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Nicholas R. Miller
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Power in Game Forms

A *game form* is a game (in the sense of game theory) with all information about the preferences, payoffs, or utilities of the players stripped out. Put otherwise, a game form is a function that maps *strategy profiles* into *outcomes*, over which players' preferences are unspecified. Game forms, rather than games, provide the appropriate representation of voting procedures, electoral systems, constitutions, and other collective decision-making institutions or power relationships. The concept of a game form was first explicitly introduced by Gibbard (1973), who proved that every 'straightforward' game form with three or more outcomes is 'dictatorial.' (A *straightforward* game form gives every player, regardless of what his preferences may be, an undominated strategy.) This general result implies that every 'strategyproof' voting procedure for choosing among three or more alternatives is also dictatorial. (A voting procedure is *strategyproof* if it never gives any voter an incentive to cast an 'insincere' or 'dishonest' vote.) By first proving the general result for all game forms, Gibbard was able neatly to sidestep the questions of what exactly constitutes a 'voting procedure' and what exactly we mean by 'insincere' or 'dishonest' voting.

Subsequently, Miller (1982, 1999) used the concept of a game form to define and analyze power. If we define power as the capacity of an actor, alone or (more likely) in combination with others, to bring about or preclude outcomes, game forms provide a natural framework for analyzing power abstractly. An n-player game form may be represented by an n-dimensional matrix such that each row corresponds to a strategy for player 1, each row to a strategy for player 2, and so forth, and each cell (or strategy profile) belongs to some outcome. (Different cells may belong to the same outcome.) From the point of view of any focal player i, such a matrix can be contracted into two-dimensions, such that rows represent i's strategies and columns represent all possible combinations of strategies for the other players. More generally, rows and columns can represent the strategy combinations for any pair of complementary coalitions of players. Given this setup, a number of definitions, observations, and propositions follow in natural ways. (In what follows, a 'coalition' may refer to any subset of players, including a single player.)

A strategy is *potent* to the extent that there are outcomes that do not appear in that row or column; such a strategy gives a coalition *preclusive power*. A strategy is *decisive* for outcome x if it is maximally potent, i.e., if it precludes all outcomes other than x; such a strategy gives a coalition *affirmative power*. A coalition has *veto power* if it has a strategy that is decisive for some single outcome x (presumably the status quo or some other default outcome). A coalition is *all-powerful* if it is decisive for every outcome, and power relations are *dictatorial* if a singe-player is all-powerful. The affirmative power of every coalition is limited *at least* by the preclusive power of its complement. If two disjoint coalitions both have affirmative power, they are both decisive for the same single outcome, i.e., they both have veto power.

It is not always possible to make power comparisons (based on set inclusion with respect to outcomes) among coalitions but, in the event that one coalition is a superset of another, the more inclusive coalition is at least as powerful as the less inclusive one. Power relations are *determinate* if the affirmative power of every coalition is limited *only* by the preclusive power of its complement. Power relations are *decisive* if the power of every coalition is equal to its affirmative power. Power relations are *simple* if they are both determinate and decisive; simple power relations generate simple games (in the sense of game theory) regardless the players' preference over outcomes. Power relations are *maximally weak* if no coalition other than the coalition of the whole has potent strategies; they are *maximally strong* if every coalition or its complement is all-powerful. As power relations become stronger, (i) determinacy increases, which implies that some coalitions become more powerful but not at the expense of others, and (2) decisiveness increases, which implies that some coalitions become more powerful at the expense of their complements.

Outcome *x* is *effectively preferred* to *y* if there is a coalition (i) with a potent strategy that precludes *y* and (ii) all of whose members prefer *all* non-precluded outcomes (including *x*) to *y*. An outcome *x* is *undominated* if no other outcome is effectively preferred to *x*. The absence of an undominated outcome implies 'social instability,' i.e., there is no outcome that satisfies every coalition that has the power to upset it. A multiplicity of undominated outcomes implies 'social bargaining,' i.e., there is conflict over which undominated outcome should prevail and no coalition has the power to resolve this in its favor. Finally, no non-dictatorial power relations can guarantee the existence of a unique undominated outcome (even if all preferences are strong). This proposition is, in effect, the coalition-based or 'cooperative' counterpart to Gibbard's 'non-cooperative' general result on game forms. Both propositions are intimately related to Arrow's General Impossibility Theorem in social choice theory.

References

Gibbard, Allan. 1973. 'Manipulations of Voting Schemes: A General Result.' *Econometrica*, 41: 587-601.

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