq (\cap V_S)$.

Recall that $F(S) = V - \cap V_S$. Substituting, we have $D(T) \subseteq V - F(S)$; and thus:

Corollary 2.1': For every pair of disjoint coalitions T and S, $D(T) \cap F(S) = \varnothing$.

This is surely reasonable; indeed it is a formal statement of our initial observation regarding the requirements of Consistency. In words, what one coalition can impose a disjoint coalition cannot preclude and vice versa. In particular, if $D(T) = V$, then $\cap V_{-T} = V$ and $F(-T) = \varnothing$; in words, the complement of an all-powerful coalition is powerless (obviously).

Suppose $D(T)$ contains two or more outcomes. Then by the first corollary, $\cap V_S$ - and likewise every element of γ_S taken alone - contains the same two or more outcomes. Thus:

Corollary 2.2: For every pair of disjoint coalitions T and S, if $D(T)$ contains at least two outcomes, then $D(S) = \varnothing$.

In words, if one coalition is decisive for as many as two outcomes, no disjoint coalition can be decisive for anything (no disjoint coalition can have any affirmative power).

Suppose $D(T) \neq \varnothing$ and also $D(S) \neq \varnothing$. Then there is (at least) one element of γ_T and also one element of γ_S containing a single outcome. By the theorem, these one-element sets intersect; thus they must contain the same single outcome. Summarizing:

Corollary 2.3: For every pair of disjoint coalitions T and S, if $D(T) \neq \varnothing$ and $D(S \neq \varnothing)$, T and S are decisive for the same single outcome, e.g., $D(T) = D(S) = \{x\}$.

In words, if two disjoint coalitions both have some affirmative power, they are both decisive for the same single outcome (they both have veto power).

7. Power in the general sense

The *power in the general sense* of a player (*not* a coalition) refers not only to what power a player has alone (he may be powerless) but also to what he can add by way of power to every coalition that he might possibly join. That is, player i has power in the general sense to the extent that there are coalitions $S \subseteq N - \{i\}$ such that $S \cup \{i\}$ is more powerful than S. Otherwise, i.e., if i is powerless and S and $S \cup \{i\}$ are equally powerful for all S, player i is *powerless in the general sense* (or a *dummy*).

Note that the various power indices discussed in this book are, in these terms, not measures of power but rather measures of power in the general sense. Thus such indices are typically used to evaluate the distribution of power among members of voting bodies, though (under almost all voting procedures) individual voters are powerless and have power only in combination with others.

8. Special properties of power relations

The foregoing discussion has suggested that in general power relations may be highly complex. We now consider certain special conditions that make power relations less complex.

In introducing the notion of Consistency, we observed that the affirmative power of a coalition must be limited *at least* by the preclusive power of its complement and vice versa. We are now interested in the case in which the affirmative power of a coalition is limited *only* by the preclusive power of its complement and vice versa. Formally:

Definition 1: The power relationship between two complementary coalitions T and $-T$ is *determinate* if $D(T) \cup F(-T) = D(-T) \cup F(T) = V$.

Recall from Corollary 2.1 that Consistency requires that $D(T) \subseteq (\cap V_{-T})$. Put otherwise, Determinancy then means that $D(T) = \cap V_{-T}$ and $D(-T) = \cap V_T$. In words, the outcomes that T can impose are precisely those that $-T$ cannot preclude, and the outcomes that $-T$ can impose are precisely those that T cannot preclude.

Power relations are *determinate* if power relationships between all pairs of complementary coalitions are determinate - that is, if all affirmative power is limited only by preclusive power and vice versa.[7]

We are also interested in the following special property:

Definition 2: The power of a coalition T is *decisive* if $V'_T \subseteq D(T)$ for all $V'_T \subset V$.

This means that each element V'_T of γ_T meets one of these conditions: (i) it contains all outcomes (s'_T is impotent), (ii) it contains precisely one outcome, say x (s'_T is decisive for x), or (iii) it is "redundant" in the sense that each of its elements also belongs to some one-element set in γ_T (while s'_T is potent for x, there is some other s''_T that is decisive for x).

Put more intuitively, the power of a coalition T is decisive if whatever power it has derives from decisive joint strategies and thus its power is identical to its affirmative power and is therefore fully specified by $D(T)$. Note that by this definition the power of a powerless coalition T is decisive, for every s'_T meets condition (i) above. This terminology may seem odd, but it is convenient for stating the following definition.

Power relations are *decisive* if the power of every coalition is decisive. It will also be convenient to introduce this definition: power relations are *almost decisive* if, for every coalition T, either the power of T is decisive or the power of $-T$ is decisive (or both).

[7]Rae [1975] introduces the still stronger condition of *Robustness* on power relations (or "control structures"): if $D(T)=\varnothing$, then there is some S such that (i) $S \cap T = \varnothing$, and (ii) $D(S)\neq\varnothing$ (given Inclusiveness, we can say simply: if $D(T)=\varnothing$, then $D(-T)\neq\varnothing$). In words, the affirmative power of a coalition is limited only by the affirmative power of its complement.

If power relations are determinate, and if all power is affirmative power (i.e., if power relations are also "decisive" and thus "simple" in the senses defined below), Robustness must hold (indeed, Robustness, Determinacy, and Consistency collapse into the same condition); but otherwise it need not. Thus Rae [1975, p.1280] is mistaken when he says that "we do not choose robustness; necessity chooses it for us." Rae overlooks the "second face" of power - "power to preclude" independent of "power to impose" - as well as the possibility of indeterminacy in power relations.

We are particularly interested in those power relations that are both determinate and decisive. We call such power relations *simple*.

Consider any complementary pair of coalitions T and $-T$. If power relations are simple, there are only three possible power relationships between them.

1. Suppose $D(T)=\varnothing$; then by Decisiveness $F(T)=\varnothing$; and then by Determinacy $D(-T)=V$. So one possible relationship is $D(T)=\varnothing$ and $D(-T)=V$.
2. Suppose $D(T)=\{x\}$; then by Decisiveness $F(T)=V-\{x\}$; and then by Determinacy $D(-T)=\{x\}$. So a second possible relationship is $D(T)=D(-T)=\{x\}$.
3. Suppose $x,y\in D(T)$ (i.e., suppose T is decisive for more than one outcome); then by Corollary 2.2 $D(-T)=\varnothing$; then by Decisiveness $F(-T)=\varnothing$; and then by Determinacy $D(T)=V$. So a third possible relationship is $D(T)=V$ and $D(-T)=\varnothing$.

By Decisiveness, $D(T)$ fully specifies the power of T and $D(-T)$ the power of $-T$, so these are the only possibilities. And apart from the labelling of the coalitions, the first and the third are identical, so in fact we have only two basic possibilities. That is, given simple power relations and two complementary coalitions, either one is all-powerful and the other is powerless, or both are decisive for the same single outcome. Accordingly, if power relations are simple, every coalition is all powerful, or has veto power only, or is powerless, and power relations can be fully specified by partitioning the set of all coalitions into these three classes.

And it follows further from the general properties of Inclusiveness and Consistency that simple power relations can be specified fully merely by indicating which coalitions are minimal all-powerful: all supersets of these are all-powerful, all complements of the all-powerful coalitions are powerless, and all other coalitions have veto power only.[8]

Power relations in general (and the game forms underlying them) may be arrayed on a rough spectrum from "weak" to "strong". Power relations are *maximally weak* if all coalitions, other than the coalition of the whole, are powerless; they are *maximally strong* if every coalition, or its complement, is all-powerful. Roughly, as power relations range from "weak" to "strong", two things happen. On one hand, some coalitions become more powerful - but not at the expense of others, i.e., power relations become more determinate. On the other hand, some coalitions become more powerful at the expense of disjoint coalitions, i.e., power relations become more decisive.

[8]What we have, of course, is the basic structure of proper simple games, where all-powerful coalitions are "winning", powerless coalitions are "losing", and coalitions with veto power only are "blocking". Cf. Shapley [1962].

At the two extremes of maximally strong and maximally weak power relations, individual players are powerless; at some intermediate points, individual players have some power. If all players have some affirmative power, a "universal veto system" exists (cf. Corollary 2.3). To the extent that individual players have considerable preclusive power, but no affirmative power, power might be characterized as "dispersed".[9]

It is worth noting that as we range over the four possible types of power relations discussed just above - maximally weak, maximally strong, "universal veto system", and "dispersed" - a player's Shapley-Shubik, Banzhaf, etc., power index values may remain constant - for example $1/n$.[10] This highlights the point made in the introduction that these are only summary indices that focus on the *distribution* of power in the general sense and take no account of very significant variations in the *structure* of power relations.

9. *Effective preference*

When we move from game forms to games by adding preferences, we shall say that outcome x is "effectively preferred" to outcome y if "society" as a whole prefers x to y in some meaningful sense. Clearly, we may expect unanimous preferences to be effective in this sense. But we also expect that some less than unanimous preferences may also be effective; i.e., in any particular case the preferences of some proper subset of actors may be effective, and the possibly conflicting preferences of other actors may be simply overridden. The way in which effective preferences are "filled out" beyond unanimity obviously depends on the nature of power relations among these actors. Thus formal consideration of effective preference involves a meshing together of preferences with the power relations we have already looked at.

I here assume that all players have (complete, transitive) preference orderings over outcomes. If player i prefers outcome x to outcome y, we write xP_iy. $P_i(y)$ designates the set of all outcomes that i prefers to y.

Our initial notion of the sense in which outcome x may be effectively preferred to outcome y is that there is a set of players who jointly prefer x to y and who in coalition have power sufficient to make x the realized outcome. Thus formally, x is effectively preferred to y if there is some coalition T such that:

[9]Such "dispersion" of power may constitute the most appropriate game-theoretical translation of Sen's [1970] "liberalism" condition in abstract social choice theory; cf. Miller [1977, esp. p. 25].

[10]Strictly speaking, though, no such indices would be defined in the fourth possible type since it does not correspond to a simple game.

(1) $x \in D(T)$
and (2) $xP_i y$ for all $i \in T$.

These two conditions suffice to meet our notion to effective preference but in general they are not necessary, since they take account only of the affirmative power of coalitions. For example, suppose that coalition *T*, while not decisive for *x*, can preclude *y* in such a way that any outcome (*x* among others) that may then be realized is preferred to *y* by all members of *T*; i.e., suppose that there is some s'_T such that $x \in V'_T$ and $y \notin V'_T$ and $V'_T \subseteq P_i(y)$ for all $i \in T$. It seems reasonable in this case to say at least that *y* is "effectively dispreferred" (to all outcomes in V'_T). We will, in fact, simply generalize our notion of effective preference and say that each outcome in V'_T is effectively preferred to *y*.

Definition 3: Outcome *x* is *effectively preferred* to outcome *y* if and only if there is some coalition *T* with some s'_T such that:

(3) $x \in V'_T$ and $y \notin V'_T$;
and (4) $V'_T \subseteq P_i(y)$ for all $i \in T$.

T is the *effective coalition* for this effective preference relationship. Other coalitions may be effective for the same relationship. In general, of course, some subsets of *T* fail to be effective because they have insufficient power (though they have the requisite preference consensus); some supersets of *T* fail to be effective because they do not have the requisite preference consensus (though, by Inclusiveness, they have sufficient power).

If *x* is effectively preferred to *y*, we represent this relationship diagrammatically thus: $x \to y$. We also say *x dominates y*. If there is an effective preference path $x \to z \to \ldots \to y$, we say *x reaches y*.

We may make these observations concerning the effective preference relation. First, even when all individual preferences are strict, the effective preference relation is not in general complete (i.e., we may have neither $x \to y$ nor $y \to x$). Second, even when all individual preferences are strict, the effective preference relation is not in general asymmetric (i.e., we may have both $x \to y$ and $y \to x$; we write this $x \leftrightarrow y$[11]). Third, the effective preference relation is not in general transitive (i.e., we may have $x \to y$ and $y \to z$ but *not* $x \to z$) and it may in fact be cyclical (e.g., we may have $x \to y$ and $y \to z$ and yet $z \to x$). Clearly, unanimous preferences are always effective, since always $D(N) = V$.

[11]But such mutual domination can occur only when effective preference works through preclusive, not affirmative, power; and see Lemma 3 below.

10. The significance of a unique undominated outcome

If a system of effective preference is a strict ordering and accordingly replicates the properties of individual preference, we should expect collective decision making to proceed in a particularly satisfactory manner. One outcome would sit at the top of the ordering and would be realized, since every other outcome is dominated - that is, for every other outcome, there is some coalition that wants to and is able to bring about a change.

On the other hand, it does not seem to be necessary for effective preference to be an ordering, in order that collective decision making proceed satisfactorily. An individual, after all, can choose properly once he has established which outcome he most prefers; it is not necessary for him to establish a full ordering. In like manner, collective decision making can proceed satisfactorily if there is one "top" outcome in the system of effective preference, even if effective preference does not generate a full ordering.

We can identify a succession of increasingly weak requirements that we might demand of effective preference.

(1) Effective preference is a strict ordering.

(2) In the system of effective preference, there is a single undominated outcome that dominates every other outcome.

(3) In the system of effective preference, there is a single undominated outcome that reaches every other outcome.

(4) In the system of effective preference, there is a single undominated outcome.

In each case, the *core*, i.e., the set of undominated outcomes, includes precisely one outcome. It is not at all clear that (4) is sufficient for satisfactory collective decision making, since (4), in the absence of any of the stronger requirements, implies that the system of effective preference is "disconnected", so the undominated outcome might never be realized.

Effective preference can fail to meet requirement (4) (or any stronger requirement) in either of two distinct ways. On the one hand, the core may be empty; pretty obviously this can occur only if the system of effective preference includes at least one cycle or symmetric relationship. On the other hand, the core may be large - i.e., include two or more outcomes; quite obviously this can occur only if the system of effective preference is incomplete, for there can be no effective preference relationship between two undominated outcomes.

An empty core implies what may be characterized as "social instability" - it is impossible for "society" to settle on an outcome that satisfies everyone (i.e., every coalition) who needs to be satisfied. For every outcome, there is some coalition with both the power and the desire to upset it. Such a society might be in constant flux, subject to an unending sequence of "realignments", "coups", "upheavals", etc., whatever term might be appropriate.

Though theorists have generally worried about empty cores more than large cores, the social consequences of the latter may be more devastating. A large core implies what may be characterized as "social bargaining" - undominated outcomes exist, but precisely because there are several, there is conflict over which *one* is to be realized. Since no coalition has power sufficient to resolve this matter in its favor, each has a strong incentive to resort to coercive influence based on threats and other bargaining tactics (such as those discussed in Schelling [1960]) in order to resolve the matter in the most favorable way. The result may be deadlock, conflict, and the realization of inefficient outcomes.

Unanimity as a decision rule (i.e., each player has veto power only) implies a non-empty core. It is often remarked that a unanimity requirement is likely to lead to social deadlock. But this is true in two distinct senses. First, it becomes very difficult to upset the status quo (or whatever is the outcome for which each player is decisive) x, since we must have

$$\bigcap_i P_i(x) \neq \varnothing$$

a quite restrictive condition. In this respect, unanimity rule may be unduly conservative, but it otherwise presents no problem. The second sense in which unanimity rule can imply deadlock is that several outcomes may be unanimously preferred to x, i.e.,

$$\bigcap_i P_i(x)$$

may include several outcomes, and among these unanimously preferred (to x) outcomes preferences are likely to conflict, in which case there is no effective preference among them.

11. The possibility of a unique undominated outcome

In this final section, we address the question of what conditions, if any, on power relations assure - at least if individual preferences are strict - the existence of precisely one undominated outcome - a condition necessary to avoid simultaneously the "social instability" and the "social bargaining" problems.

There can be no more than one undominated outcome - and thus the "social bargaining" problem is avoided - if effective preference is complete. What circumstances assure completeness of effective preference? One answer is the existence of an all-powerful coalition all the members of which have identical strict preferences. But this is a condition on preferences, as well as on power relations. We are looking for a condition on power relations alone that will assure completeness regardless of the nature of players' preferences, at least provided they are strict. (Clearly individual indifference can lead to incomplete effective preference, regardless of the nature of power relations.) One answer is fairly obvious.

Lemma 1: Given strict individual preferences, effective preference is complete if power relations are maximally strong.

Consider two outcomes x and y. If all preferences are strict, the set of all players can be partitioned into two subsets according to their preferences between x and y:

$$T = \{i \mid xP_i y\} \text{ and } -T = \{j \mid yP_j x\}.$$

Given maximally strong power relations, by definition either T or $-T$ is all-powerful and accordingly we have either $x \to y$ or $y \to x$.

What is less obvious and more important is that, if there are three or more outcomes, *only* maximally strong power relations assure completeness for all strict preferences.

Lemma 2: Given three or more outcomes, effective preference is complete for all strict preferences only if power relations are maximally strong.

Suppose that power relations are not maximally strong. This implies that there is some T such that $D(T) \subset V$ and $D(-T) \subset V$. Thus, provided that there are three or more outcomes, we can find a pair of outcomes x and y such that $x \notin D(T)$ and $y \notin D(-T)$ or vice versa. (This must be possible unless (i) either $D(T) = V$ or $D(-T) = V$, which is precluded by supposition, or (ii) both $V - D(T)$ and $V - D(-T)$ contain the same single outcome, which by Consistency is possible only if there are no more than two outcomes.)

Now suppose that the members of T are precisely those players who prefer x to y, and suppose further that all members of T also prefer y to each other outcome in $V - \{x, y\}$. Likewise suppose that the members of $-T$ are precisely those players who prefer y to x, and suppose further that all members of $-T$ also prefer x to each other outcome in $V - \{x, y\}$.

It now follows that there can be no effective preference relationship between x and y. T is not decisive for x; thus every element of γ_T that contains x but not y contains some other outcome as well. But all members of T prefer y to such a third outcome.

Thus neither T, nor any subset of T, can be effective for $x \to y$ and all players not in T prefer y to x; accordingly x is not effectively preferred to y. We can make an identical argument for $-T$; accordingly y is not effectively preferred to x. Thus effective preference is incomplete.

The preceding argument in fact supports a somewhat stronger conclusion than Lemma 2.

Lemma 2': Given three or more outcomes, there is no more than one undominated outcome for all strict preferences only if power relations are maximally strong.

In sum, we can be sure of avoiding the "social bargaining" problem only if we require maximally strong power relations.

We now turn to consider conditions necessary and/or sufficient to avoid the other problem, i.e., an empty core and resulting "social instability". Clearly every outcome can be dominated only if there is a cycle in effective preference or if there is a symmetric effective preference relationship. But it turns out that symmetric effective preference cannot itself result in an empty core since, if each of two outcomes dominates the other, there is a third outcome that dominates both.

Lemma 3: If $x \leftrightarrow y$, there is some z that dominates both x and y.

Let $T = \{i \mid xP_i y\}$ and let $S = \{j \mid yP_j x\} \subseteq -T$. By Inclusiveness and the definition of effective preference, there is some s'_T such that $x \in V'_T$, $y \notin V'_T$, and $V'_T \subseteq P_i(y)$ for all $i \in T$; likewise there is some s'_S such that $y \in V'_S$, $x \notin V'_S$, and $V'_S \subseteq Pj(x)$ for all $j \in S$. By Consistency, there is some $z \in V'_T \cap V'_S$. Thus we have $z \to x$ and $z \to y$ (T being effective for the first relationship and S for the second.)

Furthermore, it turns out that the condition of maximally strong power relations necessary to avoid "social bargaining" is more than sufficient to avoid mutual domination.

Lemma 4: If power relations are almost decisive, effective preference is asymmetric.

Consider two outcomes x and y, and let $T = \{i \mid xP_i y\}$ and $S = \{j \mid yP_j x\} \subseteq -T$. Since power relations are almost decisive, either the power of T is decisive or the power of $-T$ is decisive. Suppose the power of T is decisive. Then we have $x \to y$ if and only if $x \in D(T)$; but then we also have $x \in D(T) \subseteq (\cap V_S)$ (by Corollary 2.1), so we cannot have $y \to x$ (by Inclusiveness and the definition of effective preference). Suppose the power of $-T$ is decisive. Then, in like manner, we have $y \to x$ only if $y \in D(-T)$; but then we also have $y \in D(-T) \subseteq (\cap V_T)$, so we cannot have $x \to y$.

Thus in either case effective preference between x and y cannot be symmetric, and the argument can be repeated for any other pair of outcomes.

The maximally strong power relations required to avoid "social bargaining" are of course also decisive, thus are also almost decisive, and thus preclude symmetric effective preference. Accordingly, "social instability" can result only from cyclical effective preference.

What conditions, then, will avoid cyclical effective preference? One sufficient condition is that the intersection of all effective coalitions (cf. Definition 3) be non-empty; this assures that effective preference is a subrelation of the preference ordering of each player in the intersection, and as such it must be acyclic. However, this is a condition on preferences, as well as power.

Given the previous lemmas, the following is a more significant proposition:

Lemma 5: Given three or more outcomes and maximally strong power relations, acyclic effective preference cannot be assured if there are two or more distinct minimal all-powerful coalitions.

Given maximally strong power relations, only all-powerful coalitions can be effective, and at least one minimal all-powerful coalition is effective for each effective preference relationship.

Consider three outcomes x, y, and z. Let us suppose that $x \to y$ and that T is a minimal all-powerful coalition effective for this relationship. And let us suppose that $y \to z$ and that S is a minimal decisive coalition effective for this relationship.

Given the transitivity of individual preference and the fact that no pair of all-powerful coalitions can be disjoint (by Consistency), at least those players in the non-empty intersection $T \cap S$ prefer x to z. However, given only that $x \to y \to z$ and in the absence of any restrictions on individual preferences, these are also the only players who *must* prefer x to z.

Therefore, we can be certain to preclude $z \to x$ only if $T \cap S$ is itself all-powerful (which assures $x \to z$). But this requires that $T = S = T \cap S$, for otherwise neither T nor S would be minimal all-powerful. And if every other outcome is dominated (for example, if each of x, y, and z is unanimously preferred to every $v \in V - \{x, y, z\}$), it follows that no outcome is undominated. Thus, we can strengthen Lemma 5 as follows:

Lemma 5': Given three or more outcomes and maximally strong power relations, the existence of an undominated outcome cannot be assured for all preferences if there are two or more distinct minimal all-powerful coalitions.

Finally, we state the following:

Lemma 6: If power relations are maximally strong, either (i) there are at least three distinct minimal all-powerful coalitions or (ii) there is a dictator.

Let power relations be maximally strong and suppose that there is exactly one minimal all-powerful coalition *T*. Then for any $i \in T$, $D(N - \{i\} = \varnothing)$, for otherwise $N - \{i\}$ would be all-powerful and some subset of $N = \{i\}$, necessarily distinct from *T*, would be minimal all-powerful. But then $D(\{i\}) = V$, and *i* is a dictator (and $T = \{i\}$).

Now suppose that there are exactly two distinct minimal all-powerful coalitions *T* and *S*. Since both are minimal, neither is a subset of the other. Then for any *i* in *T* but not in *S* and any *j* in *S* but not in *T*, we have $D(N - \{i, j\}) = \varnothing$, for otherwise $N - \{i, j\}$ would be all-powerful and some subset of $N - \{i, j\}$, necessarily distinct from both *T* and *S*, would be minimal all-powerful. But then $\{i, j\}$ is all-powerful; and either $\{i, j\}$, distinct from both *T* and *S*, is a third minimal all-powerful coalition or one of *i* and *j* is a dictator.

In sum, maximally strong power relations are required to assure that there is no more than one undominated outcome, i.e., to avoid "social bargaining". But the same maximally strong power relations always entail the possibility that there is no undominated outcome, i.e., permit "social instability", if there are two or more distinct minimal all-powerful coalitions. And in fact maximally strong power relations always entail three or more minimal all-powerful coalitions, unless they are dictatorial. Thus:

Theorem 3: No non-dictatorial power relations can assure for all strict individual preferences the existence of a unique undominated outcome.

Put otherwise, it is impossible to devise a game form, and by extension a set of political institutions, that simultaneously avoids "social instability" and "social bargaining" for all preferences.

Clearly, this theorem is closely related to Arrow's [1963] General Impossibility Theorem, and especially its reformulation due to Wilson [1971, 1972]. (Wilson shows that a consistent subset of Arrow's condition implies a strong simple game, i.e., maximally strong power relations, from which it readily follows that the core may be empty.) And, in a sense, it is the cooperative counterpart of Gibbard's theorem on straigthforward game forms: every game form involving three or more outcomes that offers every player, regardless of his preferences, a dominant strategy is dictatorial. In parallel form, the present theorem says: every game form involving three or more outcomes that assures for all strict preferences the existence of a unique undominated outcome is dictatorial.

References

Arrow, K.J. (1963), *Social Choice and Individual Values*, 2nd ed., New York: John Wiley and Sons.

Banzhaf, J.F. (1965), "Weighted Voting Doesn't Work: A Mathematical Analysis", *Rutgers Law Review*, 19, 317-343.

Barry, B. (1974), "The Economic Approach to the Analysis of Power and Conflict", *Government and Opposition*, 9, 189-223.

Barry, B. (ed.) (1976), "Power: An Economic Approach", in: B. Barry (ed.), *Power and Political Theory*, London: John Wiley, 67-101.

Barry, B. (1980), "Is It Better to be Powerful or Lucky?", *Political Studies*, 28, 183-194, and 338-352.

Brams, S.J., and M. Lake (1978), "Power and Satisfaction in a Representative Democracy", in: P. Ordeshook (ed.), *Game Theory and Political Science*, New York: New York University Press, 529-562.

Coleman, J.S. (1971), "Control of Collectivities and the Power of a Collectivity to Act", in: B. Lieberman (ed.), *Social Choice*, New York: Gordon and Breach, 269-300.

Deegan, J.Jr., and E.W. Packel (1979), "A New Index of Power for Simple *n*-Person Games", *International Journal of Game Theory*, 7, 113-123.

Farquharson, R. (1969), *Theory of Voting*, New Haven: Yale University Press.

Gibbard, A. (1973), "Manipulation of Voting Schemes: A General Result", *Econometrica*, 41, 587-601.

Goldman, A.I. (1972), "Toward a Theory of Social Power", *Philosophical Studies*, 23, 221-268.

March, J.G. (1957), "Measurement Concepts and the Theory of Influence", *Journal of Politics*, 19, 202-226.

March, J.G. (1966), "The Power of Power", in: D. Easton (ed.), *Varieties of Political Theory*, Englewood Cliffs, N.J.: Prentice-Hall, 39-70.

Miller, N.R. (1977), "'Social Preference' and Game Theory: A Comment on 'The Dilemma of a Paretian Liberal'", *Public Choice*, 30, 23-28.

Rae, D.W. (1971), "Political Democracy as a Property of Political Institutions", *American Political Science Review*, 65, 111-119.

Rae, D.W. (1975), "The Limits of Consensual Decision", *American Political Science Review*, 69, 1270-1297.

Schelling, T.C. (1960), *The Strategy of Conflict*, Cambridge, Mass.: Harvard University Press.

Sen, A.K. (1970), "The Impossibility of a Paretian Liberal", *Journal of Political Economy*, 78, 152-157.

Shapley, L.S. (1953), "A Value for n-Person Games", in: H.W. Kuhn and A.W. Tucker (eds.), *Contributions to the Theory of Games*, Vol. II, Princeton: Princeton University Press, 307-317.

Shapley, L.S. (1962), "Simple Games: An Outline of the Descriptive Theory", *Behavioral Science*, 7, 59-66.

Shapley, L.S., and M. Shubik (1954), "A Method for Evaluating the Distribution of Power in a Committee System", *American Political Science Review*, 48, 787-792.

Straffin, P.D. Jr. (1976), "Power Indices in Politics", *Modules in Applied Mathematics*, The Mathematical Association of America.

Straffin, P.D, Jr., M.D. Davis, and S.J. Brams, "Power and Satisfaction in an Ideologically Divided Body", in: M.J. Holler (ed.), *Power, Voting, and Voting Power*, Würzburg-Wien, Physica-Verlag, 239-255.

Tawney, R.H. (1952), *Equality*, 4th ed., New York: Capricorn.

Von Neumann, J., and O. Morgenstern (1953), *Theory of Games and Economic Behavior*, 3rd ed., Princeton: Princeton University Press.

Wilson, R.B. (1971), "A Revision of Arrow's General Possibility Theorem", Working Paper No. 181, Stanford Business School.

Wilson, R.B. (1972), "The Game-Theoretic Structure of Arrow's General Possibility Theorem", *Journal of Economic Theory*, 5, 14-20.