ELECTION INVERSIONS
UNDER PROPORTIONAL REPRESENTATION

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Abstract

Kurrild-Klitgaard (2013) has shown that proportional representation can produce "election inversions" such that a coalition of parties collectively supported by a majority of voters fails to win a majority of seats, and he identifies several empirical examples under the Danish electoral system. However, Kurrild-Klitgaard's examples result from imperfections in its proportional representation system deliberately introduced to serve goals other than proportionality. Here I carry Kurrild-Klitgaard's analysis a step further by showing that election inversions can occur even under the purest types of proportional representation — namely, those with (i) a single national constituency, (ii) no explicit threshold requirement, and (iii) a highly proportional electoral formula. Inversions result from the "whole number problem" that no proportional representation system can in practice avoid. I examine the most recent election data from Israel and the Netherlands to look for empirical examples of inversions under their relatively pure PR systems. I also look for inversions after recalculating seat allocations without a threshold requirement and on the basis of the most proportional electoral formulas and examine whether inversions occur when the analysis is restricted to seat-winning parties. Finally I reexamine the Danish data in these respects and also examine the most recent U.S. apportionment of House seats. Many empirical examples of inversions are found.

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ELECTION INVERSIONS UNDER PROPORTIONAL REPRESENTATION

For more than a hundred years political scientists and other commentators have observed that outcomes under "majoritarian" electoral systems — in particular, single-member district plurality systems (or First-Past-the-Post, as the British say) — do not stack up well when evaluated by proportional criteria. Some twenty years ago, van Deemen (1993) turned this long-standing argument on its head by observing that proportional representation (PR) systems may not do well when evaluated by "majoritarian" (in particular, Condorcet) criteria. Using a combination of election and survey data, Van Deemen and Vurgunst (2008) and Kurrild-Klitgaard (2008) later provided empirical examples of "paradoxical" outcomes under the Dutch and Danish PR systems,

More recently, Kurrild-Klitgaard (2013) has shown that proportional systems do not always comply with a criterion that seems compelling by both proportional and majoritarian standards: more votes should mean no fewer seats. In particular, Kurrild-Klitgaard has shown that proportional representation systems can produce "election inversions" of the sort that awarded a majority of electoral votes (and the U.S. Presidency) to George W. Bush in 2000, despite the fact that more people had voted for Albert Gore. Since proportional representation systems typically produce parliaments in which no party wins a majority of either votes or seats, in this context an election inversion can be defined as a situation in which a coalition of parties collectively supported by a majority of voters fails to win a majority of seats and, conversely, the complementary coalition supported by only a minority of voters wins a majority of seats. Kurrild-Klitgaard identifies several empirical examples of such inversions under the Danish proportional representation system over recent decades.

However, Kurrild-Klitgaard's examples result from *avoidable* imperfections in the Danish (and most other) proportional representation systems. While these imperfections are deliberately introduced to serve goals other than proportionality (e.g., providing local representation, deterring excessive party proliferation), they do mean that the proportional representation system is not as pure as it might be. In this note, I carry Kurrild-Klitgaard's analysis a step further by showing that election inversions can and do occur even under the purest types of proportional representation —namely, ones that (i) use a single national constituency, (ii) impose no explicit threshold requirement, and (iii) employ a highly proportional electoral formula. This is because even the purest proportional representation systems have *unavoidable* imperfections that result from the "whole number problem" — that is, the fact parties must be awarded seats in terms of relatively small whole numbers, while their vote support comes in terms of far larger numbers, making party vote shares essentially continuous quantities. As a result, perfect proportionality can be achieved only in very special cases.

After establishing this theoretical point, I examine recent election data from Israel and the Netherlands to look for empirical examples of election inversions under proportional representation. Both countries are noted for having relatively pure PR systems, by having a single nationwide constituency combined with relatively low threshold requirements. However, both countries use the less proportional D'Hondt formula. I therefore also look for inversions after recalculating seat allocations on the basis of the highly proportional Largest Remainder and Sainte-Laguë formulas and without an explicit threshold requirement, and I examine whether inversions occur when the analysis is restricted to seat-winning parties only. Finally I reexamine Kurrild-Klittgaard's Danish data in

these respects, and I also examine the most recent apportionment of U.S. House seats. Empirical examples of inversions are found in most circumstances.

1. Election Inversions

Following Miller (2012, p. 93), "an *election inversion* occurs when the candidate (or party) that wins the most votes from the nationwide electorate fails to win the most electoral votes (or parliamentary seats) and therefore loses the election." Discussions of the U.S. Electoral College have used such terms as "misfire" (Peirce and Longley, 1981), "wrong winner" (Abbot and Levine, 1991), and "reversal of winners" (Edwards, 2004) to refer to this phenomenon, while the more theoretical literature on voting and social choice has used such varied terms as "compound majority paradox" (Nurmi, 1999), "referendum paradox" (Laffond and Laine, 2000), "majority deficit" (Felsenthal and Machover (1998), and "representative inconsistency" (Chambers, 2008). As noted above, the U.S. Electoral College produced an inversion in 2000; in like manner, the postwar Labour Government in the U.K. was turned out of office in 1951 by the Conservative Party even though Labour candidates won more votes nationwide than the Conservative candidates. Each of these instances occurred in essentially two-party contexts and entailed what we may call a *partywise* inversion, i.e., party A wins more votes, while party B wins more seats. Partywise inversions may also occur in a multi-candidate or multi-party context. For example, in the four-candidate 1860 U.S. Presidential election, Douglas (Northern Democrat) received more popular votes than either Brenkinridge (Southern Democrat) or Bell (Constitution Union), but Brenkenridge and Bell each won more electoral votes than Douglas. These inversions were inconsequential, since Lincoln (Republican) won a 39.65% plurality of the popular vote and a majority of electoral votes and the presidency.[1] Inconsequential partywise inversions have also occurred in British parliamentary elections; for example in 1997, the Ulster Unionist Party won 10 seats with less than half the electoral support of the Scottish National Party, which won only 6 seats.

If the number of seats is even (as is true of the present U.S. Electoral College), even a strictly two-party (or two-candidate) contest may produce a tie with respect to seats, even though almost certainly one party (or candidate) wins more votes than the other. Regardless of whether the number of seats is even or odd, if three or more parties win seats, two (or more) parties may win equal numbers of seats, even though almost certainly one wins more votes. We refer to such events as *partial* inversions.[2]

But, as previously noted, in the context of proportional representation it make sense to focus (as Kurrild-Klitgaard does) on *coalitionwise* inversions, in which a coalition of parties collectively supported by a majority of voters fails to win as many seats as the complementary coalition of all

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[1] However, as noted in Miller (2012), Lincoln would have won a majority of electoral votes even if supporters of the three other candidates had united behind a single ticket, which would have thereby won 60.35% of the popular vote and produced a massive election inversion.

[2] The opposite type of partial inversion — when two parties are tied with respect to votes but one wins more seats than the other — is logically possible but obviously far less likely.

other parties supported by a minority of voters. As we shall see, coalitionwise inversions can occur under even the purest types of PR systems, even though partywise inversions cannot occur under undistricted systems.

2. Election Inversions Under the Danish Proportional Representation System

Kurrild-Klitgaard's (2013) empirical examples of election inversions under proportional representation occurred under the Danish electoral system, which provides for (i) 135 seats apportioned on the basis of population among about a dozen multi-member districts of varying magnitudes, (ii) 40 national adjustment seats, and (iii) two seats for each of two autonomous overseas territories (Greenland and the Faroe Islands). The latter four seats are guaranteed regardless of population and give the overseas territories disproportionate representation. The 40 national adjustment seats effectively create a single "continental" constituency with 175 seats allocated among political parties on the basis of the (modified) Sainte-Laguë formula with a 2% threshold.[3] Therefore, while the regional districts provide a measure of geographical representation, their existence has almost no impact on the overall allocation of seats to parties.[4] Thus, the Danish system departs from pure PR in primarily in two respects: the "federal" status (to use Kurrild-Klitgaard's terminology) of Greenland and the Faroe Islands and the 2% threshold. The instances of election inversions that Kurrild-Klitgaard identifies are due to what he calls "federal effects" and "threshold effects," which result from these two imperfections in the Danish PR system.

"Federal effects" are a special case of what Miller (2012, pp. 108ff) calls *apportionment effects* in an electoral system based on states, regions, or other geographically defined districts, and they cannot occur in undistricted systems. A districted electoral system is *perfectly apportioned* if the number of votes cast in each district is precisely proportional to the number of seats (or electoral votes) awarded to each district. Given a system with *uniform* districts (each having the same number of seats), perfect apportionment requires that precisely the same number of votes is cast in each district. In practice, perfect apportionment cannot hold because districts inevitably are not precisely equal in population (or eligible voters) and voting turnout inevitably varies somewhat across districts. Given a system with *non-uniform* districts (such as states in the U.S. Electoral College), the same considerations lead to imperfect apportionment. Moreover, non-uniform districts cannot be allocated seats precisely proportional to their population (or eligible voters) for the same reason that parties cannot be allocated seats precisely proportional to their votes, i.e., the "whole number problem."[5] In addition, a districted system may be "federal" in nature in that it deliberately favors some (typically smaller) districts in apportioning seats, e.g., small states in the U.S. Electoral College and the

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[3] See Kurrild-Klitgaard (2013) and Elklit (1993). Our concern here is only how seats are allocated among parties, not how individual candidates fill these seats.

[4] However, a party retains whatever seats it wins at the district level, even if this gives it more seats than it is entitled to on the basis of its national vote (Elklit, 1993, pp. 42, 44).

[5] However, fractional electoral (or other weighted) votes, in contrast to electors or seats, are feasible, and it is possible to recalculate U.S. Presidential election results on the basis of perfectly apportioned electoral votes; see Miller, 2012, pp. 111-114).

overseas territories in the Danish electoral system. If different parties have differing strengths in different (e.g., smaller vs. larger, low-turnout vs. high-turnout) districts, imperfect apportionment may create partisan bias in the electoral system that in turn can produce election inversions.

Kurrild-Klitgaard (2013, Table 3) provides a hypothetical example of federal effects causing a partywise inversion in a proportional representation system with two districts, the smaller of which has disproportionate representation. He also tracks down a probable empirical example of an election inversion due to federal effects (Table 4). In 1971, the coalition of all center-right parties gained a bare majority of both votes and seats in "continental" Denmark. But a left-of-center coalition secured the support of the three of the four overseas representatives, thereby securing a bare majority seats in parliament without also securing a majority of the vote nationwide. However, due to peculiarities in the party systems in the two territories, there may be some ambiguity as to whether this represented a true coalitionwise inversion.

Kurrild-Klitgaard's more compelling example of an election inversion was produced by "threshold effects," which constitute a special case of what Miller (2012: pp. 112ff) calls *distribution effects*. In non-PR systems, distribution effects result when one party's vote is more "efficiently" distributed over districts than another's. For example, in a two-party system, party A may win a minority of districts (or states with a minority of the electoral votes) by large margins, while party B wins a majority of districts (or states with a majority of the electoral votes) by small margins, with the result that party B wins a majority of the seats (or electoral votes) with a minority of the votes.

Under proportional representation, the principal source of "inefficiently" distributed votes occurs when several small parties fail to win any seats, whereas a single larger party with the same total vote would win one or more seats.[6] Kurrild-Klitgaard (2012, Table 2) first provides a wholly hypothetical example of such an inversion. A less hypothetical but (thus far) unrealized example of an election inversion under PR due to threshold effects might occur in the context of contemporary German politics, where recent elections have typically been contested between rival center-right (CDU/CSU+FDP) and center-left (SPD+Greens) prospective governing coalitions. If the smaller party in one coalition but not the other were to fall below the 5% threshold of the German electoral system, an inversion might occur, as is illustrated in Table 1.[7]

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[6] Even in the absence of an explicit threshold requirement, small parties may fail to win seats by virtue of the normal operation of the electoral formula applied to the numbers of seats available. Such an "implicit threshold" cannot be specified as a particular percent of the vote (in the manner of an "explicit" threshold), because under every PR formula the number of seats a party wins (and whether a small party wins even one seat) depends not only on its vote share but also on how the residual vote is distributed among the other parties. It is precisely this fact that implies that coalitionwise, but not pairwise, inversions can occur under pure PR; it also leads to the distinction between "thresholds of representation" and "thresholds of exclusion" (Rae et al., 1971).

[7] However, anticipation of such an event would encourage some supporters of the larger party to vote "tactically" for the smaller party in the coalition. A special clause in the German Federal Electoral law guarantees that a party (but not a coalition) that wins a majority of the votes must receive a majority seats (Friedrich Pukelsheim, personal communication).

Kurrild-Klitgaard's (2012: Table 5) empirical example pertains to the 1990 election in which, with respect to "continental" Denmark only, the parties of the left collectively won a bare majority of the votes but the parties of the center-right won a decisive majority of the seats, while no plausible accounting for votes in the overseas territories could give the center-right party an overall majority of the vote. The inversion resulted because a number of small leftist parties (but only one rightist one) each fell below the 2% threshold and failed to win any seats, despite collectively winning 4.4% of the total vote. Like supporters of minor parties under First-Past-the-Post, supporters of these parties "wasted" their votes; if they had voted "tactically," they could have concentrated their votes on a single leftist party that would have met the 2% threshold and won an (approximately) proportional share of the seats and given the left coalition a parliamentary majority.[8]

3. Election Inversions under "Pure" Proportional Representation

We now show that neither federal (or other apportionment) effects nor threshold effects are necessary for election inversions under proportional representation. The claim is that, *provided that there are three or more parties, coalitionwise election inversions can occur under even the purest types of proportional representation, i.e., those with a single nationwide district, no threshold requirement, and a highly proportional electoral formula*. The possibility of coalitionwise inversions results from the unavoidable "whole number problem."

At this point, it is in order to note that the word "coalition" can be used in two distinct ways in formal political theory. In the more general sense, a *coalition* refers merely to any set of players (e.g., voters or parties); this is the standard terminology in the theory of simple games (e.g., Shapley, 1962) and in voting power theory (e.g., Felsenthal and Machover (1998). In the narrower sense, a *coalition* is a particular set of players who enter into an agreement, e.g., a "governing coalition." We use the term here in the more general sense but should note that Kurrild-Klitgaard's examples of coalitionwise inversions in Danish politics pertain to sets of ideologically compatible parties that became (governing) coalitions in the narrower sense.

We now discuss the specifics of several PR formulas.[9] Let us first define party K's *quota* Q_k of seats as the "quantity" (as opposed to "number") of seats that is precisely proportional to its vote share, i.e., $Q_k = S \times V_k / \Sigma V$, where S is the number of seats in parliament, V_k is the number of votes for party K, and ΣV is the total vote for all parties.

In the extraordinarily unlikely event that the quota for every party is a whole number, every party can be awarded seats equal to its quota and a perfectly proportional allocation of seats is

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[8] Monroe and Rose (2002) show how threshold effects in conjunction with non-uniform districts can produce partisan bias (and therefore potential election inversions) due to the "variance effect" resulting from non-uniform district magnitudes (even if the non-uniform districts are perfectly apportioned). "Reinforced" PR (as in Italy and Greece), which gives the party winning the plurality of votes a seat bonus, can straightforwardly produce election inversions.

[9] This following discussion relies heavily on Balinski and Young (1982), who discuss PR formulas primarily in the context of apportioning House seats among the U.S. states.

possible (i.e., the "whole number problem" does not arise); in this event, all PR formulas produce the perfect apportionment.[10] Otherwise, some or (almost certainly) all parties cannot be awarded their precise quota of seats, and different formulas may produce different apportionments.

The most common PR formulas are the Largest Remainder-Hare (LR-H), Sainte-Laguë, and D'Hondt systems. LR-H is a *quota* method: it gives each party its quota rounded down to the nearest whole number of seats and allocates any remaining seats to the parties with the "largest remainders," where a party's *remainder* is the difference between its quota and its quota rounded down. Sainte-Laguë and D'Hondt are *divisor* methods: they divide each party's vote by a common divisor approximately equal to the "ratio of representation" (i.e., $\Sigma V / S$) but adjusted up or down so that, when the resulting *quotients* are rounded according to some rule, they add up to the number of seats available. Divisor methods differ according to the rounding rule used: under D'Hondt, quotients are always rounded down; under Sainte-Laguë, quotients are rounded in the normal manner, i.e., up or down to the nearest whole number. The D'Hondt formula favors large parties and often violates "upper quota" by giving (large) parties more seats than their quotas rounded up; while it never violates "lower quota" by giving (small) parties fewer seats than their quotas rounded down, the quotas of small parties are very likely to be rounded down. In contrast, LR-H and Sainte-Laguë formulas exhibit no tendency to favor either large or small parties (Schuster et al., 2003). By design, LR-H "stays in quota," i.e., gives every party its quota rounded up or down. Like every divisor method, Sainte-Laguë may violate quota, but it is less likely to do so than any other divisor method and in fact almost always stays in quota. (A number of nations, including Denmark, use a modified Sainte-Laguë formula that is slightly more favorable to large parties and slightly more likely to violate quota.) LR-H and (unmodified) Sainte-Laguë have been reckoned to be the "most proportional" PR formulas, taking account of a variety of proportionality criteria simultaneously (Pennisi, 1998), and they usually (but not always) allocate seats the same way.

We are now in a position to demonstrate the central claim: given three or more parties, coalitionwise inversions can occur under even the purest types of PR. Since the claim is only that such inversions are possible, it is sufficient to provide a hypothetical example. Table 2 provides two closely related examples, both involving just three parties and 35 seats. To implement the LR-H formula, quotas are calculated for each party. In the first example, the quotas rounded down sum to 34; party A has the largest remainder and gets the extra seat for a total of 18 out of 35, despite having fewer votes than the coalition of B and C. In the second example, the quotas rounded down sum to 33; parties B and C have the two largest remainders and get the two extra seats for a collective total of 18 out of 35, despite the coalition of B and C having fewer votes than party A. The Sainte-Laguë formula produces the same seat allocations and thus the same inversion examples.[11]

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[10] A formula that did not do this could hardly be deemed "proportional." Balinski and Young (1982, p. 97) call this minimal property *weak proportionality*.

[11] To apply the Sainte-Laguë formula, suppose there are 1000 voters, so the ratio of representation is $1000/35 \approx 28.571$. In the first example, any divisor between 28.1 and 28.4 gives quotients that equal the seats shown when rounded in the normal manner. In the second example, any divisor between about 28.743 and 28.952

It is important to note that PR formulas treat "coalitions" of parties and "fusions" of parties differently, and therefore permit coalitionwise inversions but not partywise inversions. If parties B and C were fused into a single party, no inversions would occur. In the first example, adding the quotas of B and C gives the fused party the largest remainder and its eighteenth seat. In the second example, adding the quotas of B and C gives the fused party 17 seats at the outset, but A now has the largest remainder and wins its eighteenth seat. Sainte-Laguë (and D'Hondt) produce the same seat allocations and the same non-inversions. This illustrates the point asserted earlier that, if party A wins more votes than party B, every PR formula awards A at least as many seats as B. It also illustrates why the central theoretical claim includes the stipulation that there must be three or more parties.

Clearly the vote profiles in Table 2 were carefully devised, by giving party A either slightly more or slightly less than half of the vote and "tuning" the division of the residual vote between parties B and C in relation to the total number of seats. Given only three parties, a coalitionwise inversion clearly is not possible if the leading party gets substantially more or less than 50% of the vote. However, if there are relatively many parties with varying levels of vote support, inversion possibilities may become more frequent.[12]

4. Finding Inversion Possibilities

In the almost certain event that parties cannot be awarded their precise quota of seats, even under the purest PR system some parties get a (fractionally) greater "quantity" of seats than their quota and other a lesser "quantity." Let us call the difference between the number of seats a party wins (under a given formula) and its quota the party's (positive or negative) *seat differential*. As an accounting identity, the seat differentials of all parties add up to zero. Since the LR-H formula stays in quota, the seat differential of every party is between +1 and −1. Since the Sainte-Laguë formula almost always stays in quota, the seat differential of every party is almost always between +1 and −1. Since the LR-H and Sainte-Laguë formulas exhibit no tendency to favor large parties over small ones, the differentials of seat-winning parties are unrelated to party size, except that the differentials of non-

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will do. D'Hondt gives the same seat allocation in the first example but gives party A an extra seat at the expense of C in the second example and thus does not produce an inversion.

[12] Table 2 is not the first published example of an election inversion under pure PR; a footnote in van der Hout and McGann (2009: p. 744) provides an example (credited to Iain McLean). But this example includes a small party that fails to win any seats, as it falls below the implicit threshold of the Sainte-Laguë formula. While van der Hout and McGann appear to attribute the possibility of inversions (characterized in terms of "manufactured majorities") exclusively to the presence of small non-seat-winning parties, Table 2 shows that the presence of such parties is not necessary. It should also be noted that van der Hout and McGann's example runs somewhat contrary to their overall theoretical analysis, which assumes that perfect proportionality, in that "seat shares are infinitely divisible, to abstract from rounding problems, " (p. 751). (Given their theoretical goal, this assumption is perfectly reasonable.) If, as their example (and our Table 2) shows, we do not abstract from rounding problems, the weak version of their Proposition 1 — which (in effect) claims that (full, though not partial) partywise inversions cannot occur under pure PR — holds, but (even) the weak version of their Proposition 2 — which (in effect) claims that (full, though not partial) coalitionwise inversions likewise cannot occur — does not hold.

seat-winning parties are all negative. Since the D'Hondt formula favors large parties and may violate upper quota, large parties may have seat differentials greater than +1, while small parties almost always have differentials that are negative but greater than −1).[13]

Given a profile of party votes and seats, we can search for possible coalitionwise inversions by calculating seat differentials and then looking for coalitions that control a small majority of seats and are composed of parties all or most of which have positive seat differentials, and which therefore are likely to have the support of less than a majority of voters. This may not be possible, because election inversions under proportional representation are close-run things, involving complementary coalitions each with very close to half of the votes and seats, which many party configurations do not allow. Generally speaking, the likelihood of finding inversions increases as the number of parties and their variability of seat shares increase. Clearly if one party has a majority of both votes and seats, no inversion is possible. If no or few parties win just one or a few seats, it is less likely that we can find coalitions of just the right seat size. The effect of increasing the number of seats is unclear. On the one hand, as the number of seats increases, all PR formulas become more proportional, the absolute magnitudes of seat differentials become smaller relative to the total number of seats, and generally the "whole number problem" becomes less significant, suggesting the likelihood of inversions may decrease. On the other hand, as the number of seats increases, the number of small seat-winning parties can be expected to increase (Taagepera, 2007), making it more likely that coalitions of just the right size exist. However, at the logical lower limit of a single seat, where every PR formula is equivalent to plurality rule, every configuration in which no party is supported by a majority of votes entails a coalitionwise inversion.

Finally, we may note that, as suggested by van der Hout and McGann (2009) and by Kurrild-Klittgaard's (2013) threshold effects example, there is one easy way to construct hypothetical examples of inversion possibilities. This is to allow the number of small non-seat-winning parties to proliferate to the extent that they collectively win a significant proportion of the total vote. It then will be easy to find coalitions of seat-winning parties that control a majority of seats with less than a majority of the total vote. Therefore, we further inquire whether we can find empirical examples of election inversions under PR where the analysis is restricted to seat-winning parties and the votes cast for them.[14]

5. Election Inversions in Israel, the Netherlands, and Elsewhere

We now examine recent election data from Israel and the Netherlands to look for empirical examples of election inversions under proportional representation. Both countries are noted for

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[13] See the analysis of seat biases under different PR formulas by Schuster et al. (2003).

[14] Note that if we reckon inversions with respect to all (vote-winning) parties, every U.S. presidential elections that produces a "minority President" (who wins with a "manufactured majority" of electoral votes) entails a coalitionwise inversion even if only two candidates win electoral votes, e.g., in 1992 the "coalition" of Bush+Perot won more votes than Clinton but Clinton won more electoral votes. However, such an event would not ordinarily be recognized as a election inversions in what remains a preponderantly two-party system.

having relatively pure PR systems, particularly by having a single nationwide constituency combined with relatively low threshold requirements.[15] However, both countries use the less proportional D'Hondt formula. We therefore also look for inversions after recalculating seat allocations without a threshold requirement and on the basis of the LR-H and Sainte-Laguë formulas. We also examine whether inversions occur when the analysis is restricted to seat-winning parties only.

Table 3 shows the results of the 2009 election in Israel.[16] To say Israel has a multiparty system is an understatement. However, only twelve parties met the 2% threshold and were awarded seats. Since the Israeli parliament has an even number of seats (120), we might not expect to find examples of (full as opposed to partial) inversions, since a coalition with electoral support under 50% must control at least two more seats than its complement (61 vs. 59). However, it turns out to be easy enough to find examples of inversions, as shown in Table 3. Coalitions whose members are marked with an "X" control 61 or even 62 seats but are nevertheless supported by less than 50% of the voters. When we restrict our attention to seat-winning parties only, seat allocations and coalition possibilities remain as before, but the vote shares of seat-winning parties increase, as all the votes cast for non-seat-winning parties are now excluded from the total vote. As a result, the vote support for every coalition also increases, with the result that all the coalitions identified in Table 3 as collecting less than half of the total vote collected more than half of the vote cast for seat-winning parties.

Next we "purify" the seat allocation by eliminating the threshold requirement and replacing D'Hondt with LR-H or Sainte-Laguë, as shown in Table 4. (In this case, LR-H and Sainte-Laguë do not produce identical seat allocations, and Sainte-Laguë does not quite stay in quota.) We can again identify a number of examples of full inversions for each formula. Given that a large number of non-seat-winning parties remain even in the absence of an explicit threshold, it might appear that our examples rest on the existence of such parties. However, one coalition under LR-H and two under Sainte-Laguë also receive less than 50% of the vote cast for seat-winning parties.[17]

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[15] The Netherlands has districts, but they do not affect the overall allocation of seats to parties; its threshold is 0.67%. The electoral threshold in Israel, originally 1%, was increased to 1.5% in 1992 and to 2% in 2003.

[16] Straightforward application of the D'Hondt formula (together with the 2% threshold) to the party votes shown does not produce precisely the seat allocation given in Table 3. This is because the Israeli electoral system allows for list "apparentements" (in effect, temporary pre-election alliance between pairs of parties both of which meet the 2% threshold), which are not indicated in the table.

[17] None of the coalitions identified in Tables 3 and 4 actually formed the government and perhaps none was a plausible governing coalition made up of ideologically compatible parties. (Even though Israeli governments are noted for sometimes being based on rather ideologically odd coalitions, it is clear that neither Hadash nor Balad would be invited into a government, most especially one led by Likud.) The question arises as to whether an actual Israeli governing coalition controlling a majority of seats has ever been supported by less than half of the total vote. Dan Felsenthal (personal communication) has examined all Israeli elections and discovered that the governing coalition (of Likud plus several small parties) following the 1981 election controlled 62 seats but gained only 48.2% of the total valid vote cast. However, this represented 50.8% of the votes cast for seat-winning parties.

Table 5 shows the results of the 2010 election in the Netherlands. Like the Israeli parliament, the Dutch parliament has an even number of seats (150), making (full as opposed to partial) inversions more difficult. However, it is again easy enough to find examples of inversions, as shown in Table 5. Moreover, one of the coalitions identified was the actual governing coalition and another also failed to win a majority of the votes cast for seat winning parties. When we "purify" the seat allocation by eliminating the threshold requirement and replacing D'Hondt with LR-H or Sainte-Laguë (which in this case produce identical seat allocations), we can still identify a number of examples of inversions (not including the governing coalition, which no longer has a majority of seats), as shown in Table 6. However, in this case the three coalitions that win 76 seats with less than half the vote come so close to the 50% mark that we cannot find coalitionwise inversions based on seat-winning parties only.[18]

We now reexamine Kurrild-Klittgaard's Danish example of an inversion due to threshold effects. Table 7 reproduces his Table 5, showing the results of the 1990 election for Denmark's 175 "continental" seats only (thereby excluding any "federal effects"). The third coalition (that won 91 seats) — an ideologically coherent center-right governing coalition — corresponds to Kurrild-Klitgaard's inversion based on threshold effects (though his calculations also includes the non-seat-winning right-of-center Justice Party). We see, however, that there are at least two other possible coalitionwise inversions. However, none of these survive as examples of inversions with respect to seat-winning parties only. Table 8 shows the seat allocation with the threshold requirement eliminated and D'Hondt replaced by either LR-H or Sainte-Laguë (which produce identical seat allocations). While the previously identified coalitions, including the governing one, lose their seat majorities, we can identify new two examples of coalitionwise inversions. Moreover, removing the threshold requirement and using the most proportional formulas means that all parties are seat-winning except the vanishingly small Humanist Party, with the result that the same coalitions provide examples of inversions with respect to seat-winning parties only.

Finally, we examine the most recent apportionment (based on the 2010 census) of the 435 seats in the U.S. House of Representatives among the 50 states. The apportionment of House seats is based on the population of each state and since 1940 has used the Hill-Huntington formula, which is a divisor method that uses "geometric" rounding. As such, it is very similar to Sainte-Laguë but slightly more favorable to small states, though this slight bias effectively disappears with this many

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Moreover, a new general was held in February 2013, producing (at least) two potential inversions, in which coalitions with about 46.4% of the total vote (and 49.9% of the vote cast for seat-winning parties) won 61 seats, at least one of which is a possible governing coalition and neither of which includes an Arab or far-left party. (The new governing coalition has not yet been formed.)

[18] The Dutch government resigned in 2012, prompting an early general election which produced a coalition government composed of the two largest (People's and Labour) parties, with a majority of both seats and votes. However, this election also produced examples of inversions, though not with respect to seat-winning parties only. However, a seat allocation purified by using LR-H without a threshold produces a coalitionwise inversion among seat-winning parties only; the (slightly different) seat allocation under Sainte-Laguë produces a inversion but not with respect to seat-winning parties only.

seats (Schuster et al., 2003, p. 675).[19] This data is useful for getting a sense of inversion possibilities when the number of seats increases somewhat and the number of (seat-winning) "parties" (i.e., states) increases substantially. State "coalition" possibilities are extraordinarily numerous and cannot be exhaustively searched, but what we can straightforwardly do is to rank states in order of their seat differentials and cumulate their seat and population shares until we reach a majority of 218 House seats. The most favored (with respect to seat differentials) states with 207 seats have 46.25% of the population. Adding the next most favored state (Michigan) produces a "coalition" with 221 seats and 49.46% of the population. We can replace Michigan with the slightly less favored but also less populous Tennessee to create a "coalition" with a bare majority of 218 seats and only 48.31% percent of the population. It is possible but unlikely that there is another even more "efficient" coalition, and certainly there are additional inversion possibilities.

The LR-H formula (which as the "Hamilton method" has been used to apportion House seats in the past) produces the same apportionment of seats as Hill-Huntington. The Sainte-Laguë formula (which as the "Webster method" has also been used in the past) produces an apportionment that differs from Hill-Huntington with respect to only two states. The most favored (with respect to seat differentials) states with 217 seats have 48.56% of the population. Adding the next most favored state (Hawaii) produces a "coalition" with 219 seats and 49.01% of the population. We can replace Hawaii with the slightly less favored but also less populous North Dakota to create a "coalition" with a bare majority of 218 seats and only 48.78% percent of the population. Again, it is possible but unlikely that there is another even more "efficient" coalition, and certainly there are additional inversion possibilities.

We can likewise examine inversions based on electoral votes, rather than House seats. As mentioned earlier, the apportionment of electoral votes deliberately favors (as a "federal effect") small states by giving each state electoral votes equal to its total representation in Congress, i.e., its House seats plus two (for its equal two seats in the Senate). In addition, the District of Columbia has three electoral votes, for a total of 538. The most favored (with respect to electoral vote differentials and the 39 smallest) states with 255 electoral votes have 40.56% of the population. Adding in the next most favored state of Georgia gives a majority of 271 electoral votes with only 43.70% of the population. We can replace Georgia with the slightly less favored but also slightly less populous North Carolina to create a "coalition" with a bare majority of 270 electoral votes and only 43.65% percent of the population. Again, it is possible but unlikely that there is another even more "efficient" coalition, and certainly there are additional inversion possibilities. The unsurprising lesson here is that, while election inversions under more or less pure proportional apportionment are always close-run things, given major "federal effects" inversions may be much more substantial.[20]

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[19] In addition, the U.S. Constitution guarantees every state at least one House seat, i.e., the "electoral formula" has a "floor" rather than a "threshold." However, this guarantee has no present effect, as every state is entitled to at least one seat based on its population, present House size, and the normal operation of the Hill-Huntington formula.

[20] The U.S. Senate provides the most notorious example. The "coalition" of the 26 smallest states controls a majority of 52 out of 100 seats with only 17.83% of the 2010 census population.

6. Conclusion

It is generally known that plurality systems based on districts can produce election inversions, in which one party or candidate wins a majority of seats or electoral votes even though another party wins more votes. While it might seem that proportional representation systems cannot produce inversions, Kurrild-Klitgaard (2013) shows that Denmark's proportional representation system has produced inversions in which a coalition of parties supported by a minority of votes won a majority of seats. However, Kurrild-Klitgaard's examples rest on imperfections in Denmark's PR system, introduced to serve goals other than proportionality, which can produce inversions through "federal" or "threshold" effects. Moreover, Kurild-Klitgaard's inversion examples do not hold when vote shares are calculated on the basis of seat-winning parties only.

Here we have shown that coalitionwise election inversions can occur even under the purest type of proportional representation and even when the analysis is restricted to seat-winning parties. They result from the unavoidable "whole-number" effect found in any practical PR system. Moreover, we have identified a number of empirical examples in elections in several countries. Indeed, every case examined included some inversion possibilities, suggesting that they are typical, not exceptional, when PR systems produce a sufficiently fragmented party system that a number of barely winning coalitions exist — in practice, this means systems with a sufficiently low effective nationwide threshold (Taagepera, 2007, pp. 247-250).

Of course, many or most barely winning coalitions, in the general sense of sets of parties, are not plausible coalitions, in the specific sense of sets of ideologically compatible parties that might form a government and, even among coalitions of the latter type, only one can actually form. Thus, realized coalitionwise inversions under proportional representation are undoubtedly rather rare and, like all such inversions, are close-run things and are probably not great threats to political legitimacy. But proportional representation systems deal only with numbers of votes and seats, not ideological relationships among parties and coalition formation, so it is worth recognizing that they also are unavoidably subject to the inversion phenomenon.

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Table 1. A Hypothetical Election Inversion in Germany Due to the 5% Threshold

Party	Party Votes	Coalition Votes	Party Seats	Coalition Seats
CDU/CSU	46.0%	50.5%	289	289
FDP	4.5%		0	
SPD	42.0%	49.5%	264	311
Greens	7.5%		47	

Table 2. Hypothetical Coalitionwise Election Inversions under Pure Proportional Representation (LR-H or Sainte-Laguë) with 35 Seats

Party	Party Votes	Coalition Votes	Party Quota	Party Seats	Coalition Seats
A	49.7%	49.7%	17.395	18	18
B	29.4%	50.3%	10.290	10	17
C	20.9%		7.315	7	

Party	Party Votes	Coalition Votes	Party Quota	Party Seats	Coalition Seats
A	50.3%	50.3%	17.605	17	17
B	30.4%	49.7%	10.640	11	18
C	19.3%		6.755	7	

Table 3. Israeli 2009 Election: Actual Seat Allocation

Party	*Vote*	*Seats*	*Vote %*	*Quota*	*Diff*	*Coalitions*				
Kadima	758032	28	22.470	26.964	1.036	X			X	X
Likud	729054	27	21.611	25.934	1.066		X	X	X	X
Yisrael Beiteinu	394577	15	11.696	14.036	.964	X	X	X		
Labor Party	334900	13	9.927	11.913	1.087	X	X	X		
Shas	286300	11	8.487	10.184	.816					
United Torah Judaism	147954	5	4.386	5.263	-.263	X				
United Arab List–Ta'al	113954	4	3.378	4.054	-.054					
National Union	112570	4	3.337	4.004	-.004					
Hadash	112130	4	3.324	3.989	.011			X		X
New Movement-Meretz	99611	3	2.953	3.543	-.543					
The Jewish Home	96765	3	2.868	3.442	-.442		X		X	
Balad	83739	3	2.482	2.979	.021		X	X	X	X
The Green Movement	27737	0	.822	.987	-.987					
Gil	17571	0	.521	.625	-.625					
Ale Yarok	13132	0	.389	.467	-.467					
The Greens	12378	0	.367	.440	-.440					
Yisrael Hazaka	6722	0	.199	.239	-.239					
Tzabar	4752	0	.141	.169	-.169					
Koah LeHashpi'a	3696	0	.110	.131	-.131					
Da'am Workers Party	2645	0	.078	.094	-.094					
Yisrael HaMithadeshet	2572	0	.076	.091	-.091					
Holocaust Survivors	2346	0	.070	.083	-.083					
Leader	1887	0	.056	.067	-.067					
Tzomet	1520	0	.045	.054	-.054					
Koah HaKesef	1008	0	.030	.036	-.036					
Man's Rights in the Family	921	0	.027	.033	-.033					
HaYisraelim	856	0	.025	.030	-.030					
Or	815	0	.024	.029	-.029					
Ahrayut	802	0	.024	.029	-.029					
Brit Olam	678	0	.020	.024	-.024					
Lev LaOlim	632	0	.019	.022	-.022					
Lazuz	623	0	.018	.022	-.022					
Lehem	611	0	.018	.022	-.022					
TOTAL	3373490	120	100.000	120.000	.000					
Coalition Seats						**61**	**61**	**62**	**61**	**62**
Coalition Vote % (based on total vote)						**48.48**	**48.59**	**49.04**	**49.43**	**49.89**
Coalition Vote % (base on seat-winning parties only)						**50.02**	**50.13**	**51.60**	**51.00**	**51.47**

Source: Israeli Parliament (www.knesset.gov.il) for seat-winning parties; Wikipedia (en.wikipedia.org/wiki/Elections_in_Israel) for other parties.

Table 4. Israeli 2009 Election:
Seat Allocations under LR-H and Sainte-Laguë without a Threshold

	Largest Remainder-Hare						*Sainte-Laguë*				
	Seats	*Diff*	*Coalitions*				*Seats*	*Diff*	*Coalitions*		
Kadima	27	.0357	X		X	X	28	1.0357	X	X	X
Likud	26	.0665		X	X	X	26	.0665			X
Yisrael Beiteinu	14	-.0357		X			14	-.0357			
Labor Party	12	.0871	X	X			12	.0871	X	X	
Shas	10	-.1841					10	-.1841			
United Torah	5	-.2629					5	-.2629			
United Arab List	4	-.0535					4	-.0535		X	
National Union	4	-.0043	X				4	-.0043	X	X	
Hadash	4	.0114	X				4	.0114	X	X	
Meretz	4	.4567	X	X		X	4	.4567	X	X	
Jewish Home	4	.5579	X	X	X	X	4	.5579	X	X	X
Balad	3	.0213	X		X		3	.0213	X		X
Green Movement	1	.0134	X				1	.0134	X		
Gil	1	.3750	X				1	.3750	X	X	
Ale Yarok	1	.5329	X	X	X		0	-.4671			
The Greens	0	-.4403					0	-.4403			
Yisrael Hazaka	0	-.2391					0	-.2391			
Tzabar	0	-.1690					0	-.1690			
Koah LeHashpi'a	0	-.1315					0	-.1315			
Da'am Workers	0	-.0941					0	-.0941			
Yisrael HaMithadeshet	0	-.0915					0	-.0915			
Holocaust Survivors	0	-.0835					0	-.0835			
Leader		0	-.0671					0	-.0671		
Tzomet	0	-.0541					0	-.0541			
Koah HaKesef	0	-.0359					0	-.0359			
Man's Rights	0	-.0328					0	-.0328			
HaYisraelim	0	-.0304					0	-.0304			
Or	0	-.0290					0	-.0290			
Ahrayut	0	-.0285					0	-.0285			
Brit Olam	0	-.0241					0	-.0241			
Lev LaOlim	0	-.0225					0	-.0225			
Lazuz	0	-.0222					0	-.0222			
Lehem	0	-.0217					0	-.0217			
TOTAL	120	.0000					120	.0000			
Coalition Seats			**61**	**61**	**61**	**61**			**61**	**61**	**61**
Coalition Vote % (total vote)			**49.09**	**49.45**	**49.82**	**49.90**			**48.70**	**48.78**	**49.43**
Coalition Vote % (seat-winning)			**49.76**	**50.12**	**50.50**	**50.58**			**49.57**	**49.64**	**50.31**

Table 5: Dutch 2010 Election: Actual Seat Allocation

Party	*Votes*	*Seats*	*Vote%*	*Quota*	*Diff*	*Coalitions*			
People's Party (VVD)	1929575	31	20.493	30.739	.261		X	X	
Labour Party (PvdA)	1848805	30	19.635	29.452	.548	X		X	X
Party for Freedom (PVV)	1454493	24	15.447	23.171	.829	X	X		X
Christian Democratic Appeal	1281886	21	13.614	20.421	.579		X		X
Socialist Party	924696	15	9.820	14.731	.269	X		X	
Democrats 66	654167	10	6.947	10.421	-.421				
Green Left	628096	10	6.671	10.006	-.006				
Christian Union	305094	5	3.240	4.860	.140	X			
Reformed (SGP)	163581	2	1.737	2.606	-.606				
Party for the Animals (PvdD	122317	2	1.299	1.949	.051	X			X
Proud of the Netherlans	52937	0	.562	.843	-.843				
Party for Human and Spirit	26196	0	.278	.417	-.417				
Piraten Party	10471	0	.111	.167	-.167				
List 17	7456	0	.079	.119	-.119				
Partij één	2042	0	.022	.033	-.033				
New Netherlands	2010	0	.021	.032	-.032				
Heel NL	1255	0	.013	.020	-.020				
Laclé List	924	0	.010	.015	-.015				
TOTAL	9416001	150	100.000	150.000	.000				
Coalition Seats						**76**	**76***	**76**	**77**
Coalition Vote % (based on total vote)						**49.44**	**49.55**	**49.95**	**49.995**
Coalition Vote % (based on seat-winning parties only)						**49.98**	**50.10**	**50.50**	**50.549**

*Governing coalition

Source: *Dutch Election Results Since 1918* (www.nlverkiezingen.com).

Table 6. Dutch 2010 Election:
Seat Allocation under LR-H/Sainte-Laguë and No Threshold

Party	*Vote*	*Vote%*	*Quota*	*Seats*	*Diff*	*Coalitions*		
VVD	1929575	20.493	30.739	31	.261	X		X
PvdA	1848805	19.635	29.452	30	.548	X	X	X
PVV	1454493	15.447	23.171	23	-.171			
CDA	1281886	13.614	20.421	20	-.421		X	
Socialists	924696	9.820	14.731	15	.269		X	X
Dem 66	654167	6.947	10.421	10	-.421			
G reen List	628096	6.671	10.006	10	-.006	X		
CU	305094	3.240	4.860	5	.140		X	
SGP	163581	1.737	2.606	3	.394	X	X	
PvdD	122317	1.299	1.949	2	.051	X	X	
Proud	52937	.562	.843	1	.157		X	
Human & Spirit	26196	.278	.417	0	-.417			
Pirates	10471	.111	.167	0	-.167			
List 17	7456	.079	.119	0	-.119			
Partij één	2042	.022	.033	0	-.033			
NwNed	2010	.021	.032	0	-.032			
Heel NL	1255	.013	.020	0	-.020			
Laclé List	924	.010	.015	0	-.015			
TOTAL	9416001	100.000	150.000	150	.000			
Coalition Seats						**76**	**76**	**76**
Coalition Vote % (based on total vote)						**49.85**	**49.91**	**49.95**
Coalition Vote % (seat-winning parties only)						**50.10**	**50.18**	**50.22**

Table 7. Danish 1990 Election: Actual Seat Allocation
("Continental" Denmark Only)

Party	*Votes*	*Vote%*	*Quota*	*Seats*	*Diff*	*Coalitions*		
Social Democrats	1221121	37.695	65.967	69	3.033	X	X	
Conservatives	517293	15.969	27.945	30	2.055			X
Liberal Party	511643	15.794	27.640	29	1.360			X
Socialist People's Party	268759	8.296	14.519	15	.481		X	
Progress Party	208484	6.436	11.263	12	.737	X		X
Center-Democrats	165556	5.111	8.944	9	.056			X
Radicals	114888	3.547	6.206	7	.794	X		X
Chirstian People's Party	74174	2.290	4.007	4	-.007		X	X
Common Course	57896	1.787	3.128	0	-3.128			
Unity List	54038	1.668	2.919	0	-2.919			
Greens	27642	.853	1.493	0	-1.493			
Justice Party	17181	.530	.928	0	-.928			
Humanist Party	763	.024	.041	0	-.041			
TOTAL	3239438	100.000	175.000	175	.000			
Coalition Seats						**88**	**88**	**91***
Coalition Vote % (based on total vote)						**47.68**	**48.28**	**49.15**
Coalition Vote % (seat-winning parties only)						**50.11**	**50.75**	**51.66**

*Governing coalition

Source: Kurrild-Klitgaard (2013) and *Danish Parliament* (www.ft.dk).

Table 8. Danish 1990 Election: Seat Allocation under LR-H and Sainte-Laguë and No Threshold ("Continental" Denmark Only)

Party	*Votes*	*Vote%*	*Quota*	*Seats*	*Diff*	*Coalitions*		
Social Democrats	1221121	37.695	65.967	66	.033		X	
Conservatives	517293	15.969	27.945	28	.055		X	
Liberal Party	511643	15.794	27.640	28	.360		X	
Socialist People's Party	268759	8.296	14.519	15	.481		X	X
Progress Party	208484	6.436	11.263	11	-.263			
Center-Democrats	165556	5.111	8.944	9	.056		X	
Radicals	114888	3.547	6.206	6	-.206			
Chirstian People's Party	74174	2.290	4.007	4	-.007	X	X	
Common Course	57896	1.787	3.128	3	-.128			
Unity List	54038	1.668	2.919	3	.081		X	X
Greens	27642	.853	1.493	1	-.493			
Justice Party	17181	.530	.928	1	.072		X	
Humanist Party	763	.024	.041	0	-.041			
TOTAL	3239438	100.000	175.000	175	.000			
Coalition Seats						**88**	**88**	
Coalition Vote % (based on total vote)						**49.66**	**49.95**	
Coalition Vote % (seat-winning parties only)						**49.67**	**49.96**	