

other democratic countries that have created constitutional courts have provided the justices with lengthy but not life terms.

41. As the experience of the Russian Constitutional Court in 1993 illustrates.

42. Harry Kantor, "Efforts Made by Various Latin American Countries to Limit the Power of the President," in Lijphart, *Parliamentary versus Presidential Government*, 101-9.

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## MAJORITY RULE AND MINORITY INTERESTS

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### I. INTRODUCTION

The nomination in the Spring of 1993 of Lani Guinier to be Assistant Attorney General for Civil Rights, and the concomitant attention focused on her law review articles,<sup>1</sup> might have generated a useful public discussion on the relationship between majority rule and minority interests. But the discussion that got under way was hardly constructive or enlightening and, in any case, it was aborted along with the nomination. This essay attempts, in what is surely a very limited and incomplete way, to advance such a discussion.

In the introductory chapter to her book, *The Tyranny of the Majority*, Guinier discusses examples of collective choice in which majority rule implies that "the numerically more powerful majority choice simply subsumes minority preferences." In such cases, "the majority that rules gains all the power and the minority that loses gets none." Thus, in some circumstances—in particular, in a "racially divided society"—"majority rule may

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be perceived as majority tyranny," operating in a "winner-take-all" fashion that cleanly (but unnecessarily) partitions society into political "winners" and "losers."<sup>2</sup> Members of the minority, though equally enfranchised in formal terms, gain nothing with respect to collective choice from their enfranchisement.

The view that there is a fundamental incompatibility between majority rule and minority interests (a view which in fact Guinier does not unconditionally endorse) is commonly advanced and has, at first blush, a compelling plausibility. *Of course*, minority interests must suffer under majority rule. The only question seems to be whether one's reaction is: "so much the worse for minority interests" or "so much the worse for majority rule."

I hope to show that this surface plausibility is less compelling on closer analysis. This chapter attempts to clarify, and in so doing substantially qualify (though not totally negate), the common argument that majority rule invariably operates on a "winner-take-all" basis that renders minority interests irrelevant.

I undertake this task in a distinctive fashion, by employing the particular style of analysis associated with formal political theory and social choice theory.<sup>3</sup> Works within this tradition have significantly clarified the logical nature of majority-rule decision making, but they have rarely addressed the issue of majority rule versus minority interests,<sup>4</sup> even though this issue has an ancient pedigree in political theory and is a standard debating point in practical political arguments. I hope to show that the logic of formal political theory in fact provides helpful insights into this long-standing issue. More specifically, I distinguish among three types of politics—here labelled "cleavage politics," "ideological politics," and "distributive politics"—and show that the issue of majority rule versus minority interests plays out quite differently in each context. In the context of cleavage politics, the common argument is fully supported if there is just one issue; but, given several issues on which preferences are crosscutting, the argument must be qualified. In the context of ideological politics, majority rule typically produces compromise outcomes, and the interests of even an ideologically distinctive minority typically receive considerable weight. In the

context of distributive politics, majority rule is chaotic and cannot be blamed or credited for any particular outcome.

Adopting the social choice framework has implications for the scope of this study that should be explicitly noted.

First, in considering majority rule within the framework of social choice theory, we are solely concerned with "majority rule" as a formal "decision rule"—that is, as a procedure or institution for collective choice; we are not concerned with the social locus of political power or electoral enfranchisement, as we would be in speaking of the "movement towards majority rule" in Europe a century ago or in South Africa much more recently. And whereas "majority rule" may have had distinctively "radical" or "progressive" implications for government policy in the past (or in present South Africa), it may have "conservative" ones in mostly affluent industrial or post-industrial societies (especially those with racial or similar minorities). Insofar as the following analysis suggests why present racial or similar minorities may not fare too badly under majority rule, it also suggests why economic minorities (e.g., the rich) also have not fared badly.

Second, it needs to be emphasized that the concept here counterposed to "majority rule" is "minority interests"—not, as is more commonly the case, "minority rights."<sup>5</sup> The issue of "majority rule versus minority rights" is an important and difficult (though probably mislabelled) one in political practice and constitutional law, but the issue need not in principle raise a problem within the context of social choice theory. Social choice theory often assumes (as we do here) that a collectivity has made a prior "constitutional agreement" concerning the domain of activities that are subject to collective decision—the scope of powers "delegated" (so to speak) to the collectivity—and concerning the decision rule (majority rule or otherwise) to be employed in making these decisions.<sup>6</sup> In the complementary domain of activities "reserved" (so to speak) for individual choice, the rights of individuals hold sway.<sup>7</sup> Thus majority rule operates in one domain, the rights of individuals—regardless (and this is why the issue may be mislabelled) of whether they belong to particular majorities or minorities—in another. The

minority interests we are concerned with here are not those interests that are secured (at least in principle) as individual rights but precisely those interests unquestionably at play within the domain of collective control and choice.

Finally, our concern is with the implications for minority interests of majority rule operating at the level of the overall political institutions of society. To bring the issue of majority rule versus minority interests most directly into focus, we assume that "simple majoritarian power relations" exist within society. This means that any coalition in society comprising a majority of the whole can bring about whatever political outcomes (within the domain of collective control and choice) it wants.<sup>8</sup> We are not, therefore, particularly concerned with the issue of "majoritarian" versus "proportional" electoral systems (which, of course, is Guinier's principal concern), since legislatures and parliaments, however elected, typically use majority rule to enact their decisions. It is worth emphasizing that, insofar as there is a conflict between majority rule and minority interests, a proportional electoral system by itself does little to mitigate the conflict. Indeed, given legislative majority rule, it follows that a society that uses proportional representation to elect its parliament is if anything even more reliably majoritarian in its overall power relations than one that uses a "majoritarian" electoral system.<sup>9</sup>

## II. THE SOCIAL CHOICE FRAMEWORK

In this section, I sketch out a social choice framework that unavoidably entails a vastly oversimplified representation of any political system. But it allows us to think clearly about the relationship between majority rule and minority interests.

We suppose that a political system has three ingredients: (1) a set of "issues" concerning activities within the domain of collective control that accordingly must be resolved by collective choice; (2) a set of "citizens" endowed with "preferences" (or "interests") with respect to these issues; and (3) a political "decision rule" that determines how citizen preferences ultimately count in resolving these issues. In these terms, we also distinguish among three types of politics.

### 1. Issues

Formally, an issue is a set of two or more mutually exclusive *alternatives* (possible policies, programs, budgets, etc.), exactly one of which must be collectively chosen by the polity.

Some issues are *discrete*, with a finite, and typically small, number of distinct alternatives. The simplest kind of issue is *dichotomous*—that is, one with just two alternatives ("pro" and "con," "X" and "not X," "NAFTA" and "not NAFTA," or whatever). A more complex kind of issue is *polychotomous*—that is, one with multiple distinct alternatives ("NAFTA," "not NAFTA," and "NAFTA with specified amendments," or whatever).

Other issues are *continuous*, with an infinite number of alternatives. Social choice theorists customarily think of such an infinite set as an *alternative space* of one or several dimensions. Suppose, for example, that an issue that must be resolved is the funding level of some collective activity or the provision of some *public good*. The "space" of possible funding or provision levels is infinite—a continuum running from zero upwards. An alternative space might also represent an *ideological* spectrum running along a continuum from left to right. Some continuous issues are multidimensional. *Allocative* issues provide an example. For example, if a fixed supply of some *private good* (benefit) or bad (cost) must be divided three ways, every point on the surface of an equilateral triangle represents a possible alternative.

### 2. Preferences

We now consider the *citizens* who may participate in the political system and who have *preferences* on issues. We suppose that these preferences are (to use a technical term) *separable* by issues. This means that preferences on one issue are not conditioned on the resolution of other issues.<sup>10</sup>

Preferences over discrete issues are described by orderings of the alternatives from most to least preferred. (For simplicity, we suppose that no one is indifferent between discrete alternatives.)

With respect to continuous one-dimensional issues, we make a more powerful (but plausible and standard) assumption. We suppose, first, that each citizen has a most preferred alternative or *ideal point* on the issue continuum and, second, that, in comparing two points on the same side of his ideal point, each citizen prefers the one closer to his ideal to the one more distant from it. Such preferences are called *single-peaked*, for reasons that are evident if one imagines drawing a "utility curve" (or "function") to describe preferences over the issue continuum.<sup>11</sup>

We need also to specify preferences in a more comprehensive way. Several issues may (at least implicitly) be considered jointly, so we must suppose each citizen has preferences over *political outcomes*, i.e., combinations of alternatives, one for each issue under consideration. If the issues are discrete, these outcomes are likewise discrete (though relatively numerous) and preferences over them can be described by an ordering. Separability restricts what orderings can occur, however. Suppose that two dichotomous issues,  $X_1$  (with alternatives  $x_1$  and  $\bar{x}_1$ ) and  $X_2$  (with alternatives  $x_2$  and  $\bar{x}_2$ ), are under simultaneous consideration. There are four possible outcomes:  $(x_1, x_2)$ ,  $(x_1, \bar{x}_2)$ ,  $(\bar{x}_1, x_2)$ , and  $(\bar{x}_1, \bar{x}_2)$ . Given separability, any citizen who has a first preference of outcome  $(x_1, x_2)$  must have a last preference of the "opposite" outcome  $(\bar{x}_1, \bar{x}_2)$ , and so forth. But, at the same time, two citizens with the same first (and last) preferences may have orderings that differ with respect to their *intermediate* (second and third) preferences. Put substantively, two citizens with the same first preferences may differently answer the question: "If you could get your way on just one of the two issues, which one would it be—that is, with respect to which issue is your preference more 'intense'?"

A multidimensional space results if a continuous issue has two or more dimensions or if several continuous issues are considered together. Given such a multidimensional space, we often suppose that citizen preferences over it have this property: preferences over the alternatives lying on any straight line through the space are single-peaked preferences in the multidimensional, as well as one-dimensional, context.<sup>12</sup>

Allocative issues—for example how to divide up benefits

(or costs) among members of society—are also continuous and multidimensional, with essentially as many dimensions as there are individuals (or groups or geographical entities) in society. But preferences over allocative issues surely are not single-peaked; plausibly, they are *individualistic*, i.e., given two alternatives, each citizen prefers the alternative that provides him with more benefits or imposes lower costs on him.

### 3. The Decision Rule

The "issues" discussed above belong to the domain of collective control and choice. The political system needs some *decision rule* by which citizen preferences over issues are aggregated into collective choices. Here we suppose that the applicable decision rule is *majority rule*—that is, in choosing between any two alternatives or outcomes, collective choice is determined by the preferences of the greater number of citizens. As a result, the political system is *majoritarian*, so that any majority of citizens is empowered to bring about any political outcome within the domain of collective choice.

The operation of majority rule is most transparent in resolving a single dichotomous issue considered in isolation, and indeed informal discussions of the relationship between majority rule and minority interests often do not look beyond this simplest case. In section III, we will see how majority rule extends (with some complexities) to polychotomous or multiple issues.

### 4. Three Kinds of Politics

Within the social choice framework outlined above, we can define three different contexts of collective choice that are relevant for our discussion, and we can link each reasonably clearly with different kinds of practical politics in terms of which we label them.

*Cleavage (or Group) Politics.* In this context, issues arise that are "naturally" discrete and in fact (we will here assume) dichotomous. Accordingly, each such issue partitions (or "cleaves") the

population of citizens into two clusters, each of which is internally homogenous, but between which there is total conflict, with respect to preferences on the issue. As a matter of sociological description (not necessarily entailed by our social choice framework), the issue may engage and the preference partition reflect some underlying "social cleavage" or stigmatic criterion that divides the population into different ethnic, linguistic, religious, territorial, occupational, etc., groups. It is in the context of cleavage politics that the issue of majority rule versus minority interests—indeed of "the majority" versus "the minority"—can arise most starkly. In this context, collective choice appears unavoidably to be a matter of winner-take-all.

*Ideological (or Public Goods) Politics.* In this context, issues arise that are "naturally" continuous and over which preferences are single-peaked. Examples include the level of provision of public goods or of government activity or the trade-off between two or more such goods or activities (e.g., "guns versus butter") within a fixed budget. Or perhaps various issues fit into one or a few ideological dimensions. In any event, in this context, collective choice is not so clearly a matter of winner-take-all.<sup>13</sup>

*Distributive (or Private Goods) Politics.* In this context, issues arise that pertain to the allocation of a fixed supply of divisible and essentially private goods. The archetypical distributive issue is often described as that of dividing \$1—though, in the age of big government, dividing \$1 trillion might be a better description. The allocation of "pork-barrel" projects, together with the sharing out of their costs, provides a more realistic description of distributive politics. Since the benefits in question can be parcelled out in any fashion, distributive politics need not be winner-take-all.

### III. PROPERTIES OF MAJORITY RULE

Research within the social choice tradition has significantly clarified the properties of majority rule and the logical nature of majority-rule decision making. Here I provide a very brief overview of some of that research.

#### 1. May's Theorem

Given a single dichotomous issue, majority rule strikes most people as fair and reasonable in the abstract. Many years ago, Kenneth May formalized our intuition, by identifying four conditions that we may want a decision rule to meet when applied to a dichotomous issue and demonstrating that majority rule, and only majority rule, meets these conditions.<sup>14</sup> May's conditions are these: *decisiveness*—however citizen preferences are distributed, there is always a clear result (even if it is a tie); *anonymity* (of citizens)—we do not need to know who has which preferences to determine the collective choice (we just count preferences); *neutrality* (between alternatives)—we do not need to know how alternatives are labelled (e.g., "status quo" versus "change in the status quo") to determine the collective choice; and *positive responsiveness*—if alternative  $x$  at least ties  $y$  and then someone changes his preference ordering so it is more favorable to  $x$ , then  $x$  is certainly the collective choice. *Unanimity rule* (as used by juries) violates decisiveness; *weighted majority rule* violates anonymity; *supra-majority rule* (such as two-thirds majority rule) violates either neutrality (if—as in practice—failure of the alternative entailing change in the status quo to receive the requisite supra-majority support results in choosing the status quo) or positive responsiveness (if failure of either alternative to receive the requisite supra-majority support results in a "tie"). May demonstrated that majority rule meets all four conditions and is the only decision rule that can do so.

#### 2. The Rae-Taylor-Straffin Theorem

Suppose citizens are negotiating a "constitutional agreement" and are choosing what decision rule they would like the political system being formed to use in resolving a series of dichotomous issues that will arise in the future. Each citizen aims to maximize the proportion of times that the collective choice agrees with his own preference (i.e., each aims to maximize what we may call his *political satisfaction* over the long run). No citizen knows how his preferences will be related to those of other citizens. From behind this "veil of ignorance," each supposes that his probabil-

ity of supporting an alternative on any issue is proportional to the overall support for that alternative. It then follows that majority rule maximizes the prospective satisfaction of every citizen.<sup>15</sup>

### 3. Multiple Alternatives and Condorcet Winners

Both May's Theorem and the Rae-Taylor-Straffin Theorem deal with collective choice on a single dichotomous issue, in which context the meaning of majority rule is straightforward. Some citizens prefer one alternative, others prefer the other, and—at least if the number of citizens is odd and they are never indifferent between alternatives—one or other alternative is the *majority winner* on the issue, i.e., is the first preference of a majority of citizens.

But if an issue has multiple alternatives, or if several dichotomous issues are considered together (generating multiple outcomes), it is likely that first preferences will be sufficiently dispersed among the multiple alternatives or outcomes that no majority winner exists. What then does majority rule imply for collective choice?

One possibility is that the *plurality winner* becomes the collective choice. The plurality winner—one weakening of the notion of majority winner—is the alternative that is the first preference of the greatest number of citizens, even if that number fails to constitute a majority. Apart from the possibility of ties, a plurality winner always exists, regardless of the number of alternatives and the nature of citizen preferences over them. But social choice theory shows that in other respects the plurality winner is an unreliable basis for collective choice from among multiple alternatives, as it changes erratically in response to small changes in the nature of alternatives, in the number of alternatives, or in citizen first preferences. Moreover, few practical political institutions and processes base collective choice on the plurality winner. Most notably this is true of parliamentary voting in a legislative setting.

Given three or more alternatives, it seems desirable—and, in a legislative setting, generally true—that collective choice meet the following conditions: (1) it should take account of citizens'

full preference orderings (not just their first preferences); and (2) it should proceed on the basis of pairwise choices, with respect to each of which majority rule can be straightforwardly implemented. (If the second condition is realized, the first also is, since many pairwise choices will necessarily involve alternatives not the first preference of many citizens.)

Two centuries ago, the Marquis de Condorcet proposed that collective choice should comply with both conditions, by being based on what contemporary social choice theory calls the "majority preference relation."<sup>16</sup> Consider any two alternatives  $x$  and  $y$ . Some citizens prefer  $x$  to  $y$ ; others prefer  $y$  to  $x$ . (We continue to rule out indifference.) Suppose the number of citizens preferring  $x$  to  $y$  exceeds the number preferring  $y$  to  $x$ . Then we say  $x$  beats  $y$  under majority rule. If we consider all possible pairs of alternatives in this way, we discover the *majority preference relation* over the entire set of alternatives. Condorcet's candidate for collective choice under majority rule is customarily dubbed the *Condorcet winner*. A Condorcet winner is an alternative that beats every other alternative under majority rule and, in that sense, stands at the top of the majority preference relation. Clearly, for any given set of alternatives and preferences, there can be no more than one Condorcet winner.<sup>17</sup>

Several points follow. First, a majority winner  $x$  is preferred by the same majority (i.e., the majority of citizens all of whom share  $x$  as their first preference) to every other alternative (though this "permanent majority" may be augmented by different sets of additional citizens in different pairwise comparisons). Therefore, a majority winner is necessarily a Condorcet winner. But the converse is not true. In general, a Condorcet winner may beat different alternatives through different (or "shifting") majorities, rather than through a single "permanent majority." Indeed (if there are at least four alternatives), these majorities may have no citizen in common. Another way to put this point is that a Condorcet winner may fail to be the first preference of any citizen (so it is clear the Condorcet winner may be distinct from the plurality winner). The Condorcet winner is therefore another weakening of the notion of the majority winner—one that has more theoretical appeal than the plurality winner.

Nevertheless, the extension of majority rule to multiple alternatives is not entirely straightforward, due to a characteristic of the majority preference relation that Condorcet was fully aware of. This is its possibly *cyclic* nature:  $x$  may beat  $y$ ,  $y$  may beat  $z$ , and yet  $z$  may beat  $x$ . This possibility is most readily illustrated by the following three citizen preference orderings over the three alternatives  $x$ ,  $y$ , and  $z$ .

1	2	3
$x$	$y$	$z$
$y$	$z$	$x$
$z$	$x$	$y$

Note that each alternative is beaten by one other alternative, so a Condorcet winner fails to exist. This phenomenon has been called the "Condorcet effect," the "Arrow problem," the "paradox of voting," and (perhaps most illuminatingly) "cyclical majorities." It evidently was first discovered by Condorcet, and it was then alternately forgotten and rediscovered until the work of Duncan Black and Kenneth J. Arrow some forty years ago firmly placed it in the minds of social choice theorists.<sup>18</sup>

Notice that cyclical majority preference requires that different majority preference relationships be effected through different majorities. Thus in the example above,  $x$  beats  $y$  through the majority of 1 and 3,  $y$  beats  $z$  through the majority of 1 and 2, and  $z$  beats  $x$  through the majority of 2 and 3. If any citizen is a "permanent winner"—that is, belongs to all majorities through which majority preference relationships are effected—majority preference must be identical to that citizen's preference ordering and is therefore non-cyclical, and the most preferred alternative of the "permanent winner" is the Condorcet winner.<sup>19</sup>

#### 4. The Median Voter Theorem

Consider a continuous issue  $X$  over which the preferences of all  $n$  citizens are single-peaked. Each citizen has an ideal point somewhere along the continuum, and these ideal points can be ordered from (let us say) "left" to "right," and we can accordingly label them  $x^1, x^2, \dots, x^n$ . If the number  $n$  of citizens is

odd, a unique ideal point  $x^m$  (where  $m = (n + 1)/2$ ) occupies the median position in this ordering. A social choice result of fundamental importance, originally due to Duncan Black and commonly known as the *median voter theorem*, is that  $x^m$  is the Condorcet winner on issue  $X$ .<sup>20</sup> To verify this, consider any point to the left of  $x^m$ ;  $x^m$  is preferred by the median citizen and all citizens to the his right, by definition a majority of citizens, so  $x^m$  beats every alternative to its left. By similar argument,  $x^m$  beats every alternative to its right. Thus  $x^m$  beats every other alternative and is the Condorcet winner.

#### 5. Multiple Dimensions and "Chaotic" Majority Rule

In a multidimensional issue space with single-peaked preferences, a unique median ideal point exists only if ideal points are distributed in a distinctly fortuitous fashion; consequently, there is almost never a Condorcet winner.<sup>21</sup> Moreover, in the almost certain event that there is no Condorcet winner, a global majority preference cycle encompasses the entire issue space.<sup>22</sup> Because of these results, majority rule on a multidimensional issue space has been characterized as "chaotic." However, further results indicate that, even in the multidimensional case, majority rule has a strong centralizing tendency, so that collective choice based on majority rule typically leads to selection of a relatively centrist (in the sense of median) alternative (e.g., one belonging to the "uncovered set"), even though we cannot say exactly which one.<sup>23</sup>

However, this centralizing tendency does not apply to allocative issues. In this context, majority rule appears to be genuinely "chaotic."<sup>24</sup> Collective choice appears to be governed not by preferences and the decision rule but by fortuitous coalition formation or by specific (and arbitrary) features of institutional context.<sup>25</sup>

#### 6. Majority Rule and Political Process

Having summarized results on the theoretical properties of majority rule, it may be worthwhile to ask to what extent the political process in (more or less) majoritarian political systems

is governed by these results, and particularly by the median voter theorem applied in the context of ideological politics.

In a legislature with weak or non-existent party discipline (e.g., the United States), we might expect that any proposal would be successfully amended until the ideal point of the median legislator was (approximately) reached. (Some majority of legislators always has the incentive to introduce and support such amendments.) Furthermore, the ideal point of the median legislator ought to be quite similar to that of the median citizen. (Details of the electoral system—in particular, the extent to which it is majoritarian or proportional—would influence the spread of legislator ideal points but would have little influence on the location of their center.)

In a legislature with two strongly disciplined parties (e.g., the United Kingdom), in the preceding electoral campaign (or in anticipation of the subsequent one), each party—whether influenced by the desire for simple electoral victory or by the desire to maximize its policy goals (or, more plausibly, by a mixture of these goals)—would offer an (approximately) median platform or establish a similarly centrist record in office.<sup>26</sup>

In a legislature with multiple disciplined parties (presumably elected by some variant of proportional representation), the median party has an overwhelming bargaining advantage in getting into a coalition government and in imposing its preferred position on that government, and in turn (given the proportional electoral system) the median party ought to represent the median citizen accurately.

Of course, participants in the political process often make mistakes and fail to exploit available opportunities, so collective choices may be made and endure even though some majority exists that prefers another outcome but “can’t get its act together.” But such “failures” in the political process can hardly be blamed on the institution of majority rule, the consequences of which we are evaluating.

#### IV. CLEAVAGE POLITICS

In this section, we examine the consequences of majority rule for minority interests in the context of cleavage politics. We

examine in turn collective choice on a single dichotomous issue considered in isolation, two dichotomous issues considered independently, two dichotomous issues considered jointly, and finally a series of such issues.

##### 1. Single-Issue Cleavage Politics

Let us designate the one issue  $X$  with alternatives  $x$  and  $\bar{x}$ . Issue  $X$  partitions the set of citizens into two complementary subsets according to their preferences on the issue: a larger subset who prefer the majority winner  $x$  and a smaller subset who prefer the minority alternative  $\bar{x}$ . Let  $p$  (between  $\frac{1}{2}$  and 1) designate the proportion of all voters who prefer the majority alternative. We call  $p$  the *level of popularity* of  $x$ . To the extent that  $p$  approaches 1, issue  $X$  is *consensual*; to the extent that  $p$  approaches  $\frac{1}{2}$ , it is *divisive*.

It is in the context of a single dichotomous issue that the problem of majority rule versus minority interests arises most starkly. The majority wins totally and the minority loses totally. Moreover, this partitioning of society into winners and losers depends in no way on the size of the minority; a large minority does no better than a tiny minority. Likewise, collective choice is unaffected by the “distance” (if we can speak of such a thing in the context of cleavage politics) between the majority and minority alternatives. Minority preferences are indeed “subsumed” by the “numerically more powerful majority choice”; it makes no difference for collective choice whether members of the minority are enfranchised or not.

Insofar as this situation may be deemed socially unfortunate, we should note that it comes about essentially because of the nature of the issue and preferences regarding it, not because of the nature of the decision rule used to resolve the issue. If there are truly only two ways to resolve the issue (i.e., if the issue is truly dichotomous), to choose the only other way that is available would almost surely be regarded as an even more socially unfortunate resolution. And if other more appealing resolutions of the issue are available, we are beyond the scope of a single dichotomous issue.<sup>27</sup>



2. Two-Issue Cleavage Politics

Suppose we have two dichotomous issues  $X_1$  and  $X_2$ . We use the same notation introduced above with the respective subscripts.

The two issues partition the set of citizens into four subsets or clusters according to their first preferences regarding issues  $X_1$  and  $X_2$ , as follows. The *majority cluster* is composed of citizens who prefer the majority alternatives on both issues; their first preference is the outcome  $(x_1, x_2)$ , their last preference is  $(\bar{x}_1, \bar{x}_2)$ , and their intermediate preferences—with respect to  $(x_1, \bar{x}_2)$  and  $(\bar{x}_1, x_2)$ —may vary. The *minority cluster* is composed of citizens who prefer the minority alternatives on both issues; their first preference is  $(\bar{x}_1, \bar{x}_2)$ , their last preference is  $(x_1, x_2)$ , and their intermediate preferences—with respect to  $(x_1, \bar{x}_2)$  and  $(\bar{x}_1, x_2)$ —may vary. The two *mixed clusters* are composed of citizens who prefer the majority alternative on one issue and the minority alternative on the other. One such cluster has the first preference of  $(x_1, \bar{x}_2)$  and the last preference of  $(\bar{x}_1, x_2)$ ; the other cluster has the reverse first and last preferences. The intermediate preferences of both mixed clusters—with respect to  $(x_1, x_2)$  and  $(\bar{x}_1, \bar{x}_2)$ —may vary (both between and within clusters).

The relative size of these clusters depends on three parameters: the *level of popularity* of each majority alternative and the *degree of association or reinforcement* between preferences on the two issues. Levels of popularity are indicated by the fractions  $p_1$  and  $p_2$ . The degree of association or reinforcement between preferences on the two issues is indicated by the (positive or negative) parameter  $r$ , which represents, as a fraction of all citizens, the amount by which the size of each of the majority and minority clusters exceeds (or falls below) the “baseline” fraction that would result if preferences on the two issues were statistically independent. (The mixed clusters necessarily deviate by the same magnitude in the opposite direction.) Using these symbols, the relative frequency distribution of citizens over the four clusters is shown in figure 1.

Examination of the expressions in the cells of figure 1 confirms (as should be intuitively evident) that: (1) the majority cluster is never empty and always exceeds the minority cluster

FIGURE 1

Preferred Position on $X_1$		on $X_2$		Popularity
		$x_2$	$\bar{x}_2$	
$x_1$	$x_1$	Majority Cluster $p_1 p_2 + r$	Mixed Cluster $p_1(1 - p_2) - r$	$p_1$
	$\bar{x}_1$	Mixed Cluster $(1 - p_1)p_2 - r$	Minority Cluster $(1 - p_1)(1 - p_2) + r$	
Popularity		$p_2$	$(1 - p_2)$	1.00

in size; and (2) the combination of the majority cluster and either mixed cluster must itself be a majority.

If preferences on the two issues are perfectly *crosscutting*,  $r$  equals zero, so the fraction of citizens in each cluster is just the “baseline” fraction. To the extent that issue preferences are *positively reinforcing*,  $r$  is positive, so the fraction of citizens in the majority and minority cluster is greater, and in each mixed cluster is less, than the “baseline” fraction. When preferences are as positively reinforcing as possible (given the popularity levels), at least one mixed cluster is empty. To the extent that issue preferences are *negatively reinforcing*,  $r$  is negative, so the fraction of citizens in each mixed cluster is greater, and in the the majority and minority cluster is less, than the “baseline” fraction. When preferences are as negatively reinforcing as possible (given the popularity levels), the minority cluster is empty.

The size of the majority cluster increases with the level of popularity of each majority alternative and the degree of positive reinforcement of preferences on the two issues. If the ma-

majority cluster is itself of majority size, we may speak of a *universal majority*. We also dub each member of the majority and minority clusters a *universal winner* and *loser*, respectively. (These terms are more natural when we move to multiple issues).

Let us use the following simple (inter-personally comparable) scale of political satisfaction (in the manner of the Rae-Taylor-Straffin analysis): a citizen's *level of political satisfaction* is equal to the fraction of issues on which his preferred alternative is collectively chosen. If we suppose for the moment that each majority alternative is collectively chosen, *average* satisfaction over two (or more) issues is simply their average popularity level and is independent of how reinforcing or crosscutting preferences are. Given our concern with minority interests, however, we must focus on how *unequally* satisfaction is distributed, and this does depend on how reinforcing or crosscutting preferences are. Specifically, *if we suppose that the majority alternative on each issue is collectively chosen*, the status of minority interests depends largely on the degree of positive reinforcement in preferences. If preferences are highly reinforcing in the positive direction (and especially if the issues are highly divisive), there are relatively many universal losers. If preferences are crosscutting (and especially if the issues are quite consensual), there are relatively few universal losers. And if preferences are sufficiently reinforcing in a negative direction, universal losers disappear entirely.

### 3. Coalitions of Minorities

Outcome  $(x_1, x_2)$  is the outcome composed of the two majority winners on the two issues considered separately. But if the issues are considered jointly, we must examine majority preference over all pairs of outcomes. Outcome  $(x_1, x_2)$  certainly beats both  $(x_1, \bar{x}_2)$  and  $(\bar{x}_1, x_2)$  under majority rule. Both pairwise comparisons involve outcomes that differ with respect to only one issue, and with respect to that issue  $(x_1, x_2)$  gives (by definition) the more popular alternative. But it does not follow that  $(x_1, x_2)$  beats  $(\bar{x}_1, \bar{x}_2)$  under majority rule.

Thus we should not jump too readily to the conclusion that the collective choice will be the pair of majority winners, for if

*the two issues are considered jointly* an effective "coalition of minorities" may exist. In terms of the majority preference relation over political outcomes, an effective *coalition of minorities* is a majority that prefers outcome  $(\bar{x}_1, \bar{x}_2)$  to outcome  $(x_1, x_2)$ , with the result that  $(x_1, x_2)$  is *not* the Condorcet winner among the four outcomes.<sup>28</sup>

Let us count up citizens who may have the required preferences to belong to a coalition of minorities. Certainly members of the minority cluster have the required preference for  $(\bar{x}_1, \bar{x}_2)$  over  $(x_1, x_2)$ . Equally certainly, members of the majority cluster have the opposite preference and cannot be part of such a coalition. Members of the mixed clusters may or may not have the requisite preference, depending on their *intermediate preferences* or, in more substantive terms, depending on which issues they care about more intensely. Therefore, whether there is an effective coalition of minorities depends on the size of the respective clusters and the intermediate preferences (or distribution of intensities) in the mixed clusters.

With respect to intermediate preferences, let the *fraction with majority intensity* be the fraction of citizens in the mixed clusters who prefer  $(x_1, x_2)$  to  $(\bar{x}_1, \bar{x}_2)$ —that is, who prefer getting their way only on the issue on which they prefer the majority alternative to getting their way only on the issue on which they prefer the minority alternative. Let the *fraction with minority intensity* be the fraction of citizens in the mixed clusters who prefer  $(\bar{x}_1, \bar{x}_2)$  to  $(x_1, x_2)$ —that is, who prefer getting their way only on the issue on which they prefer the minority position to getting their way only on the issue on which they prefer the majority position.<sup>29</sup>

Holding the distribution of intermediate preferences in the mixed clusters constant, the possibility of an effective coalition of majorities decreases with the size of the majority cluster, which in turn increases with the level of popularity of majority alternatives and with the degree of positive reinforcement. However, increasing the popularity of majority alternatives increases average satisfaction with the outcome  $(x_1, x_2)$  and, in that sense, makes the possibility a coalition of minorities less important. In this respect, therefore the degree of positive reinforcement is probably most important.<sup>30</sup>

The condition for an effective coalition of minorities is that

the minority cluster plus the fraction of the mixed clusters with minority intensity must be greater than the majority cluster plus the fraction of the mixed clusters with majority intensity. From this, we can readily identify two *necessary* conditions for an effective coalition of minorities. First, a coalition of minorities is effective only if the majority cluster is of less than majority size. Turning the proposition around, one condition that precludes an effective coalition of minorities is the existence of a universal majority, which can occur if one or both issues are quite consensual or preferences are quite positively reinforcing. Otherwise, the effectiveness of a coalition of minorities depends on the intermediate preferences of voters in the mixed cluster. An effective coalition of minorities can exist only if the fraction with minority intensity is greater than the fraction with majority intensity.<sup>31</sup> (In this sense, an "impartial" distribution of intensities means that coalitions of minorities can never be effective.) The required advantage of the fraction with minority intensity over the fraction with majority intensity depends in turn on the size of the majority cluster relative to the minority.

Finally, we observe that if one issue is of overwhelming *salience*, so that *all citizens care about it more intensely than about the other issue*, it follows that all members of mixed clusters with majority intensity belong to the *same* mixed cluster, that this mixed cluster together with the majority cluster is a majority, and that therefore there can be no effective coalition of minorities.

In the absence of an effective coalition of minorities,  $(x_1, x_2)$  is the Condorcet winner among the four outcomes generated by considering the two issues jointly. But if there is an effective coalition of minorities, i.e., if  $(\bar{x}_1, \bar{x}_2)$  beats  $(x_1, x_2)$  under majority rule, it does not follow that  $(\bar{x}_1, \bar{x}_2)$  is the Condorcet winner. Indeed, it is clear that both  $(x_1, \bar{x}_2)$  and  $(\bar{x}_1, x_2)$  beat  $(\bar{x}_2, \bar{x}_1)$ , for both pairwise comparisons involve outcomes that differ with respect to only one issue, and with respect to that issue  $(\bar{x}_1, \bar{x}_2)$  gives (by definition) the less popular alternative. Therefore, *an effective coalition of minorities implies that majority preference is cyclical and that there is no Condorcet winner at all.*<sup>32</sup> Collective choice with an effective coalition of minorities is therefore unpredictable and depends on factors independent of the institution of

majority rule. Given two-issue cleavage politics, we cannot say, even if the conditions for an effective coalition of minorities holds, that the two minorities will be substantially satisfied. But we can say that either minority or their intersection *may* be fully satisfied; nothing in the institution of majority rule implies that they must be dissatisfied.

#### 4. Multiple-Issue Cleavage Politics

Suppose we have a series of dichotomous issues  $X_1$  through  $X_k$ . The general nature of the previous analysis may be extended to this general case. As before, there is a single majority cluster (whose members prefer the majority alternative on every issue) and a single minority cluster (whose members prefer the minority alternative on every issue). But once the number of issues exceeds two, the majority cluster may be empty (and, once the number of issues exceeds three, may be empty even when the minority cluster is not). In addition, there are now many mixed clusters, which run from being *imbalanced in the minority direction* (made up of citizens who favor minority alternative on most issues) through being *balanced* (made up of citizens who favor majority alternatives on about half the issues) to being *imbalanced in the majority direction* (made up of citizens who favor majority alternatives on most issues). Unless preferences are systematically and positively reinforcing in a high degree, almost all citizens belong to mixed clusters.

Increasing the number of issues has no systematic impact on the average level of satisfaction, which (if majority alternatives are consistently chosen) is simply the average level of popularity of those alternatives. But, to the extent that preferences are crosscutting, increasing the number of issues does have a systematic effect on the distribution of satisfaction (even if majority alternatives are consistently chosen). The more issues there are, and the more preferences on these issues are crosscutting, the more citizens bunch up in mixed clusters and particularly mixed clusters that are balanced or (to the extent that majority alternatives are popular) somewhat imbalanced in the majority direction, so the more equally political satisfaction with outcomes is distributed. If some pairs of issues are negatively reinforcing,

universal (or even near-universal) winners and losers disappear, further reducing inequality in satisfaction. On the other hand, systematically and positively reinforcing preferences tend to polarize citizens into universal winners and losers.<sup>33</sup>

At the same time, if preferences are crosscutting, the more issues there are, the more likely it is that at least one coalition of minorities will be effective and consequently that majority alternatives may *not* be consistently chosen. In the manner described in the previous subsection, we can check whether there is an effective coalition of minorities with respect to any pair of issues. And even if there is no effective coalition of minorities on any *pair* of issues, there may be an effective coalition of minorities with regard to some *larger set* of issues. As we consider the effectiveness of coalitions of minorities with respect to an expanding set of issues, two competing considerations arise. On the one hand (unless preferences are positively reinforcing in high degree), the size of the *potential* coalition increases, as more and more citizens find themselves in the minority on at least one issue. On the other hand, finding the right distribution of intermediate preferences (intensities) to support an *actual* coalition of minorities becomes more complex. Citizens in many mixed clusters prefer majority alternatives on most issues, and it is unlikely that they will care enough about the smaller number of other issues to prefer to get their way on those at the cost of not getting their way on the greater number of issues on which they prefer the majority alternative. On the other hand, citizens in other mixed clusters prefer minority alternatives on most issues, and it is likely that they will care enough about these issues to prefer to get their way on them at the cost of not getting their way on the small number of issues on which they prefer the majority alternative. Finally, citizens in balanced mixed clusters face tradeoffs similar to citizens in the two mixed clusters in the two-issue case. To the extent that issues tend to be consensual, the first class of mixed clusters will be the largest, making an effective coalition of minorities with respect to the expanded set of issues rather unlikely. On the other hand, as the set of issues expands, it evidently becomes more likely that a coalition of minorities will be effective with respect to *some* subset of issues.

The upshot is that citizens with minority interests on a particular issue *X* are unlikely to be in a minority with respect to all or even most issues, if there are a number of issues and preferences are at least somewhat crosscutting. Moreover, under the same circumstances, there is a reasonable probability that the minority in question will find itself part of an effective coalition of minorities with respect to *X* (and one or more other issues), in which case the minority alternative on issue *X* itself may well end up being collectively chosen under majority rule.

#### V. IDEOLOGICAL POLITICS

While cleavage politics partitions citizens into discrete clusters on the basis of their first preferences, ideological politics spreads citizens over a continuum with respect to their first preferences (or ideal points). One point on the continuum must be collectively chosen and becomes binding on the whole collectivity.

It is possible and useful in this context to introduce another crude measure of citizen satisfaction. We may suppose (very justifiably) that a citizen is totally satisfied if the collective choice is identical to his ideal point and is increasingly dissatisfied as the distance between the collective choice and his ideal point increases. (This is no more than the single-peakedness assumption stated in terms of satisfaction.) For simplicity (but with somewhat less justification), we shall further suppose that *dissatisfaction* for each citizen is a simple linear function of this distance. Moreover we shall continue to suppose that dissatisfaction is interpersonally comparable, so that we can speak of the minimum, maximum, average, etc., level of dissatisfaction with collective choices (by equating each with the minimum, maximum, average, etc., distance from each ideal point to the collective choice).

We should also observe that the notion of "winning" versus "losing" is greatly softened in the ideological context. It is true that, if the political process converges on the Condorcet winner through a series of parliamentary style votes (as suggested in section III.6 with respect to a political process with weak parties), winners and losers are generated on each pairwise vote.

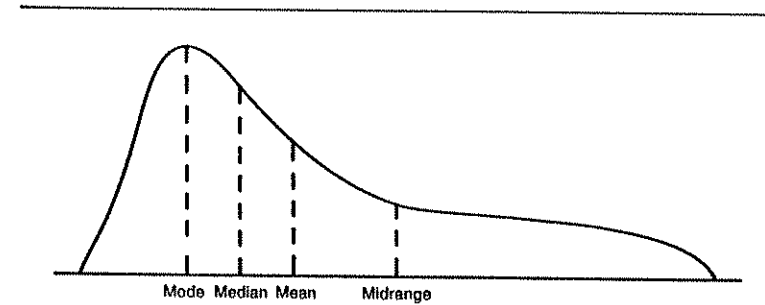
But the tally of such wins and losses will have little to do with how satisfied or dissatisfied citizens are with the ultimate collective choice. For example, if the initial proposal is far to the right and is then progressively amended toward the median, extreme leftwing citizens will consistently "win" and rightwing citizens will consistently "lose" in the sequence of votes, but the final (centrist) choice will be (more or less) equally unsatisfactory to both.

### 1. Ideological Politics with a Single Preference Distribution

In the context of ideological politics, with ideal points spread over a continuum, the notion of a "majority group" versus a "minority group" is rather murky. This is especially true if the distribution of ideal points is given by a single, more or less bell-shaped, frequency curve. In this subsection, we suppose that this is the case, although we do not suppose that the frequency curve is necessarily normal or even symmetric in shape. In this case, an ideological minority consists of citizens whose ideal points put them at one extreme of the distribution. But there are necessarily two extremes and two ideological minorities, one at each end of the distribution. Clearly if any change in collective choice gives greater satisfaction to members of one ideological minority, it must at the same time—and in approximately the same magnitude—give less satisfaction to members of the other ideological minority. Apparently the conflict here is not so much majority interests versus minority interests but (one set of) minority interests versus (another set of) minority interests.

Given ideological politics on a single dimension, collective choice of the median ideal point (i.e., the Condorcet winner) has this desirable property: it *minimizes the average dissatisfaction of citizens*—any other collective choice results in greater total (and average) dissatisfaction.<sup>34</sup> But of course, to the extent that preferences are dispersed, levels of satisfaction will likewise be dispersed about this (optimal) average. And if our concern is to minimize the damage done to minority interests, we might want to proceed in a Rawlsian manner by minimizing the maximum level of dissatisfaction in society.<sup>35</sup> How does majority rule fare

FIGURE 2



in this respect? This "minimax" criterion dictates that the collective choice should be the *mid-range*, i.e., the point exactly halfway between the ideal points of the most leftwing and the most rightwing citizens. Whatever the collective choice, it is clear that one or other of these two extreme citizens must be the most dissatisfied. The midrange makes them equally dissatisfied and any change in collective choice must increase the dissatisfaction of one or the other. It follows that majority rule conforms with this "minimax" criterion if the distribution of preferences is symmetric (so that the mirror image of the distribution is indistinguishable from the original distribution), in which case the median and midrange coincide.

Figure 2 shows a (bell-shaped but) asymmetric distribution of ideal points over an ideological continuum, and it shows the resulting discrepancy between the median and midrange. A compromise collective choice is the *mean*, which has the property that the average algebraic deviation from it is minimum and in fact equals zero; put more substantively, if the collective choice is the mean, the total "leftwing" dissatisfaction is just equal to the total "rightwing" dissatisfaction.<sup>36</sup>

Collective choice could conceivably be made by dividing the continuum up into small intervals of equal length, asking citizens to vote for one interval, with collective choice being the *plurality winner*. Provided that everyone votes "sincerely," this would produce the *mode* as the collective choice. Collective choice as the mode has the property of maximizing the proportion of citizens who are (essentially) fully satisfied. It can treat

one or other ideological extreme very badly, however, as the distribution in figure 2 illustrates.

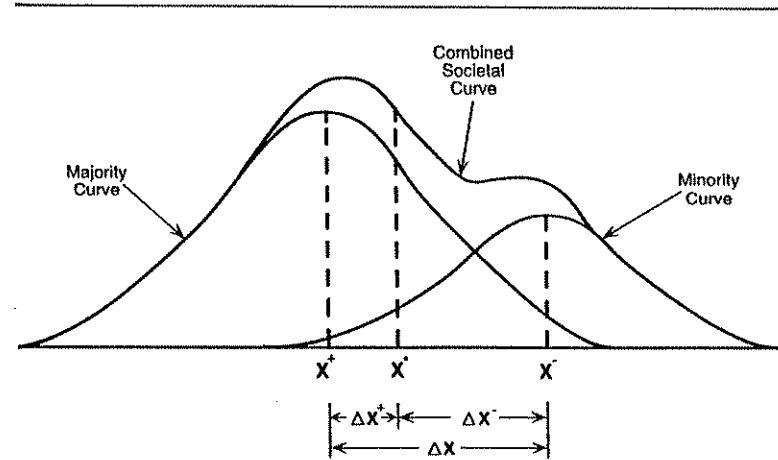
Finally, we may note that if the ideal points of voters at either ideological extreme (say the right in figure 2) were to shift outwards and become even more extreme, collective choice as the mean or midrange would move somewhat in the same direction. In some respects, and in terms of being solicitous of minority interests in particular, this might be regarded as a desirable trait in collective choice. But, in another respect, it could be a severe drawback, for by responding to extreme preferences in this way, it would give citizens with moderately extreme preferences an incentive to *express* far more extreme preferences, in the hope that the resulting collective choice would be close to their true (and more moderate) preferences. Since citizens on both extremes could play this game, there would be a widespread tendency to feign extreme preferences.<sup>37</sup> In any event, collective choice as the median is resistant to (real or feigned) changes in extreme preferences. This gives majority rule in the context of ideological politics a quality of "strategyproofness."

Moving from one to two or more ideological dimensions has two effects. First, there are no longer just two ideological minorities; rather ideological minorities populate the entire "boundary" of the distribution. But such minorities come in "antipodal" pairs with directly opposed interests, with respect to which similar considerations arise as in the one-dimensional case. Second, it is very unlikely a Condorcet winner exists, but the literature cited in section III.5 suggests majority rule is still likely to produce outcomes near the center of the distribution.

### 2. Ideological Politics with a Distinctive Minority

As we have seen, if ideological politics involves a single (bell-shaped) distribution of ideal points for the whole society, the issue of majority rule versus minority interests somewhat disappears from view. However, we can recover much of the dualistic conflict between majority and minority interests by assuming that a minority group of citizens exists that is (in some degree) *ideologically distinct* from the larger majority group, in that (1) each group has its own (bell-shaped) preference distribution

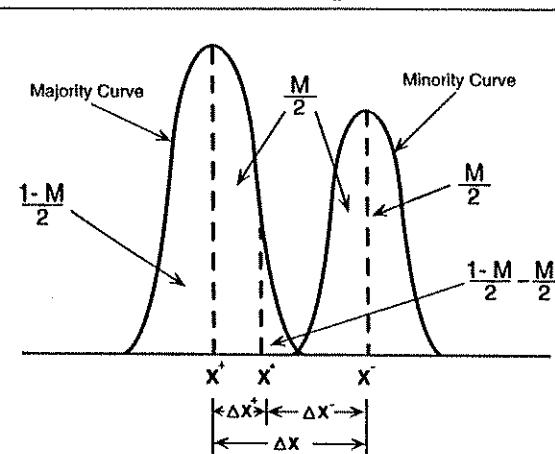
FIGURE 3



and (2) the two distributions are differently centered. The preference distribution of the whole society therefore is combination of the two distributions, and this combined societal distribution is typically *bimodal* and *asymmetric*, as illustrated in figures 3 and 4.

For purposes of specific illustrations and calculations, it is

FIGURE 4



convenient to assume that the majority and minority groups have symmetrically—and indeed normally—distributed ideal points. However, the general thrust of the conclusions reported here holds for any bell-shaped curves.

The collective choice situation that results from ideological politics with a distinctive minority is described by a number of parameters, which we now consider.

The first is *relative size of the minority group*. Let the size of the minority group relative to the total society be given by the fraction  $M$ , where  $M$  lies strictly between zero and  $1/2$ . Geometrically,  $M$  is the area under the minority curve as a proportion of the area under the societal curve. In figure 3,  $M$  is about 0.25; in figure 4,  $M$  is about 0.4.

The next parameters are the *centers* (means) of the majority and minority distributions, which we designate  $x^+$  and  $x^-$  respectively. Given that both distributions are symmetric, these means are also medians. Thus, by the median voter theorem,  $x^+$  is the (hypothetical) *majority collective choice*—that is,  $x^+$  is what the collective choice would be if the majority group were choosing (by majority rule) unilaterally (e.g., if the minority were disenfranchised). Likewise,  $x^-$  is the (hypothetical) *minority collective choice*—that is,  $x^-$  is what the collective choice would be if the minority group were choosing (by majority rule) unilaterally (e.g., if the majority were disenfranchised).

The two means may be collapsed into a single parameter—the magnitude of *ideological polarization* between the majority and minority groups, given by the length of the interval along the ideological continuum from  $x^+$  to  $x^-$ , which we designate  $\Delta x$ .

The third and fourth parameters pertain to the level of *ideological cohesion* of the majority and minority groups, respectively. A perfectly cohesive group is monolithic or homogenous in its preferences; a less cohesive group is dispersed or heterogenous in its preferences. Let  $D^+$  and  $D^-$  designate the dispersion of the majority and minority distributions, respectively—specifically, the standard deviations of their ideal points;  $D^+$  and  $D^-$  measure (inversely) the cohesiveness of the two groups.

We are concerned here with majority and minority groups that are ideologically distinct. This means that each group has

its own preference distribution differently centered on the ideological continuum, so that there is some degree of ideological polarization between them. *Ideological distinctiveness* is a matter of degree, however, and it depends on the cohesiveness of the two groups as well as their polarization. For example, if we compare figures 3 and 4, polarization is greater in figure 3 but the majority and minority are more distinctive in figure 4 because both groups are more cohesive. Ideological distinctiveness determines the amount of “overlap” between the two frequency curves. Thus the minority depicted in figure 4 is essentially *totally distinctive*, in that there is virtually no overlap between the two curves.

The notion of ideological distinctiveness implies that, for fixed levels of cohesion for the majority and minority groups, there is a *critical threshold* of polarization just sufficient (virtually) to eliminate overlap between the two frequency curves, so that the minority becomes totally distinctive. The polarization depicted in figure 4 is just about at this critical threshold; if it were diminished at all, the frequency curves would begin to overlap substantially.<sup>38</sup>

Finally, we label as  $x^*$  the societal collective choice—that is,  $x^*$  is what the collective choice actually is given that the whole society is choosing by majority rule. By the median voter theorem,  $x^*$  is the median of the combined societal distribution, and it is apparent that  $x^*$  lies somewhere in the interval between  $x^+$  and  $x^-$ . Therefore the interval  $\Delta x$  can be subdivided into the two subintervals from  $x^+$  to  $x^*$  and from  $x^*$  to  $x^-$ , as shown in figures 3 and 4, which we label  $\Delta x^+$  and  $\Delta x^-$  respectively.

The first subinterval  $\Delta x^+$  may be characterized as the *minority impact* on collective choice, as it represents the impact that the presence of the enfranchised minority has on collective choice under majority rule. To say that “the numerically more powerful majority choice simply subsumes minority preference” is to claim, in this context, that  $\Delta x^+$  is (essentially) zero.

The second subinterval  $\Delta x^-$  may be characterized as *collective minority dissatisfaction*, as it represents the gap between the collective choice the minority would make if it constituted its own polity and the collective choice that binds it when it is part of the larger polity.<sup>39</sup>



The focus of our analytical concern in the remainder of this subsection is the location of the societal collective choice  $x^*$  between  $x^+$  and  $x^-$  (and thus the magnitude of minority impact  $\Delta x^+$ ) as a function of the parameters we have just discussed.

It may be worth observing that collective minority dissatisfaction is in large measure a function of the magnitude of the polarization between the majority and minority groups and, in particular, that (given majority rule) minority dissatisfaction can be reduced to zero only if polarization is reduced to zero. But if polarization is reduced to zero, the minority impact on collective choice is necessarily reduced to zero also. That is, the only kind of minority that can have an impact on collective choice under majority rule is a distinctive one that is also bound to be at least somewhat dissatisfied with that collective choice (even after having its impact on it).

The next point is that there is almost always a minority impact, and that the magnitude of this impact almost always depends on the size of the minority. That is, minority interests are almost never entirely "subsumed" in ideological politics. In this respect, ideological politics, even with a distinctive minority, differs from single-issue cleavage politics.

Let the cohesion of the majority and minority groups be fixed and let polarization reach the critical threshold. The magnitude of minority impact is the distance between the majority and societal medians. Since the minority is (essentially) totally distinct, (essentially) the entire minority frequency lies on the  $x^-$  side of  $x^*$  (as is true in figure 4 but not figure 3). Thus the interval from  $x^+$  to  $x^*$  is essentially the interval required to make the area under the majority frequency curve that lies between  $x^+$  and  $x^*$  equal to half the total area under the minority curve.<sup>40</sup>

If we measure minority impact in terms of the *area* under the majority curve that lies between  $x^+$  and  $x^*$ , it follows that this impact, as a function of minority size  $M$ , is  $M/2(1-M)$ . Thus a small minority, with for example  $M=0.1$ , has a very small area impact (i.e., 0.056). But as minority size increases, area impact increases faster, until a minority of maximum size (i.e.,  $M$  just short of 0.5) has a maximum area impact (likewise just short of 0.5).

Our real concern, however, is minority impact measured in terms of the *displacement of collective choice* along the ideological continuum toward the minority distribution, i.e., the interval  $\Delta x^+$ . Since the height of the (normal) majority frequency curve is declining in this range—and declines especially rapidly from about  $0.5D^+$  to about  $1.5D^+$  away from  $x^+$ , increases in minority size  $M$  (and in area impact  $M/2(1-M)$ ) produce increasingly large increases in the displacement of  $x^*$  along the continuum. Minority impact  $\Delta x$ , where  $\Delta x$  is expressed in terms of  $D^+$ , increases roughly linearly from 0 to about  $1D^+$  as  $M$  increases from 0 to 0.4 and increases rapidly to about  $2D^+$  as  $M$  increases to 0.49.

Suppose ideological polarization increases while group cohesion is constant, so that polarization goes beyond the critical threshold. *For fixed minority size, minority impact does not further increase as polarization increases beyond the critical threshold.* However, the relationship between minority size and minority impact remains as before, so *larger minorities have greater impact than small ones no matter the degree of ideological polarization.* In the first respect, ideological politics with a distinctive minority resembles—but, in the latter respect, contrasts with—single-issue cleavage politics.

Finally, suppose that polarization is reduced while the cohesion of both groups remains the same, so that polarization falls below the critical threshold, and the two frequency curves overlap substantially. The minority curve now straddles  $x^*$  (and perhaps  $x^+$  as well, as in figure 3). Since a portion of the minority lies in the majority side of the societal median  $x^*$ , the area under the majority curve between  $x^+$  and  $x^*$  is now less than  $M/2$ . Moreover, area impact is now less responsive to variation in minority size. As polarization vanishes (and the majority and minority become ideologically indistinguishable), minority impact likewise vanishes (and necessarily becomes unresponsive to minority size), though collective minority dissatisfaction disappears.

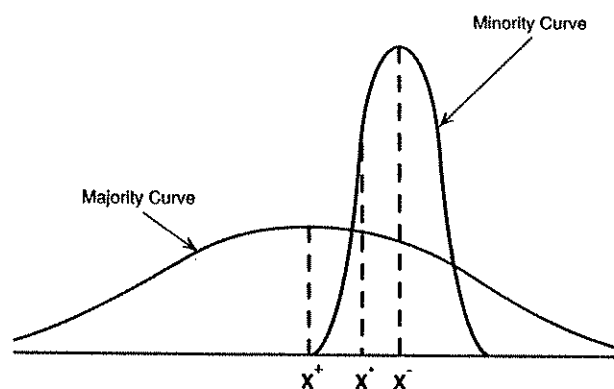
To this point, we have assumed that majority (and minority) cohesion are fixed, and we have described minority impact in terms of  $D^+$ -units. For example, we have seen that, with polarization at or beyond the critical threshold,  $M=0.4$  produces a



minority impact of about  $1D^+$ . But the actual magnitude of this impact, i.e., the amount of displacement along the ideological continuum, obviously depends in turn on the magnitude of  $D^+$ . That is, *minority impact depends*, not only on minority size, but also on the cohesion of the majority group. Other things constant, *minority impact on collective choice decreases as majority cohesion increases*. Further, *minority impact becomes less responsive to variation in minority size as majority cohesion increases*. Both effects reach their logical maximum when the majority become fully cohesive (i.e.,  $D^+ = 0$ ); in this circumstance, no minority of any size (and regardless of the level of minority cohesion or polarization between majority and minority) can have any impact on collective choice. In this special case, we are in effect back to the realm of single-issue cleavage politics.<sup>41</sup>

So long as ideological polarization is at or beyond the critical threshold, *minority cohesion has no effect on collective choice*. However, below the critical threshold of polarization, both minority impact on collective choice and the responsiveness of that impact to minority size increase (though only modestly) with minority cohesiveness. In fact, if polarization is low and the minority is so much more cohesive than the majority that the majority distribution "straddles" the minority distribution in the manner of figure 5, societal collective choice  $x^*$  can lie closer to  $x^-$  than to  $x^+$ .

FIGURE 5



We have seen that group cohesion—and especially majority cohesion—is an important determinant of minority impact on collective choice. This raises the question of a *collusive majority*—that is, a majority in which a high degree of (apparent) cohesion is artificially enforced. More particularly, members of the majority group might agree on a common alternative (perhaps  $x^+$ ) and then act as if they all shared this alternative as their ideal point; then, just as if the majority were in fact fully cohesive, "majority choice simply subsumes minority preferences."<sup>42</sup>

Such collusion, however, is intrinsically difficult to enforce. Whatever instruments of enforcement may be available are beyond the scope of our social choice framework, and the institution of majority rule itself tends to undermine such collusion. Whatever the parameters of the situation may be, the majority frequency curve straddles  $x^*$ —that is, some members of the majority are highly satisfied with  $x^*$ . More importantly, a significant minority of them prefer  $x^*$  to  $x^+$  and—since  $x^*$  is the Condorcet winner—this minority of the majority group together with the minority group itself constitute a societal majority. Therefore, there is no reason within the abstract logic of the situation why these critical members of the majority group would join a collusive agreement on  $x^+$  (or, for that matter, any alternative on the  $x^+$  side of  $x^*$ ).<sup>43</sup>

## VI. DISTRIBUTIVE POLITICS

Under majority rule, citizens (and the votes they cast) are *formally interchangeable* (or "fungible"); May's Anonymity Condition expresses this formal property. Essentially, this means that any one citizen (and vote) is as good as any other in building or sustaining a majority coalition. In this formal sense, majority rule entails "equality of political opportunity."

But, given any actual occasion for collective choice, citizens (and their votes) are attached to preferences (or interests), and the preferences to which they are attached are typically *not* interchangeable—some preferences are compatible with one another, others are not.<sup>44</sup> In this substantive sense, one citizen (and one vote) may not be as good as another in building or sustaining a majority coalition *in support of a particular policy*.

This is most strongly the case in single-issue cleavage politics, or multiple-issue cleavage politics with positively reinforcing preferences or with one issue of overwhelming salience. It is somewhat less strongly the case with multiple-issue cleavage politics with cross cutting preferences and dispersed intensities or in ideological politics, especially if preferences are reasonably dispersed.

In the realm of distributive politics, however, citizens (and the votes they cast) in fact are fully interchangeable. While citizens (and votes) are attached to (individualistic) preferences, such preferences are all identical from the point of view of third parties looking around for coalition partners to support some allocative alternative. Put otherwise, given any present majority controlling collective allocation, anyone outside that majority (presumably receiving nothing at the moment) can advantageously make an offer to displace any member of the present majority that will be appealing to other members of that majority. Thus we may expect to see a constant shifting (or generic instability) of majorities and allocative outcomes. This expectation is confirmed by formal results in social choice on the structure of majority rule over the space of allocative alternatives. First, there is no Condorcet winner. Second, a global majority preference cycle encompasses virtually all allocative alternatives (even inefficient ones that waste a portion of the good being allocated). Third, even such refined "solutions" as the "uncovered set" that give (even in the face of global cycles) considerable coherence to majority rule on multidimensional alternative with single-peaked preferences spaces effectively break down in the allocative context with individualistic preferences by expanding to include essentially all efficient alternatives.<sup>45</sup> In sum, in the context of distributive politics, majority rule is chaotic and cannot be blamed or credited for any particular outcome.

A consequence of the interchangeability of preferences in distributive politics is that, within our social choice framework, we have no way to identify or define a minority or minority interests. In cleavage and ideological politics, minorities and minority interests may be defined in terms of preference distributions, and exactly for this reason members of such minorities may not make desirable coalition partners. But in the context of

distributive politics, all citizens have, in a relevant sense, identical preferences, so the only way minority status may be defined is in terms of some exogenously fixed stigmatic criteria (such as race, language, religion, and so forth).

Given the chaotic nature of majority rule in the context of distributive politics, majority rule itself cannot really be blamed if outcomes systematically discriminate against stigmatic minorities by excluding them from collectively chosen allocations. But it is also true that the chaotic nature of majority rule may give room for such seemingly arbitrary factors to exert an influence that they would not (directly) exert in the realm of cleavage or ideological politics, where collective choice under majority rule is more clearly determined by preferences.<sup>46</sup>

Three sorts of "solutions" have been proposed for the problem of allocation by majority rule. Consider the archetypical divide-the-dollar game with three players. Many years ago, John von Neumann and Oskar Morgenstern proposed that certain "standards of behavior" would emerge to constrain collective choice to a particular "solution" (subset of allocative alternatives) that would exhibit two types of stability. First, such a solution would exhibit *internal stability* in the sense that no alternative in the solution would be beaten by any other alternative in the solution. Second, such a solution would exhibit *external stability* in the sense that any alternative outside the solution that beats an alternative inside the solution would in turn be beaten by another alternative in the solution.<sup>47</sup> They then showed that the three-player allocation game possesses two types of solutions. The first is the *main simple solution*, which consists of the three allocations that give each member of each two-player majority half of the spoils and gives the remaining player nothing. The second is an infinite family of *discriminatory solutions*, each containing an infinite number of alternatives all of which have this property: two players collude, by conceding some small amount of spoils to the third player and implicitly agreeing to deal no further with him, and then bargaining over the remainder.<sup>48</sup>

A third type of solution has been proposed in the more empirically oriented work on legislative choice (with the U.S. Congress especially in mind). This is the *universalistic solution*

that consists of the single alternative the gives each player an equal share of the spoils. The universalistic solution is internally stable in a trivial way. But it is externally stable only in a long-run sense and with additional assumptions (e.g., about attitudes toward risk). And its realization may depend on characteristics of the organization of a legislature that compromise its majoritarian character.<sup>49</sup>

The main simple and universalistic solutions treat all players symmetrically, but—as its name suggests—the discriminatory solution does not. Indeed both the nature and name of this solution suggest (but only suggest) how stigmatic characteristics might influence collective choice of allocative outcomes under chaotic majority rule. But it bears repeating that nothing in the logical character of majority rule implies that an exogenously defined minority must do poorly in distributive politics.

#### VII. CONCLUSION

On the basis of formal analysis, we have seen that the argument that majority rule renders minority interests irrelevant—that “the numerically more powerful majority choice simply subsumes minority preferences”—must be importantly qualified. The argument in effect assumes that politics is invariably cleavage politics with a single issue or with highly reinforcing preferences. In other political contexts—cleavage politics with cross-cutting preferences, ideological politics (even with a distinctive minority), and distributive politics—minority preferences significantly influence collective choice. Real-world politics of course is a complex mixture of cleavage, ideological, and distributive politics; such complexity probably gives minority interests additional influence, but taking account of such complexity is largely beyond the present power of formal political theory. Real-world political outcomes clearly are also influenced by factors other than preferences and decision rules. Whether and how such additional factors affect the fortune of minority interests must await further analysis; clearly they are likely to affect different minorities (sociologically defined) in different ways.

#### NOTES

1. “The Triumph of Tokenism: The Voting Rights Act and the Theory of Black Electoral Success,” *Michigan Law Review*, 89 (1991): 1077–154; “No Two Seats: The Elusive Quest for Political Equality,” *Virginia Law Review*, 77 (1991): 1413–514; “The Representation of Minority Interests: The Question of Single-Member Districts,” *Cardozo Law Review*, 14 (1993): 1135–74; “Groups, Representation, and Race-Conscious Districting: A Case of the Emperor’s Clothes,” *Texas Law Review*, 71 (1993): 1593–642.

2. *The Tyranny of the Majority* (New York: The Free Press, 1994). This book contains the law review articles, together with an introductory chapter and some other material.

3. “Classic” works in this genre most notably and relevantly include: Kenneth A. Arrow, *Social Choice and Individual Values*, 2nd ed. (New York: Wiley, 1963); Duncan Black, *The Theory of Committees and Elections* (Cambridge: Cambridge University Press, 1958); Anthony Downs, *An Economic Theory of Democracy* (New York: Harper & Row, 1957); and James M. Buchanan and Gordon Tullock, *The Calculus of Consent* (Ann Arbor: University of Michigan Press, 1962). A survey (together with a somewhat idiosyncratic thesis) is provided by William H. Riker, *Liberalism Against Populism: A Confrontation between the Theory of Democracy and the Theory of Social Choice* (San Francisco: Freeman, 1982).

4. A partial exception is my own “Pluralism and Social Choice,” *American Political Science Review*, 77 (1983): 734–47, which this essay in some ways extends. Another partial exception is Brian Barry, “Is Democracy Special?” in *Philosophy, Politics, and Society*, 5th series, edited by Peter Laslett and James Fishkin (New Haven: Yale University Press, 1979). I should say that both of my essays owe much to Barry’s work.

5. For example, Henry Steele Commager, *Majority Rule and Minority Rights* (New York: Oxford University Press, 1943).

6. See, in particular, Buchanan and Tullock, *The Calculus of Consent*.

7. Some social choice theorists have formulated the problem of collective decisions and individual rights in another way that has raised all sorts of theoretical complexities. This formulation is due to Amartya K. Sen, “The Impossibility of a Paretian Liberal,” *Journal of Political Economy*, 78 (1970): 152–57, and it has generated an enormous literature.

8. See Nicholas R. Miller, “Power in Game Forms,” in *Power, Vol-*

ing, and Voting Power, edited by Manfred J. Holler (Vienna: Physica-Verlag, 1982); and Miller, "Pluralism and Social Choice."

9. Under a typical "majoritarian" (Anglo-American style) electoral system based on single-member districts, some majority coalitions are impotent because of their particular geographical configurations. Proportional representation assures that any majority coalition in the electorate, whatever its geographical configuration, can secure majority representation in the legislature and thus (given legislative majority rule) can control legislative outcomes.

10. Of course, preferences on many distinct issues (as the term is ordinarily used) are not separable. (For example, how much money you want to spend on Project X likely depends on how much is spent on Project Y and also on what the Tax Rate Z is.) Our assumption can be justified on two grounds. First, the distinction between majorities and minorities, defined in terms of their preferences on issues, becomes murky if preferences are not separable, so informal discussions of political majorities and minorities tend implicitly to make the same assumption of separability. Second, we can simply define a set of alternatives as constituting a distinct issue only if preferences on it are in fact separable.

11. Such preferences were first formally described and named by Duncan Black, "On the Rational of Group Decision-Making," *Journal of Political Economy*, 56 (1948): 23-34; also see *The Theory of Committees and Elections*, 6-8. With respect to alternative funding levels, the implication of single peakedness is that a citizen who most prefers a low level also prefers a moderate to a high level, and a citizen who most prefers a high level also prefers a moderate to a low level. It should be emphasized that single peakedness is neither a logical necessity nor always plausible. For example, a citizen who most prefers a high funding level for a program may also believe that, in the event such funding is not provided, little or no funding would be preferable to an intermediate amount that would not accomplish its purpose.

12. In the terminology of formal political theory, such multidimensional preferences are "strictly quasi-concave"—a term that seems worthwhile to avoid in the text of this essay.

13. Insofar as the label "ideological politics" may suggest a politics in which participants are rigid and uncompromising, the label is misleading. The reader should bear in mind that it is the structure of collective choice (i.e., selecting a point along a continuum), not the behavior of the participants, that is being characterized.

14. "A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision," *Econometrica*, 20 (1952): 680-84.

15. This problem was first formally posed by Douglas W. Rae, "Decision-Rules and Individual Values in Constitutional Choice," *American Political Science Review*, 63 (1969): 40-56. Rae assumed that each citizen expects to have a 0.5 probability of supporting a given alternative on any issue, and his argument proceeded in a way that was persuasive but not logically conclusive. Shortly thereafter Michael Taylor, "Proof of a Theorem on Majority Rule," *Behavioral Science*, 14 (1969): 228-31, provided a general proof for all probabilities. Philip D. Straffin, Jr., "Majority Rule and General Decision Rules," *Theory and Decision*, 8 (1977): 351-60, subsequently provided a proof that was both more concise and covered a more comprehensive domain of possible decision rules.

16. *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix* (Paris, 1795). See Black, *The Theory of Committees and Elections*, 159-80.

17. If we allow for citizen indifference and/or an even number of citizens, so that "majority preference ties" may result, there may be several alternatives not beaten by any other.

18. Black, "On the Rational of Group Decision-Making"; Kenneth A. Arrow, *Social Choice and Individual Values*, 1st ed. (New York: Wiley, 1951); Black, *The Theory of Committees and Elections*.

19. A "permanent loser" also ensures non-cyclical majority preference, and his least preferred alternative is the Condorcet winner.

20. Black, "The Rationale of Group Decision-Making," and *The Theory of Committees and Elections*, 16. If the number of citizens is even, ideal points  $x^{(n/2)}$  and  $x^{(n/2)+1}$  jointly define the median; all alternatives in the interval connecting these points tie each other, every alternative in the interval at least ties every other alternative, and every alternative outside the interval is beaten by some alternative in it. Regardless of whether  $n$  is even or odd, single-peakedness precludes cyclical majority preference.

21. Charles R. Plott, "A Notion of Equilibrium and Its Possibility under Majority Rule," *American Economic Review*, 57 (1967): 787-806.

22. Richard D. McKelvey, "Intransitivities in Multidimensional Voting Models and Some Implications for Agenda Control," *Journal of Economic Theory*, 12 (1976): 472-82; and "General Conditions for Global Intransitivities in Formal Voting Models," *Econometrica*, 47 (1979): 1085-112.

23. Nicholas R. Miller, "A New Solution Set for Tournaments and Majority Voting," *American Journal of Political Science*, 24 (1980): 68-96; Richard D. McKelvey, "Covering, Dominance, and Institution Free Properties of Social Choice," *American Journal of Political Science*, 30

(1986): 283–314; Gary W. Cox, "The Uncovered Set and the Core," *American Journal of Political Science*, 31 (1987): 408–22; Scott L. Feld, Bernard Grofman, and Nicholas R. Miller, "Centripetal Forces in Spatial Voting Games: On the Size of the Yolk," *Public Choice*, 59 (1988): 37–50; Craig A. Tovey, "The Instability of Instability," Naval Postgraduate School Working Paper, Monterey, California, May 1991, and "The Almost Surely Shrinking Yolk," Political Economy Working Paper No. 161, School of Business and Center in Political Economy, Washington University, January 1992.

24. Benjamin Ward, "Majority Rule and Allocation," *Journal of Conflict Resolution*, 5 (1961): 379–89; Nicholas R. Miller, "The Complete Structure of Majority Rule on Distributive Politics," paper presented to the Annual Meeting of the Public Choice Society, San Antonio, Texas, March 1982; David L. Epstein, "Uncovering Some Subtleties of the Uncovered Set," paper presented to the American Political Science Association, Washington, D.C., September 1991.

25. On the latter point, see for example David P. Baron and John A. Ferejohn, "The Power to Propose," in *Models of Strategic Choice in Politics*, edited by Peter C. Ordeshook (Ann Arbor: University of Michigan Press, 1989).

26. Here we invoke the well-known result concerning "Downsian" party competition (Downs, *An Economic Theory of Democracy*, chapter 8), embellished in many subsequent works. For the extension to policy-motivated parties, see Randall L. Calvert, "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence," *American Journal of Political Science*, 29 (1985): 69–95. The same results, applied at the level of single-member districts, help justify the conclusion in the weak-party case that the median legislator closely resembles the median citizen.

27. Consider an example that Guinier uses (*Tyranny of the Majority*, 2). Responding to a problem posed in a children's magazine (what game would a group of six children play when four vote to play tag and two vote to play hide-and-peek?), her four-year son proposed this "untraditional" but appealing solution: they would take turns (perhaps playing tag about two-thirds and hide-and-peek about one-third of the time). But what has happened here is that an issue originally posed as dichotomous has been converted into a continuous one (i.e., what proportion of the time should each game be played?). And, if preferences on this issue are reasonably spread out over the resulting continuum, majority rule will produce (on the basis of the median voter theorem) a "taking turns" solution. (It seems doubtful, however, that a "taking turns" solution is practicable for many substantial and divisive

issues of public policy. We really can't make abortion legal on Mondays, Wednesdays, and Fridays, and illegal on Tuesdays and Thursdays.) However, if the four children are tag-playing fanatics, *all* of whom want to play tag *all* the time every day, majority rule will *not* lead to the compromise of "taking turns," even if such a compromise is available. (We will reach this conclusion more formally in section V.2.)

We might note that even a fanatical tag-playing majority might choose to compromise with the minority because (we may suppose) the whole group of children share a common desire to play *something* together, and the disgruntled minority of two might (or might threaten to) secede from the group if they were not accommodated in some degree. The bargaining power of minorities to withdraw cooperation or even secede is indeed a check on the power of majorities, but it obviously is not a check that derives from the nature of majority rule itself.

28. Stated otherwise, distinct "intense minorities" can "trade votes" or "logroll" to bring about an outcome that includes the minority alternative on the issue each cares about the most. The term "coalition of minorities" is due (I believe) to Anthony Downs (*An Economic Theory of Democracy*, 55–62). Robert Dahl's discussion of "minorities rule" in *A Preface to Democratic Theory* (Chicago: University of Chicago Press, 1956), 128–29, focuses on the same phenomenon.

29. The fraction in each category need not be the same in the two mixed clusters; all that matters in determining the effectiveness of a coalition of minorities is the overall fraction.

30. In "Pluralism and Social Choice," I argued that crosscutting preferences made effective coalitions of minorities more likely and reinforcing preferences made them less likely. An analysis by Tse-min Lin ("The Paradox of Voting and the Equilibrium Cycle," paper presented to the 1991 Annual Meeting of the American Political Science Association, Washington, D.C., August 29–September 1, 1991) shows that this proposition needs to be refined. The original distinction in sociological literature between "reinforcing" and "crosscutting" patterns of cleavage (to which "Pluralism and Social Choice" referred) concerned cleavages (for example, religion, language, region, and so forth) that had no common polarity such as "low" and "high." In this context, one could talk only about pairs of cleavages being either (in some degree) reinforcing or crosscutting—no distinction between positive and negative reinforcement was appropriate. However, when we extend the notion of crosscutting versus reinforcement to preferences on dichotomous issues, in conjunction with a focus on the operation of majority rule that leads us to label issue alternatives as "majority"

versus "minority," a common polarity is introduced, so it becomes possible and natural to distinguish between positive versus negative reinforcement, as I have done explicitly in this essay. While it was not explicit, the discussion of "reinforcing preferences" in "Pluralism and Social Choice" referred to *positively* reinforcing preferences only—for example, "reinforcing divisions of a population into majority and minority groups" (740). It is clear that the possibility of an effective coalition of minorities increases (i.e., the required fraction with minority intensity decreases) monotonically as the degree of positive reinforcement decreases towards crosscuttingness (popularity remaining constant). But the required fraction with minority intensity is not at its minimum when preferences are perfectly crosscutting. It continues to decrease (though only slightly) as preferences become negatively reinforcing, and the minimum is achieved at maximum negative reinforcement.

31. Downs, *An Economic Theory of Democracy*, 64–69, refers to the ineffectiveness of a coalition of minorities as *rule of the passionate majority*. The terminology may be slightly misleading. First, no interpersonal comparisons of preference intensity are relevant—the question is how *individual citizens* make issue tradeoffs within their own preference orderings. Secondly, how citizens within either the majority or minority clusters make such tradeoffs is also irrelevant—the question is only how citizens within the mixed clusters make such tradeoffs. Finally, it is not necessary for all citizens in the mixed cluster to prefer to get their way on the issue on which they prefer the majority alternative in order to produce "rule of a passionate majority"; it is always sufficient that at least one-half of such voters have such preferences (we may refer to this condition as a "generalized passionate majority"), but even this is almost never necessary. How great this proportion must be depends on the size of the several clusters.

32. An early result in formal political theory, provided independently and more or less simultaneously by many different researchers, was this demonstration that an effective coalition of minorities entails cyclical social preference. For a general review, see Nicholas R. Miller, "Logrolling, Vote Trading, and the Paradox of Voting: A Game-Theoretical Overview," *Public Choice*, 30 (1977), 51–75.

33. Note that preferences cannot be systematically and negatively reinforcing, since if issues 1 and 2 are negatively reinforcing and issues 2 and 3 are negatively reinforcing, issues 1 and 3 are positively reinforcing.

34. This follows because the median has the property that the average absolute deviation from the median is less than the average

absolute deviation from any other point. See Herbert Weisberg, *Central Tendency and Variability* (Newbury Park, CA: Sage Publications, 1992), 25–26. Start at the median ideal point  $x^m$  and move any distance  $D$  toward the left. You are *increasing* the dissatisfaction of the median citizen and every citizen to the his right by the same amount  $D$  for each of these  $(n+1)/2$  citizens. *At best* you are *reducing* dissatisfaction of the left-of-center citizens by the same amount  $D$  for each of these  $(n-1)/2$  citizens. (If you move beyond the ideal point of any left-of-center citizen, you reduce his dissatisfaction by less than  $D$  or even increase it.) So the overall effect of the move is to increase total dissatisfaction by at least  $D$  and average dissatisfaction by at least  $D/n$ . Obviously the same argument holds for any movement to the right.

35. John Rawls, *A Theory of Justice* (Cambridge: Harvard University Press, 1971).

36. Weisberg, *Central Tendency and Variability*, pp. 28–29. We might alternatively suppose that dissatisfaction increases, not linearly with distance from one's ideal point, but with the square of that distance. (Such "quadratic loss functions" are sometimes used in formal political theory.) It then follows that collective choice as the mean, rather than median, minimizes average dissatisfaction.

37. A "split-the-difference" arbitration rule (i.e., the midrange rule) is widely recognized to encourage such behavior in bilateral choice.

38. Given that the majority and minority distributions are normal with standard deviations of  $D^+$  and  $D^-$  respectively, we may deem the critical threshold of polarization to be about  $2.5(D^+ + D^-)$ . (In a normal distribution, fewer than 1 percent of the points lie more than 2.5 standard deviations in either direction from the mean.)

39. We are characterizing both intervals from the minority point of view. From the majority point of view,  $\Delta x^-$  may be characterized as the "majority impact" and  $\Delta x^+$  as "collective majority dissatisfaction." Note that we refer to  $\Delta x^-$  (or  $\Delta x^+$ ) as *collective* minority (or majority) dissatisfaction, since some individual citizens (though necessarily a minority) within the minority (or majority) group may actually prefer  $x^*$  to  $x^-$  (or  $x^+$ ). (If the minority group is highly distinctive, however, this minority within the minority must be vanishingly small; the corresponding minority within the majority would usually be relatively substantial. See figure 4.)

40. Then the total area under the curves on the  $x^+$  side of  $x^*$  is equal to  $(1-M)/2 + M/2$ , and the area on the  $x^-$  side is  $[(1-M)/2 - M/2] + M$ , and elementary manipulation shows that these magnitudes are equal. If polarization falls much below the critical threshold, a significant portion of the minority distribution lies on the  $x^+$  side  $x^*$ , so

the area under the majority curve between  $x^1$  and  $x^*$  can be less than  $M/2$ .

41. The case of the fanatical tag-playing majority discussed in footnote 27 is an example of a fully cohesive majority that precludes minority impact.

42. The term "white primary" comes to mind; see V. O. Key, Jr., *Southern Politics* (New York: Alfred A. Knopf, 1949), chapter 29.

43. As the previous footnote suggests, one can think of reasons that might well arise in real social situations to promote such collusion. It follows from the previous discussion that there is no incentive for minority collusion if polarization is at or beyond the critical threshold. There is an incentive for minority collusion if polarization falls below the critical threshold, but in that case minority collusion presents the same dilemma as majority collusion invariably does.

44. See Charles R. Beitz, "Equal Opportunity in Political Representation," in *Equal Opportunity*, edited by Norman E. Bowie (Boulder, Colo.: Westview Press, 1988).

45. See footnote 24.

46. Such stigmatic criteria may account for the cleavages in cleavage politics or for a distinctive minority in ideological politics, of course.

47. *The Theory of Games and Economic Behavior* (Princeton: Princeton University Press, 1944).

48. Note that such collusion is far more likely to succeed than that discussed in the previous section, because this is not collusion that attempts to displace a Condorcet winner.

49. Barry R. Weingast, "A Rational Choice Perspective on Congressional Norms," *American Journal of Political Science*, 23 (1979): 245-62; Morris P. Fiorina, "Universalism, Reciprocity, and Distributive Policy Making in Majority Rule Institutions," in *Research in Public Policy Analysis and Management*, edited by J. P. Crecine (Greenwich: JAI Press, 1981); Gary J. Miller and Joe A. Oppenheimer, "Universalism in Experimental Committees," *American Political Science Review*, 76 (1982): 561-74; Barry Weingast and William Marshall, "The Industrial Organization of Congress," *Journal of Political Economy*, 96 (1987): 132-63.

## 10

## DELIBERATIVE EQUALITY AND DEMOCRATIC ORDER

THOMAS CHRISTIANO

Deliberation has acquired a privileged status in recent work in democratic theory. Many have claimed that democracy without deliberation is an unstable system wherein the desires of citizens clash without regard to the common good and even without concern for the reasonableness of the desires themselves. Some, inspired by the ideas of Jürgen Habermas and John Rawls, have argued that the process of social discussion among equals is itself intrinsically valuable and ought to be thought of as the preeminent value underlying democratic institutions.<sup>1</sup> Others, such as this author, give deliberation a more instrumental role in enhancing citizens' understanding of their and others' interests and in encouraging individuals to abandon the pursuit of naked self-interest. Moreover, equality in this process of discussion is itself essential to democratic equality and thus is required by the principles of justice underlying democracy.

Contemporary theorists are certainly right to point to the importance of open discussion among equals in a democratic society. But placing deliberation among equals at the heart of democratic values entails important theoretical and perhaps practical costs. Though a principle of equality in the process of deliberation can be defined, the principle is such that in a pluralistic democratic society it is practically impossible for citizens