VOTING

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Virtually all economic doctrines prescribe that certain activities -- for example, the provision of public goods — be undertaken by government. Accordingly, such doctrines implicitly prescribe that certain allocative decisions — for example, determining the level of supply of public goods -- be made by political rather than market processes. Thus voting (and government decision making generally), though logically a part of political science, is of clear relevance to economic theory.

Historically, economists have contributed at least as much as political scientists to the pure theory of voting. The theory of voting has its origins in the work of such enlightenment philosophers and mathematicians as Borda, Condorcet, and Laplace. Little further progress was made until some forty years ago when the economist Duncan Black wrote a series of articles (most notably Black, 1948) on the logic of committees and elections, which were subsequently consolidated into a book (Black, 1958). Since Black revived the subject, a number of economists and political scientists have made important contributions. Indeed, the theory of voting has to some extent been subsumed by the more recent and abstract theory of social choice, which was virtually invented by the economist Kenneth Arrow (1951).

Here we review the generic voting problem of selecting, on the basis of the declared preferences of several individuals, one alternative out of a set of alternatives. The voting body may be a small committee, a legislature, or a mass electorate. The alternatives may be proposed budgets, programs, policies, or candidates for some single office — the common problem is that several alternatives are available from which exactly one must be chosen. (We exclude, therefore, the somewhat different problem of voting for representative bodies, to which several candidates may be elected simultaneously.)

The simplest voting problem is that in which there are just two alternatives, one of which is to be chosen. In this case, voting by simple majority rule strikes most people as fair and reasonable. Each voter votes for one or other alternative (or abstains), and whichever alternative receives more votes is selected. May (1952) formalized our intuition concerning majority rule: he identified four conditions that we probably want a voting rule to meet in a two-alternative contest, and he demonstrated that majority rule, and only majority rule, meets these conditions. May's conditions are: *decisiveness* -- however people vote, there is always a clear outcome (even if that is "social indifference," i.e., a tie); *anonymity* (of voters) -- we do not need to know who cast which votes to determine the outcome; *neutrality* (between alternatives) — if everyone voted in the opposite fashion (or continued to abstain), the other alternative would win (or, if the outcome were initially a tie, it would remain a tie); and *positive responsiveness* — if alternative *A* at least ties *B* and someone then changes his vote to make it more favorable to *A* (i.e., by voting for *A* instead of abstaining or voting for *B*, or by abstaining instead of voting for *B*), A then wins. May demonstrated that majority rule meets these four conditions and is the only decision rule that does so. (Decision rules distinct from majority rule can meet any three of them.)

In sum, voting based on majority rule to choose between two alternatives is essentially straightforward, though objections can still be raised against it. One common objection is that, on any particular decision, the winning majority may be, in some sense, "wrong" or misinformed. Another objection, stated in terms of political theory, is that an "apathetic" majority (with only weak preferences for alternative *A*) may override an "intense" minority (with strong preferences for alternative *B*); in economic terms, there is no assurance — supposing that some interpersonal accounting of costs and benefits is possible — that selection of *A* provides to the group as a whole greater benefits net of costs than selection of *B*. Finally, it may be remarked that, in some circumstances, one or more of May's conditions — and thus also majority rule itself — may not seem so fair and reasonable; an example may be provided if alternative *A* represents a fundamental change in constitutional arrangements and *B* represents maintenance of the constitutional status quo, in which case it may well seem appropriate to treat the alternatives in a non-neutral fashion (by, for example, requiring greater than majority support for the selection of *A*).

But more vexing problems arise when the domain of choice is expanded to three or more alternatives. Many different apparently fair and reasonable voting procedures are possible (and in actual use), all of which reduce to simple majority rule in the event there are just two alternatives, but which operate differently in the event there are three or more alternatives. It is not clear which, if any, of these procedures is the "natural" or appropriate extension of simple majority rule. On closer inspection, they all have serious flaws — that is, they turn our not to be so fair and reasonable; indeed such flaws appear to be unavoidable in the general case.

With three or more alternatives, a procedure may require voters to declare their preferences either "nominally" or "ordinally" -- a distinction that collapses when just two alternatives are being voted on. Under a *nominal* procedure, each voter divides the alternatives into two sets -- those he votes for and (implicitly) those he votes against. Under an *ordinal* procedure, each voter ranks orders the alternatives according to his preferences. (There are other ballot forms, but they are rarely used in practice.)

For descriptive purposes, we may assign commonly used voting procedures that select one alternative out of many to three broad types (for a more extended recent discussion see Dummett, 1984), which we may label *aggregation* procedures, *elimination* procedures, and *sequential binary* procedures. To simplify the following discussion, we sidestep the question of how procedures may break ties and we suppose voters are never indifferent between alternatives.

An aggregation procedure takes declared preferences and aggregates them in a single step to determine the selected alternative; thus only one vote is taken. The simplest voting procedure is *plurality* (or "first-past-the-post") voting: on a nominal ballot, each voter votes for no more than one alternative; the aggregation rule selects the alternative with the most votes. A recently proposed variant is *approval* voting (Brams and Fishburn, 1982): on a nominal ballot, each voter votes for any number of alternatives; the aggregation rule is the same as plurality. The most common aggregation procedure using an ordinal ballot is *preferential* (or *Borda count*) procedure. The aggregation rule is this: if there are m alternatives altogether, an alternative is awarded m points for each ballot on which it is ranked first, $m-1$ points for each on which it is ranked second, and so forth; the alternative with the most points is selected.

A elimination procedure initially aggregates declared preferences in some fashion, on the basis of which weaker alternatives are eliminated. A new vote is then taken on the remaining alternatives. (If an ordinal ballot was used at the outset, the original ballots can be reaggregated with the eliminated alternatives deleted from each ranking.) Elimination and revoting (or reaggregation) continue until every alternative but one has been eliminated. *Plurality plus runoff* voting initially aggregates in the manner of plurality voting, eliminates all alternatives except those receiving the most and second most votes, and holds a simple majority vote runoff between these two. The *alternative vote* procedure also aggregates in the manner of plurality voting, but only the alternative with the fewest number of votes is eliminated at each stage; thus $m - 1$ votes are required altogether. *Exhaustive* (or *Coombs*) procedure uses an ordinal ballot and eliminates from among the remaining alternatives the one with the most last-place, rather than the fewest first-place, preferences. Still other elimination procedures aggregate in the manner of preferential voting.

A sequential binary procedure is a voting procedure of the parliamentary type, in which a sequence of binary choices (e.g., yes or no) is put to the voters. A very simple sequential binary procedure — which approximately (but not exactly) mimics Anglo-American parliamentary voting — is what Black called *ordinary committee* procedure and is now generally referred to as standard *amendment* procedure: two alternatives are paired for a simple majority vote, the winner is paired with a third alternative for a second vote, and so forth until every alternative has entered the voting. The alternative that wins the final vote is selected. Another sequential procedure is variously referred to as *sequential elimination* or *successive* procedure: each alternative in turn is voted up or down on a simple majority vote; the first alternative to receive majority support is selected; if every alternative but one has been rejected, the one remaining alternative is selected by default. Under any sequential procedure, the alternatives must be placed in some kind of voting order; this raises the possibility that such procedures may violate the spirit of May's neutrality condition, in that whether an alternative is selected may depend on when it enters the voting.

The reader may easily check that each procedure described above reduces to simple majority rule in the event that there are just two alternatives. Moreover, at first blush, they all appear to be fair and reasonable -- in any case, certainly not perverse. Thus each procedure appears to be a natural extension of simple majority rule when the domain of choice is expanded beyond two alternatives. However, the reader may also check that, for given declarations of preferences by voters, each procedure may imply a different selected alternative. By way of partial illustration, consider the following declaration of preferences over four alternatives (the number above each ordering indicates the number of voters declaring such preferences):

Example 1	4	4	2	9
first preference	*A*	*B*	*B*	*C*
second preference	*B*	*A*	*D*	*D*
third preference	*D*	*D*	*A*	*A*
fourth preference	*C*	*C*	*C*	*B*

Under plurality voting, *C* is selected (with 9 votes, as opposed to 6 for *B*, 4 for *A*, and none for *D*). Under approval voting, if we suppose that each voter votes for his top two alternatives, *D* wins (with 11 votes, as opposed to 10 for *B*, 9 for *C*, and 8 for *A*). Under plurality plus runoff voting, *B* is selected (the runoff is between *B* and *C* and the four voters whose first preference *A* has been eliminated prefer *B* to *C*). The alternative vote, in this case, works in just the same way as plurality plus runoff. Exhaustive voting selects *D* (*C*, with 10 last-place preferences, is eliminated first, then *B* with the 9 last-place preferences, and then *A*). Preferential voting selects *A* (with 50 points, as opposed to *D* with 49 points, *C* with 46 points, and *B* with 45 points). With respect to sequential binary procedures, voting under both amendment and successive procedures voting can select any alternative other than *C* (which loses every possible pairwise vote), depending on the voting order (specifically, the alternative other than C that enters the voting last is selected).

In choosing among competing voting procedures, an appealing approach is to do what May did for simple majority rule — that is, identify a set of attractive criteria and then determine which procedure uniquely meets them. (See, for example, Young, 1974.) The problem here is that different procedures meet different sets of criteria, and no procedure meets all criteria that we might regard as necessary for a fair and reasonable system to meet. (In effect, voting theory runs up against Arrow's "general impossibility theorem" in social choice theory; cf. Arrow, 1951.)

A particularly severe flaw that affects all these voting procedures is that they are subject to *agenda manipulation* — that is, individuals who can add alternatives to, or delete alternatives from, the agenda of choice can influence the outcome *even if the alternatives that may be added or deleted cannot themselves win*. (It is this property of plurality voting that makes the presence or absence of "third-party" candidates, who cannot themselves win, so significant in British parliamentary elections or U.S. Presidential elections.) Consider Example 1 one again. If all four alternatives are on the agenda, *C* is selected under plurality voting, but if *A* is removed from the agenda (and thus deleted from each preference ordering), *B* is selected. (This is why *B* wins under plurality plus runoff voting. More generally, it is only because the elimination of alternatives can alter the relative strength of surviving alternatives under aggregation procedures that there is any reason to devise elimination versions of these procedures.) Similar illustrations could be provided for other procedures. Thus voting under such procedures violates the Weak Axiom of Revealed Preference — which economists usually take to be an aspect of rational choice -- and indeed weaker consistency criteria as well.

Let us now consider one apparently attractive approach to extending simple majority rule to the multi-alternative case that none of the procedures described above exactly implements (for good reason, it turns out). Let us consider the *majority preference relation* — that is, simple majority rule between all pairs of alternatives. Consider the following declaration of preferences by five voters (we will discuss the "social ordering" momentarily):

Example 2	2	1	2	*social ordering*
first preference	*A*	*B*	*C*	*B*
second preference	*B*	*A*	*B*	*A*
third preference	*C*	*C*	*A*	*C*

We may note that B, though it has the fewest first preferences and would lose under many procedures, has a particular strength and perhaps a strong claim to be the alternative that should be selected. This is due to the fact the B can defeat each other alternative in a pairwise vote (or "straight fight") under simple majority rule. An alternative that can do this is called the *Condorcet winner*, and the criterion for voting procedures which requires that the Condorcet winner be the selected alternative is called the *Condorcet criterion*. Every procedure described above, other than the standard amendment procedure, violates this criterion -- that is, we can find some declaration of preferences such that the procedure selects an alternative other than the Condorcet winner.

The approach of looking at pairwise contests based on majority rule apparently has this further attraction: for the example above, we can identify not only the Condorcet winner but a "social ordering" based on majority rule, as shown above -- that is, A is majority preferred to both B and C, and B is majority preferred to C. Given such a "social ordering," if it turned out that A was in fact not a feasible alternative, the group could simply move to B as its second "social preference." The majority preference relation, moreover, is quite immune to agenda manipulation, as majority preference between two alternatives depends only on individual preferences between the same two alternatives and is unaffected by the presence or absence of other alternatives or by changes in individual preferences among alternatives other than the two in question.

The majority preference relation has further descriptive significance. Most electoral and legislative voting rules are *majoritarian* in nature — that is, they empower any majority of voters acting in concert to select whatever alternative that majority agrees upon. Thus to say *A* is majority preferred to *B* is equivalent to saying, in the language of cooperative game theory, that *A dominates B*, i.e., that there is a coalition of individual who all prefer *A* to *B* and who collectively have the power to bring about *A*. The Condorcet winner is, therefore, the *undominated* or *core* alternative. Thus, if voters treat voting as a game of strategy in which coalitions can form freely, the outcome will be determined by the majority preference relation, independent of the particular (majoritarian) voting procedure nominally in use.

It may appear, therefore, that we have satisfactorily solved the problem of generalizing majority rule to the multi-alternative case, but unfortunately we have not. The reason is that majority preference (like game-theoretic domination) does not in general generate a "social ordering." This is illustrated by Example 1, in which it may be checked that, in pairwise votes, *A* defeats *B*, *B* defeats *D*, and D defeats *A*. (It was for this reason that the selected alternative under amendment procedure depended on the order of voting.) This phenomenon is variously called the "paradox of voting," the "Condorcet effect," the "Arrow problem," and the phenomenon of "cyclical majorities." It is most simply illustrated by the following three voter, three alternative example.

<u>Example 3</u>	<u>1</u>	<u>1</u>	<u>1</u>
first preference	*A*	*B*	*C*
second preference	*B*	*C*	*A*
third preference	*C*	*A*	*B*

This phenomenon evidently was first discovered by Condorcet, and it was then alternately forgotten and rediscovered until the work of Black and Arrow appeared in the late 1940s. It results from some declarations of preferences (e.g., Example 3) but not others (e.g., Example 2). The question naturally occurs of whether we can specify general conditions on preference declarations under which the paradox of voting does, and does not, occur.

The most obvious condition that excludes the paradox is *majority consensus*, i.e., a majority of voters declare the same preferences; but we may note that this does not explain the absence of paradox in Example 2. What is true in Example 2 is that the declared preferences are — to use the term introduced by Black (1948) — *single-peaked*. (See Sen, 1966, for generalization of this concept.) What this means is that the declared preferences are consistent with the supposition that the alternatives are perceived by all voters as arrayed along a single dimension of evaluation. For example, three alternatives might be arrayed along an ideological dimension such that one is the (relatively) "leftist" alternative, another is the (relatively) "rightist" alternative, and the third is the "centrist" alternative that represents a compromise between the other two. If all voters structure their preferences accordingly, it follows that there is some alternative — namely, the centrist one — that no voter ranks last. Then in turn it follows that either an absolute majority of voters prefers one or other extreme alternative or the centrist alternative defeats each extreme alternative in a pairwise majority vote (since the voters who most prefer one extreme alternative prefer the centrist alternative to the other extreme); in any event there is a Condorcet winner. It may be checked that the declared preferences in Example 2 meet the single-peakedness condition (with B the alternative that no one ranks last), while the preferences in Example 3 do not.

The notion of single-peaked preferences extends readily to a continuum of alternatives. Each voter has an *ideal point* of highest preference or maximum utility somewhere along the continuum and his utility declines as distance from his ideal point increases in either direction.

Whether alternatives are discrete points along a dimension or a continuum of points, if preferences are single-peaked voter ideal points can be rank ordered from left to right (or whatever is the nature of the evaluative dimension). It then follows that the alternative *M* corresponding to the median of voter ideal points is the Condorcet winner. This is the *median voter theorem* due originally to Black (1948, 1958). Consider any point *A* to the left of *M*. *M* is preferred to *A* by the median voter and all voters whose ideal points lie to the right of *M* and, by definition of a median point, this is a majority of the voters. Obviously the same argument can be made for any point *B* to the right of *M*. Thus *M* defeats every other point and is the Condorcet winner.

The notion of single-peaked preferences can be generalized to a multidimensional space of alternatives, where each point in the space represents a different combination of policies, programs, appropriations, points on distinct evaluative dimensions, or whatever. Generalized to this setting, the notion requires that all voter preferences with respect to sets of alternatives lying on any straight line through the space be single-peaked. This is equivalent to the standard economic assumption that individual preferences on a space (of, for example, commodity bundles) be convex. But, in the multidimensional case, there almost never is a point that is the median ideal point in all directions, so there is almost never a Condorcet winner, and cyclical majorities almost always exist (Plott, 1967). Moreover, it turns out that, in the almost certain event that there is no Condorcet winner, a massive majority cycle encompasses all points in the space (McKelvey, 1979). Despite all this, recent work indicates that, even in the multidimensional case, common voting processes, in particular those of a competitive nature, lead to selection of more or less centrist alternatives.

Throughout the discussion thus far, we have consistently sidestepped one further complexity in voting. Voting procedures operate on the *declared preferences* on voters. The question arises of whether it always is expedient for voters to declare (or reveal) their "honest" or "sincere" preferences. In fact, it well known to both students and practitioners of politics that, under common voting procedures, voters who cast "honest" votes may regret doing so. For example, suppose the preferences displayed in Example 1 are actually the honest preferences of all voters. Under plurality voting, alternative *C* is selected, if preferences are honestly revealed. But it would be to the advantage of the four voters whose preference ordering appears in the first column to declare their preferences otherwise, specifically by ranking *B* first, for then *B* — which they all prefer to *C* — would be selected. For another example, suppose the preferences displayed in Example 3 are actually honest preferences. Under standard amendment procedure with the alternatives voted on in alphabetical order, *A* defeats *B* in the initial vote and *C*, which defeats *A* in the second vote, is ultimately selected. However, if the voter whose preference ordering appears in the first column were to vote insincerely for *B* instead of *A* at the first vote, *B* would be ultimately selected and that voter prefers *B* to *C*. In general, if voting is treated as a game of strategy, voting in a manner that reveals true preferences may not be the best strategy.

Several questions then naturally occur. First, if voting is treated as a game of strategy, is it possible to identify "best" strategies for all voters? If so, and if all voters use their best strategies, is the selected alternative different from what would be selected if all voters used honest strategies? (Note that, in the two examples above, we did not consider possible counter-strategies of the remaining voters.) If the outcomes are different, how do the "strategic" and "honest" outcomes compare? Finally, it is possible to design a voting procedure such that best and honest strategies always coincide for all voters — that is, can we devise a "strategyproof" voting procedure?

The first question was first systematically treated by Farquharson (1969), who introduced the concept of *sophisticated* voting, i.e., voting that is strategically optimal, which is in general different from *sincere* voting, i.e., voting that honestly reveals preferences. Farquharson stated a theorem that says this: if no voters are indifferent between alternatives, sophisticated voting under any sequential binary procedure is determinate, i.e., the game of strategy has a definite solution. However, Farquharson's method for solving such voting games, based on successive elimination of dominated strategies, is cumbersome to employ in even the simplest situation and, for all practical purposes, impossible to employ if there are more than a few voters or alternatives. Fortunately, an alternative definition of sophisticated voting under sequential binary procedures and an alternative and much easier method of solution, exist. This is the *multistage* or *tree* method, which has been definitively characterized by McKelvey and Niemi (1978).

Using this method, sophisticated voting outcomes under binary procedures may easily be identified and compared with sincere outcomes. First, sincere and sophisticated outcome often diverge — that is, strategic behavior on the part of all voters does not necessarily "cancel out." Second, and perhaps contrary to "common sense" expectations, sophisticated voting outcomes are, by several criteria, superior to sincere outcomes. (For example, sophisticated voting, but not sincere, always complies with the Condorcet criterion.) Third, if voting is sincere, alternatives are favored by being placed later in the voting order; if voting is sophisticated, the reverse is true. Finally, these differential effects are magnified to the extent that majority preference is cyclical.

With respect to the final question, voting theorists conjectured for many years that a strategyproof voting procedure could not exist, but two fundamental problems stood in the way of decisively demonstrating this. First, it is not at all clear how to define the class of objects that we might call "voting procedures." Thus, no matter how many procedures we can demonstrate to be vulnerable to strategy, there seems always to be the logical possibility that something else exists that we might be willing to call a "voting procedure" and that is strategyproof. Second, especially with more exotic procedures (e.g., approval voting), it is not always clear what constitutes "sincere" or "honest" voting.

Gibbard (1973) neatly sidestepped both of these problems and proved the conjecture. He did this by solving a much more general problem in game theory. First, he said, however we define the set of all voting procedures, it is certainly a subset of all "game forms," where a *game form* is a game (in the sense of game theory) minus the preferences of players over outcomes. A game form is *dictatorial* if there is some player who, for every outcome of the game, has a strategy that is *decisive* for that outcome, i.e., its selection guarantees that outcome, regardless of the strategy selections of the other players. In a game, a strategy is *dominant* for a player if he would never regret selecting it, regardless of the strategies selected by other players. A game form is *straightforward* if it gives every player, for all possible preferences over outcomes, a dominant strategy. Gibbard then proved (using Arrow's theorem) that every straightforward game form with three or more outcomes is dictatorial.

Now suppose that a voting procedure is strategyproof. Then no voter, regardless of his preferences, can ever have reason to regret voting sincerely, regardless of how other voters vote. But this means that every voter, regardless of his preferences, must always have a dominant strategy (which, moreover, must be a sincere strategy). But, even apart from the requirement that the dominant strategies be sincere, this requires that the voting procedure be a straightforward game form. Thus, once we move beyond choice between just two alternatives, and at the same time make selection depend on the declared preferences of more than one individual, we cannot avoid the possibility that individuals may have an incentive to declare other than their true preferences.

REFERENCES

Arrow, K. (1951). *Social Choice and Individual Values*, Cowles Foundation monograph no. 17, New York, Wiley.

Brams, S., and P. Fishburn (1983). *Approval Voting*, Boston, Birkhauser.

Black, D. (1948). On the rationale of group decision-making, *Journal of Political Economy*, February.

Black, D. (1958). *The Theory of Committees and Elections*, Cambridge, Cambridge University Press.

Dummett, M. (1984). *Voting Procedures*, Oxford, Clarendon Press.

Farquharson, R. (1969) *Theory of Voting*, New Haven, Yale University Press.

Gibbard, A. (1973). Manipulation of voting schemes: A general result, *Econometrica*, July.

May, K. (1952). A set of independent necessary and sufficient conditions for simple majority rule, *Econometrica*, January.

McKelvey, R. (1979). General conditions for global intransitivities in formal voting models, *Econometrica*, September.

McKelvey, R., and R. Niemi (1978). A multistage game representation of sophisticated voting for binary procedures, *Journal of Economic Theory*, January.

Plott, C. (1967). A notion of equilibrium and its possibility under majority rule, *American Economic Review*, September.

Sen, A. K. (1966). A possibility theorem on majority decisions, *Econometrica*, April.

Young, H. P. (1974). An axiomatization of Borda's rule, *Journal of Economic Theory*, September.