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THE COMPLETE STRUCTURE OF MAJORITY RULE ON DISTRIBUTIVE POLITICS

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In an important class of social interactions, agents may undertake certain actions or not, the benefits of which are concentrated on a single agent (or group) while their costs are spread over many or all agents (or groups). Actions that cause pollution of one sort or other provide a salient example.

Let us formalize this in an admittedly oversimplified fashion. Each agent i of n agents can either undertake an action ($x_i = 1$) or not ($x_i = 0$). If $x_i=1$, benefits of magnitude B are received by i alone and costs of total magnitude C are equally shared by all n agents. In "the state of nature," or under "laissez-faire," each agent is empowered to undertake actions unlaterally; he compares his private benefits B with his private costs C/n resulting from the prospective action and chooses $x_i=1$ if $B>C/n$ and $x_i=0$ if $B<C/n$. (If $B=C/n$, he may or may not undertake the action.) In any case, the individual agent ignores the external costs of his action, which are borne by the other n-1 agents. Accordingly, he will act even if total social costs (equaling private costs plus external costs) fall below total social benefits (equaling private benefits alone), i.e. even if $B<C$. And there is symmetry among all agents, who will decide in like fashion. Thus if $C/n \leq B<C$, we have an n-Prisoners' Dilemma, since for each agent i $x_i=1$ is a dominant strategy but the net payoff to each agent from the resulting outcome is $B-C<0$, while if each had chosen his dominated strategy $x_i=0$ the payoff to each would be 0. (Among many possible references, see in particular Hardin, 1971; Schelling, 1974; and Orbell & Wilson, 1978.) In the "state of nature"/"laissez-faire" context, the situation usually presented is the "dual" of that just described, where each action entails widely shared benefits and concentrated costs, i.e., the problem of public goods provision. The two versions are essentially equivalent in their characteristics, and all of the subsequent discussion of the concentrated benefits/dispersed costs case applies just as well to the dispersed benefits/concentrated costs case with suitable interchange of notation and concepts. The former version is presented here because we are mainly concerned with the problem under collective - specifically majority rule - decision making, not laissez-faire, and in the former context the phenomenon of distributive politics, i.e. the politics of concentrated benefits and dispersed costs, is especially prevalent.) Of course, if $B<C/n$, all will abstain and the resulting outcome is socially optimal; and if $B>C$, all will act and the resulting outcome is socially optimal.

In "the state of nature," therefore, if $C/n \leq B < C$, there is an incentive for agents to enter into a "social contract" collectivizing - on the basis of majority rule or otherwise - the decision on what actions to undertake. The rationale for collectivization is apparent; the "laissez-faire" outcome is Pareto-inefficient and can be improved on by collective decision making to everyone's benefit. But to say that collectivization (or "government intervention") <u>can</u> improve things is not to say that collectivization <u>will</u> improve things. Making this clear has been the great substantive contribution of public choice theory. Before we can say that collectivization will (or will not) improve things, we must understand how collective decision making, political institutions, and the political process work. The purpose of paper is to examine closely how one particular institution - simple majority rule - works in the context of concentrated benefits and dispersed costs. We study the application of a highly simplified institution to a highly simplified politics, beleiving that "the sound procedure is to obtain first utmost precision and mastery is a limited field, and then to proceed to another, somewhat wider one, and so on" (Von Neumann and Morgenstern, 1953, p. 7).

Collective decision making in the type of interaction described above results in what political scientists have come generally to refer to as a distributive politics, i.e. the politics of concentrated benefits and dispersed costs, and more specifically as pork-barrel politics in the special case in which B<C.

In this work, I am following the lead most especially of Orbell and Wilson (1978), who examine the operation of several types of political institutions - majority rule included - in making collective decisions about distributive politics. A number of other papers and articles, in addition to Orbell and Wilson, have examined characteristics of majority rule on distributive politics, the dominant conclusion being that it likely to be chaotic, i.e. that cycling is typically pervasive. (These additional papers and articles include Hardin, 1971; Fiorina, 1979; Weingast, 1979; Shepsle and Weingast, 1980; Shepsle and Weingast, 1981a; Weingast, Shepsle, and Johnsen 1981.) But, so far as I can tell, the complete structure of majority rule on distributive politics has not yet been fully analyzed. It is the essentially technical purpose of this paper to complete such analysis, including consideration of a new solution set - the "uncovered set" -- in this majority rule structure, as this set appears to be a significant solution set in majority voting games (Miller, 1980; Shepsle and Weingast, 1981b; McKelvey, 1981).

Finally it should be noted that much of the literature cited just above is concerned with explaining the empirical prevalence of "universalism" (i.e., giving something to everyone) since universalism is by no means dictated, or even supported, by the basic structure of majority rule. The present analysis only reinforces this seeming anomaly.

We introduce the following notation and assumptions.

Let x, y, etc. designate elements in the set V of all logically possible outcomes.

Let 1, 2,...,i,...,n designate elements of the set N of all agents (e.g., legislators and their districts), each with a corresponding proposal (e.g., proposed project) that provides $B_i > 0$ in benefits received entirely by i at a cost $C_i > 0$ shared equally by all members of N. For the most part we are concerned with the symmetric case in which $B_i=B_j$ and $C_i=C_j$ for all $i,j \in N$ and so can drop the subscripts. We assume that $n \geq 3$ and is an odd number.

An outcome $x \in V$ is a package of proposals, i.e., a vector $x = \langle x_1, x_2, \ldots, x_n \rangle$ where $x_i=1$ if the package provides for the i-th project and $x_i=0$ otherwise.

Let $S_x = \{i: x_i=1\}$, i.e., the set of voters provided projects at package x. Let $V^k \subset V$ designate the set of all packages providing for exactly k projects, i.e., $x \in V^k$ if and only if $|S_x|=k$. Note that $V^0 = \{x^0\}$ (the <u>null</u> package) and $V^k = \{x^n\}$ (the <u>universal</u> package) are one-element sets; each V^k for $1 \leq k \leq n-1$ is a multi-element set.

If $x \in V^k$, $y \in V^h$, and $h < k$, x is more <u>expansive</u> than y. If $S_y \subset S_x$, x is more <u>inclusive</u> than y. If $k \geq \frac{n+1}{2}$, x is a <u>majority</u> package. If $k \leq \frac{n-1}{2}$, x is a <u>minority</u> package. Packages x and y are <u>complementary</u> if $S_x = N - S_y = \bar{S}_y$.

The utility of a package to voter i is its benefits net of costs. Thus in general:

$$u_i(x) = \begin{cases} B_i - \sum_{j \in S_x} \frac{c_j}{n} & \text{if } x_i = 1 \\ -\sum_{j \in S_x} \frac{c_j}{n} & \text{if } x_i = 0. \end{cases}$$

In the symmetric case, where $x \in V^k$:

$$u_i(x) = \begin{cases} B - \frac{kC}{n} & \text{if } x_i = 1 \\ -\frac{kC}{n} & \text{if } x_i = 0. \end{cases}$$

In either case, $x R_i y$ if and only if $u_i(x) \geq u_i(y)$. In the conventional manner, let P_i and I_i designate the asymmetric and symmetric components of the individual preference relation

R_i. Also in the conventional manner, we define the (relative) majority rule relation R: $x R y$ if and only if $|\{i: x P_i y\}| \geq |\{j: y P_i x\}|$, and let P and I designate the asymmetric and symmetric components of R.

We now consider, in the symmetric case, the utilities and preferences of voters for two arbitrary non-null packages $x \in V^k$ and $y \in V^h$ where $1 \leq h \leq k$. Voters can be classified according to whether they belong to S_x or $\bar{S}_x$ and likewise to S_y or $\bar{S}_y$. We examine the resulting cross-classification. First we introduce the following notation enumerating the categories:

	S_x	$\bar{S}_x$	Total
S_y	n_{xy}	$n_{\bar{x}y}$	$n_y = h$
$\bar{S}_y$	$n_{x\bar{y}}$	$n_{\bar{x}\bar{y}}$	$n_{\bar{y}} = n-h$
Total	$n_x = k$	$n_{\bar{x}} = n-k$	n

Next we show the utilities and preferences of the voters in each category in the following <u>Preference Table:</u>

$1 \leq h \leq k$	S_x	$\bar{S}_x$
S_y	$u_i(x) = B - \frac{k}{n}C$ $u_i(y) = B - \frac{h}{n}C$ Thus $y R_i x$, specifically $y I_i x$ iff $h = k$ $y P_i x$ otherwise $(h < k)$	$u_i(x) = -\frac{k}{n}C$ $u_i(y) = B - \frac{h}{n}C$ Thus $y P_i x$
$\bar{S}_y$	$u_i(x) = B - \frac{k}{n}C$ $u_i(y) = -\frac{h}{n}C$ Thus $x P_i y$ iff $B > \frac{k-h}{n}C$ $x I_i y$ iff $B = \frac{k-h}{n}C$ $y P_i x$ iff $B < \frac{k-h}{n}C$	$u_i(x) = -\frac{k}{n}C$ $u_i(y) = -\frac{h}{n}C$ Thus $y R_i x$, specifically $y I_i x$ iff $h = k$ $y P_i x$ otherwise $(h < k)$

The following points are immediate, and hold also for $h = 0$.

LEMMA 1. For any $x \in V^k$ and $y \in V^h$ with $h \leqslant k$

(a) $x\ P_i\ y$ if and only if (i) $i \in S_x \cap \bar{S}_y$ and (ii) $B > \frac{k-h}{n}C$;

(b) $x\ I_i\ y$ if and only if (i) $i \in (S_x \cap S_y) \cup (\bar{S}_x \cap \bar{S}_y)$ and $h = k$ or (ii) $i \in S_x \cap \bar{S}_y$ and $B = \frac{k-h}{n}C$.

From the above we can deduce the following:

LEMMA 2. For any $x \in V^k$ and $y \in V^h$ with $h \leqslant k$

(a) $x\ P\ y$ if and only if (i) $n_{x\bar{y}} \geqslant \frac{n+1}{2}$ and (ii) $B > \frac{k-h}{n}C$;

(b) $x\ I\ y$ if and only if (i) $h = k$ or (ii) $y = x^0$, $x = x^n$ and $B = C$;

(c) $y\ P\ x$ otherwise.

(a) Forward Implication. By definition, $x\ P\ y$ only if there is some i for whom $x\ P_i\ y$. By Lemma 1(a), $i \in S_x \cap \bar{S}_y$ and $B > \frac{k-h}{n}C$. Always $B > 0$; thus $h < k$. Thus by Lemma 1(b) there is no j for whom $x\ I_j\ y$. Thus we must have $\{i: x\ P_i\ y\} = n_{x\bar{y}} \geqslant \frac{n+1}{2}$.

(a) Reverse Implication. Immediate from the Preference Table.

(b) Forward Implication. Suppose $x\ I\ y$, i.e., $|\{i:x\ P_i\ y\}| = |\{j:y\ P_j\ x\}| = n^*$. Necessarily $0 \leqslant n^* \leqslant \frac{n-1}{2}$. We consider two cases separately: (1) $n^* \geqslant 1$ and (2) $n^* = 0$.

(1) Suppose $h < k$. By Lemma 1(a), $B > \frac{k-h}{n}C$ and $\{i:x\ P_i\ y\} = S_x \cap \bar{S}_y$. Thus $n^* = n_{x\bar{y}}$. And by Lemma 1, $\{j:y\ P_j\ x\} = N - S_x \cap \bar{S}_y$. Thus $n^* = n_{\bar{x}y} = n_{xy} + n_{\bar{x}y} + n_{\bar{x}\bar{y}}$. But then $n^* = n/2$, contradicting both the assumption that n is odd and the requirement that $n^* \leqslant \frac{n-1}{2}$. Thus $h = k$.

(2) Suppose $h < k$. Then $S_x \cap \bar{S}_y = N$ and $n_{x\bar{y}} = k = n - h = n$ and $h = 0$, so either $y = x^0$ and $x = x^n$ or $h = k$.

(b) Reverse Implication. Suppose $h = k$. Then for all

$i \in (S_x \cap S_y) \cup (\bar{S}_x \cap \bar{S}_y)$, $x\ I_i\ y$. Since always $B > 0$, for all $i \in S_x \cap \bar{S}_y$, $x\ P_i\ y$. For all $i \in \bar{S}_x \cap S_y$, always $y\ P_i\ x$. And always $n_{x\bar{y}} + n_{\bar{x}\bar{y}} = n - h$ and $n_{\bar{x}y} + n_{\bar{x}\bar{y}} = n-k$; so if $h=k$, $n_{x\bar{y}} = n_{\bar{x}y}$. Thus equal numbers of voters prefer x to y and vice versa, and all others are indifferent. Thus x I y. Finally suppose that $x = x^n$ and $y = x^0$ and $B = C$. Then it is immediate that $u_i(x) = u_i(y)$ for all i, so x I y.

LEMMA 3. For any $x \in V^k$ and $y \in V^h$ with $h \leqslant k$, x P y only if $h < n/2 < k$.

In words, a more expansive package dominates a less expansive package only if the former is a majority package and the latter is a minority package.

Suppose x P y. By Lemma 2(a), $n_{x\bar{y}} \geqslant \frac{n+1}{2}$. Always $k \geqslant n_{x\bar{y}}$, so $k \geqslant \frac{n+1}{2}$. Likewise always $n-h \geqslant n_{x\bar{y}}$, so $n-h \geqslant \frac{n+1}{2}$ and $h \leqslant \frac{n-1}{2}$.

It is useful also to state the complement of Lemma 3.

LEMMA 3'. For any $x \in V^k$ and $y \in V^h$ with $h < k \leqslant \frac{n-1}{2}$ or $\frac{n+1}{2} \leqslant h < k$, y P x.

In words, comparing either two minority packages or two majority packages, the less expansive package always dominates the other. The substantive corollary here is that successful distributive legislation always involves "omnibus" bills and indeed majority packages.

The following refines Lemma 3:

LEMMA 4. For any V^h and V^k with $h < n/2 < k$, there is some $y \in V^h$ and $x \in V^k$ such that x P y if and only if $B > \frac{k-h}{n}C$.

This follows directly from Lemma 2(a) if we let $S_x \subseteq \bar{S}_y$ in the event $h+k \leqslant n$ and let $\bar{S}_y \subseteq S_x$ in the event $h+k \geqslant n$. In the first event, $S_x \cap S_y = \emptyset$, so $n_{xy} = 0$, so $n_{x\bar{y}} = k \geqslant \frac{n+1}{2}$. In the second event, $\bar{S}_x \cap \bar{S}_y = \emptyset$, so $n_{\bar{x}\bar{y}} = 0$, so $n_{x\bar{y}} = n-h \geqslant \frac{n+1}{2}$.

LEMMA 5. For any $x \in V^k$ and $y \in V^h$ such that $S_y \subset S_x$ and $1 \leq k-h \leq \frac{n-1}{2}$, y P x.

In words, comparing a less inclusive package with a more inclusive package (which may be a minority and majority package respectively), the former always dominates the latter provided the number of additional projects provided by the latter is less than $\frac{n+1}{2}$. (Cf. Orbell and Wilson, 1978, p. 417; but also note their footnote on the same page.)

Since $S_y \subset S_x$, $S_y \cap \bar{S}_x = \emptyset$, i.e., $n_{\bar{x}y} = 0$. Thus $n_{xy} = 0$ and $n_{x\bar{y}} = k-h$. By supposition, $k-h \leq \frac{n-1}{2}$, so $n_{x\bar{y}} \leq \frac{n-1}{2}$, so by Lemma 2, y P x.

The following further refines Lemma 3:

LEMMA 6. If and only if $k-h \geq \frac{n+1}{2}$ and $B > \frac{k-h}{n}C$, then for every $y \in V^h$ and every $x \in V^k$, x P y.

Suppose that $k-h \geq \frac{n+1}{2}$. This can be restated as $(n_{xy} + n_{x\bar{y}}) - (n_{xy} + n_{\bar{x}y}) \geq \frac{n+1}{2}$, so $n_{x\bar{y}} - n_{\bar{x}y} \geq \frac{n+1}{2}$. Since $B > \frac{k-h}{n}C$, x P y by Lemma 2(a). Conversely, suppose that $k-h < \frac{n+1}{2}$. Then by Lemma 5, there is some $y \in V^h$ and $x \in V^k$ (with $S_y \subset S_x$) such that y P x.

Let $F(x) = \{y: y \text{ P } x\}$, i.e., the set of outcomes that dominate x. Now if we let $h = k-1$, Lemma 5 immediately supports the following fundamental theorem on distributive politics:

THEOREM 1. For every $x \in V^k$ such that $k \geq 1$, $F(x) = \emptyset$.

In words, every non-null or "omnibus" package is dominated by some other package. Thus letting V^{***} designate the <u>core</u> of the majority voting game, we have:

COROLLARY 1.1. $V^{***} \subseteq V^0$.

In words, the core of distributive politics is either empty or includes only the null package x^0--put otherwise x^0

is the Condorcet proposal if there is one. Note also that if x^0 is undominated, then also, by Lemma 2(b), x^0 P y for all $y \in V - \{x^0, x^n\}$ (also x^0 P x^n by the conjunction of Lemma 2(b) and Corollary 3.1 below).

Since distributive politics involves n (binary) issues on which (despite external effects on the cost side) preferences are separable, Theorem 1 and its corollary are themselves corollaries of the well-known theorem of Kadane (1972).

Transitivity of majority rule is sufficient for the existence of an undominated or Condorcet proposal, but it is not necessary. Even if x^0 is undominated, majority rule on distributive politics may fail to be fully transitive. Instead, we have the following:

THEOREM 2. Majority rule on distributive politics is transitive if and only if $B \leqslant C/n$.

Note that this is the same condition necessary (and sufficient if $B < C/n$) to avoid the n-Prisoners' Dilemma under laissez-faire.

<u>Sufficiency</u>. Suppose that $B \leqslant C/n$ and, for distinct x, y, z, x R y and y R z. We must show that x R z.

(a) If x I y and y I z, then,by Lemma 2(b), $x, y, z \in V^k$ for some $1 \leqslant k \leqslant n-1$ and x I z.

(b) If x I y and y P z, then, by Lemma 2(b), $x, y \in V^k$ for some $1 \leqslant k \leqslant n-1$ and $z \in V^h$ where $h \neq k$. Suppose $h < k$; then $k-h \geqslant 1$. Since $B \leqslant C/n$, also $B \leqslant \frac{k-h}{n}C$. But then by Lemma 2, z P y. So it must be that $k < h$. Then by Lemma 2, x P z.

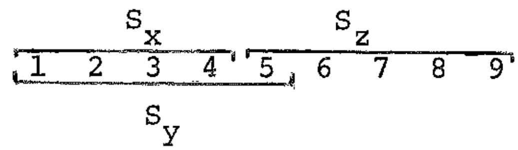
(c) If x P y and y I z, then by reasoning parallel to that in (b) above, x P z.

(d) If x P y and y P z, then by Lemma 2(b) $x \in V^k$, $y \in V^h$, and $z \in V^g$ where $k \neq h$, $k \neq g$, and $h \neq g$. Again by reasoning parallel to that in (b), we must have $g < h < k$, and so x P z.

So in any case, x R z.

<u>Necessity</u>. We show that if $B > C/n$, R is intransitive for some triple of packages.

Consider a triple of packages $x \in V^h$ and $y,z \in V^k$ where $h = \frac{n-1}{2}$ and $k = \frac{n+1}{2}$ and x and z are complementary, i.e., $S_x = \overline{S}_y$, and $S_x \subset S_y$. (Cf. the diagram for an example with n = 9.)



By Lemma 5, x P y. By Lemma 2(b), y I z. Since k-h = 1 and by supposition $B > C/n$, then in the manner of the proof of Lemma 4, z P x. Thus we have x R y and y R z but not x R z.

We may note that in the example constructed just above, transitivity (and quasitransitivity) is violated, but not acyclicity. Indeed, we can generate cyclic triples (and, it appears but I do not prove, longer cycles), if and only if $B > 2C/n$.

First, we state and prove the following:

LEMMA 7. Suppose $x \in V^k$, $y \in V^h$, and $z \in V^g$ and $g < h < k$. Then we cannot have both x R y and y R z.

Since $g < h < k$, $n \neq h$ and $h \neq 0$. Thus x R y implies x P y (Lemma 2(b)). In turn Lemma 2(a) implies $n_{x\bar{y}} \geq \frac{n+1}{2}$; thus $n_{\bar{y}} = n-h \geq \frac{n+1}{2}$.

Also since $g < h < k$, $h \neq g$ and $h \neq n$. Thus y R z implies y P z; thus $n_{y\bar{z}} \geq \frac{n+1}{2}$; and thus $n_y = h \geq \frac{n+1}{2}$, contradicting the implication in the previous paragraph.

THEOREM 3. Majority rule on distributive politics is triple acyclic if and only if $B \leq 2C/n$.

Sufficiency. Suppose that $B \leq 2C/n$ and x P y and y P z, where $x \in V^k$, $y \in V^h$, and $z \in V^g$. We must show that x R y.

By Lemma 2, $k \neq h$ and $h \neq g$. We consider the four possible cases in turn:

	$h < k$	$k < h$
$g < h$	(1)	(2)
$h < g$	(3)	(4)

(1) In this case, $g < h < k$. By Lemma 7, we cannot have both x P y and y P z. But by supposition we do. So this case

cannot arise.

(2) Suppose $k < g < h$. Since y P z, by Lemma 7, we must have x P z. Suppose $g < k < h$. Thus $h-g \geqslant 2$. Since y P z, by Lemma 1(b), $B > \frac{h-g}{2}C$. But $h-g \geqslant 2$. Thus $B > 2C/n$, contradicting the supposition of the theorem. Thus this case cannot arise. Finally suppose that $k = g$. Then by Lemma 1(b), x I z.

(3) By reasoning parallel to (2) above, we conclude that in any case that can arise x R z.

(4) In this case, $k < h < g$; thus $g - k \geqslant 2$. By Lemma 2, x P z.

Thus if $B \leqslant 2C/n$ and x P y and y P z, we must have x R z.

<u>Necessity</u>. Suppose x P y, y P z, and z P x for some $x \in V^k$, $y \in V^h$, and $z \in V^g$. By Lemma 2(b), we cannot have $k = h$, $k = g$, or $h = g$. By Lemma 7, we cannot have $g < h < k$, $h < k < g$, or $k < g < h$. Thus we must have (i) $g < k < h$ and $h-g \geqslant 2$, (ii) $h < g < k$ and $k-h \geqslant 2$, or (iii) $k < g < h$ and $h-k \geqslant 2$. By Lemma 2(a), if (i), since y P z and $h-g \geqslant 2$, we have $B > 2C/n$; if (ii), since x P y and $k-h \geqslant 2$, we have $B > 2C/n$; if (iii), since z P x and $h-k \geqslant 2$, we have $B > 2C/n$. So in any case, $B > 2C/n$.

Neither transitivity nor even acyclicity is required, of course, for "consistent" or "stable" majority rule on a fixed set of alternatives, i.e., for $V^{***} \neq \emptyset$ or for the existence of a Condorcet proposal. And indeed the condition for distributive politics to have non-empty core or a Condorcet winner is considerably weaker than the condition, identified in the previous theorems, for full transitivity or acyclicity. We have already seen (Corollary 1.1) that, if there is a Condorcet winner, it is x^0. Thus the condition for a Condorcet winner is simply the condition for $F(x^0) = \emptyset$. And this condition is a corollary of the following theorem, which is itself an immediate consequence of Lemma 2 in the special case in which $h = 0$.

THEOREM 4. For any $x \in V^k$, x R x^0 if and only if

(i) $\frac{n+1}{2} \leqslant k$; and

(ii) $B \geqslant kC/n$.

More specifically x P x^0 if and only if

(iii) $\frac{n+1}{2} \leqslant k$; and

(iv) $B > kC/n$.

And $x \; I \; x^0$ if and only if

(v) $k = n$; and

(vi) $B = C$.

COROLLARY 4.1 (Orbell and Wilson). $F(x^0) = \emptyset$ if and only if $B \leqslant \frac{n+1}{2n}C$.

Cf. Orbell and Wilson (1978, p. 418), who consider in effect a continuum of voters; thus the condition becomes $B < C/2$.

Hardin (1971, p. 477) calls an outcome realizable if it gives every individual at least the (maximin) utility he could guarantee himself in the "state of nature," i.e., $u_i(x) \geqslant B - C$, the minimum payoff i could receive by choosing $x_i = 1$. Thus $x \in V^k$ is realizable if and only if for all i

$$u_i(x) = \begin{Bmatrix} B - kC/n & \text{iff } x_i = 1 \\ -kC/n & \text{iff } x_i = 0 \end{Bmatrix} \geqslant B - C.$$

This implies that x is realizable if and only if (i) $x = x^n$ or (ii) $-kC/n \geqslant B - C$, which implies $B \quad \frac{n-k}{n}C$.

COROLLARY 4.2 (Hardin). If $B < C$, x^0 dominates every realizable outcome.

Note that $B < C$ is required for an n-Prisoners' Dilemma to exist in the "state of nature."

Suppose that $x \; R \; x^0$ for some $x \in V^k$ with $k > 0$. Then by the theorem $B \geqslant kC/n \geqslant \frac{n+1}{2}C > \frac{n-1}{2}C \geqslant \frac{n-k}{n}C$, so $B > \frac{n-k}{n}$. Thus $x \in V^k$ is not realizable unless $k = n$. But given $B < C$, $x^0 \; P \; x^n$. So x^0 dominates every realizable outcome.

Hardin's theorem can, therefore, be readily demonstrated, given Theorem 4. Its theoretical significance is not so clear, however. An agent can indeed guarantee himself B-C in the "state of nature" and any outcome providing an "individually rational" (cf. Luce and Raiffa, 1957, p. 193) agent with less is indeed "unrealizable." But, in the absence of special constitutional restrictions, full "collectivization" under majority rule reduces each agent's security level to the logical minimum of $-\frac{n-1}{n}C$ (this general point is fundamental in the theory of Buchanan and Tullock, 1962), thus rendering every outcome realizable. Thus Hardin's theorem seems to have no theoretical import in the absence of specific constitutional limitations on the power of majorities.

Let us define $P(x) = \{y: x \; P \; \ldots \; P \; y\}$, i.e., $P(x)$ is the set of outcomes reachable from x by a P-path of majority domination. Then we have:

THEOREM 5. For every $x,y \in V$, $x \in P(y)$ and $y \in P(x)$ if and only if $F(x^0) \neq \emptyset$.

That is, every pair of packages is mutually reachable by a P-path if and only if the core of distributive politics is empty.

Sufficiency. Suppose $F(x^0) \neq \emptyset$; then by Corollary 3.1 $B > \frac{n+1}{2}C$. Consider any arbitrary $x \in V^k$ and $y \in V^h$. We must show that $x \in P(y)$. We consider the four possible cases: (1) $h,k \leq \frac{n-1}{2}$; (2) $h,k \geq \frac{n+1}{2}$; (3) $k < n/2 < h$; and (4) $h < n/2 < k$.

(1) Pick $z \in V^g$ such that $g = \frac{n+1}{2}$ and $S_x \subset S_z$. Then x P z (Lemma 5), $z \; P \; x^0$ (Theorem 4), and $x^0 \; P \; y$ (Lemmas 3 and 2(b)).

(2) Pick $z \in V^g$ such that $g = k-\frac{n+1}{2}$. Then x P z (Lemma 6). Pick $w \in V^h$ and $u \in V^j$ such that $h = \frac{n-1}{2}$, $j = \frac{n+1}{2}$ and $S_z \subseteq S_w \subset S_u \subseteq S_y$. Then z P w P y (Lemma 5).

(3) Pick $z \in V^g$ and $w \in V^h$ such that $g = \frac{n-1}{2}$, $h = \frac{n+1}{2}$, and $S_z \subset S_w \subseteq S_y$. Then x P z P w P y (Lemma 5).

(4) Pick any $z \in V^g$ where $g = k-\frac{n+1}{2}$. Then x P z (Lemma 5). Pick any $w \in V^h$ where $h = \frac{n-1}{2}$. Then z P w (Lemma 3). Pick $u \in V^j$ such that $j = \frac{n+1}{2}$ and $S_w \subset S_u$. Then w P u (Lemma 5) and $u \; P \; x^0$ (Theorem 4) and $x^0 \; P \; y$ (Lemmas 3 and 2(b)).

Necessity. Suppose $F(x^0) = \emptyset$. Then $x^0 \; R \; x$ for all $x \in V - \{x^0\}$ (Lemma 2). So for any $x \in V^k$ where $k > 0$, $x^0 \notin P(x)$.

Designate as $V^* \subseteq V$ the minimal undominated set of outcomes, i.e., (i) for every $x \in V^*$ and $y \notin V^*$, x R y; and (ii) no proper subset of V^* meets (i) (cf. Miller, in progress, and Miller, 1977, p. 775). All outcomes in V^* are mutually reachable by a P-path, i.e., V^* includes a complete closed sequence. Thus, from Theorem 5, we get:

COROLLARY 5.1. In distributive politics

(a) $V^* = V$ if and only if $B > \frac{n+1}{2}C$; and

(b) $V^* = V^{***} = \{x^0\}$ otherwise.

We are now in a position to characterize the complete structure of majority rule on distributive politics. We introduce this further notation:

$V^h \longrightarrow V^k$ iff for some $y \in V^h$ and $x \in V^k$, $y\ P\ x$;

$V^h \Longrightarrow V^k$ iff for every $y \in V^h$ and $x \in V^k$, $y\ P\ x$; and

$V^h \Longleftrightarrow V^k$ iff $V^h \longrightarrow V^k$ and $V^k \longrightarrow V^h$.

Observation I. For all $h < k \leq \frac{n-1}{2}$ and for all $\frac{n+1}{2} \leq h < k$, $V^h \Longrightarrow V^k$ (from Lemma 3').

Observation II. For all $1 \leq k-h \leq \frac{n-1}{2}$, $V^h \longrightarrow V^k$ (from Lemma 5).

Note that the two observations above hold regardless of the relative magnitudes of B, C, and n. The following two observations depend on these relative magnitudes.

Observation III. For all $h < n/2 < k$, $V^k \longrightarrow V^h$ if and only if $B > \frac{k-h}{n}C$ (from Lemma 4).

Observation IV. For all $h < k$, $V^k \Longrightarrow V^h$ if and only if $k-h \geq \frac{n+1}{2}$ and $B > \frac{k-h}{n}C$ (from Lemma 6).

Figure 1 shows all package sets, $V^0, V^1, \ldots, V^n$. Each pair V' and V" of sets may be related thus: $V' \Longrightarrow V''$, $V' \Longleftrightarrow V''$, or $V'' \Longrightarrow V'$. By Observations I and II we have at least those relationships shown in the figure. How the remaining arrows are filled in depends on the parameters B, C, and n or, more concisely, on the single parameter λ relating them in the expression $B = \lambda C/n$.

If $\lambda \leq 1$, i.e., $B \leq C/n$, the structure is fully transitive (from Observation III) with $V^0 = V^* = V^{***}$.

If $1 < \lambda \leq 2$, then $V^{\frac{n-1}{2}} \Longleftrightarrow V^{\frac{n+1}{2}}$ but otherwise $h < k$ implies $V^h \Longrightarrow V^k$ (also from Observation III) so intransitivities (but not cycles) occur among triples of outcomes in $V^{\frac{n-1}{2}} \cup V^{\frac{n+1}{2}}$ (cf. the proof of necessity for Theorem 2).

If $2 < \lambda \leq 3$, then in addition $V^{\frac{n-3}{2}} \Longleftrightarrow V^{\frac{n+1}{2}}$ and $V^{\frac{n-1}{2}} \Longleftrightarrow V^{\frac{n+3}{2}}$ but otherwise $h < k$ implies $V^h \Longrightarrow V^k$, so that cycles can occur among triples of outcomes in these package sets (cf. the proof of necessity for Theorem 3).

In general, if $m < \lambda \leqslant m+1$, then $V^h \Longleftrightarrow V^k$ for all $h < n/2 < k$ with $k-h \leqslant m$. Thus as B increases relative to C/n an "infestation" of cycles that originates in the vicinity of the dividing line between majority and minority packages spreads in the directions of both V^0 and V^n (but, until it reaches them, we still have $V^0 = V^* = V^{***}$).

Once $\frac{n+1}{2} < \lambda$, $V^{\frac{n+1}{2}} \Longrightarrow V^0$ and $V^n \Longrightarrow V^{\frac{n-1}{2}}$ (from Observation 4), so cycles now infest the entire structure (in the sense of Theorem 5) and $V^{***} = \emptyset$ and $V^* = V$.

Although cycles now infest the entire structure, intransitivity is not yet maximal and increases further as λ increases beyond $\frac{n+1}{2}$, as we get $V^{\frac{n+3}{2}} \Longrightarrow V^0$ and $V^n \Longrightarrow V^{\frac{n-3}{2}}$, then $V^{\frac{n+5}{2}} \Longrightarrow V^0$ and $V^n \Longrightarrow V^{\frac{n-5}{2}}$, and so forth until $\lambda = n$ and $x^n \; I \; x^0$ and finally $\lambda > n$ and $x^n \; P \; x^o$ so we get $V^n \Longrightarrow V^0$. At this point, intransitivity is maximal.

Increasing λ beyond $\frac{n+1}{2} + \epsilon$ does not change the size of V*, which jumps from V^0 to V once $\lambda > \frac{n+1}{2}$. But the increase in intransitivity resulting from further increases in λ does have the effect of expanding the size of the Pareto-optimal set of outcomes and also the size of the "uncovered set" of outcomes. To these matters we now turn.

Outcome x Pareto dominates y (x PD y) if and only if $x \; R_i \; y$ for all $i \in N$ and $x \; P_i \; y$ for some $i \in N$. Let P(V) designate the set of all Pareto-optimal outcomes, i.e., all outcomes not Pareto dominated.

LEMMA 8. Given any $x \in V^k$ where $1 \leqslant k \leqslant n-1$, x^0 PD x if and only if $B < kC/n$.

For all $i \in N$: $u_i(x^0) = 0$.

For all $i \in N$: $u_i(x) = \begin{cases} -kC/n < 0 & \text{iff } i \in \bar{S}_x \\ B - kC/n & \text{iff } i \in S_x. \end{cases}$

In any case, $u_i(x) < 0 = u_i(x^0)$ for all $i \in \bar{S}_x$. Since $k \leqslant n-1$,

$\bar{S} \neq \emptyset$ and there is some i such that $x^0 \; P_i \; x$.

Sufficiency. If $B \leq kC/n$, then $u_i(x) \leq 0 = u_i(x^0)$ for all $i \in S_x$. Thus $x^0 \; R_i \; x$ for all i and x^0 PD x.

Necessity. If $B > kC/n$, then $u_i(x) > 0 = u_i(x^0)$ for all $i \in S_x$. Since $k \geq 1$, $S_x \neq \emptyset$ and there is some i such that $x \; P_i \; x^0$. So x^0 does not Pareto dominate x.

LEMMA 9. x^n PD x if and only if $x = x^0$ and $B > C$.

The reverse implication is immediate. Now consider any $x \in V^k$ with $1 \leq k \leq n-1$.

$u_i(x^n) = B - C$ for all $i \in N$; and

$u_i(x) = B - kC/n < B-C$ for any $i \in S_x$.

Since $k \geq 1$, $S_x \neq \emptyset$ and there is some i such that $x \; P_i \; x^n$. So x^n does not Pareto dominate x.

LEMMA 10. $\{x^0, x^n\} \cap P(V) \neq \emptyset$

In words, either the null package or the universal package is Pareto-optimal. This is immediate from Lemmas 8 and 9.

LEMMA 11. For any $x \in V^k$ with $k > \frac{n+1}{2}$, x^0 P x if and only if x^0 PD x.

The reverse implication is immediate. Now suppose x^0 P x. Since S_x constitutes a majority, it must be that $u_i(x^0) \geq u_i(x)$ for all $i \in S_x$. Certainly $u_i(x^0) > u_i(x)$ for all $i \in \bar{S}_x$. So x^0 PD x.

LEMMA 12. For any $x, y \in V - \{x^0, x^n\}$, y PD x only if $y \in V^h$ and $x \in V^k$ with $h < k$.

Suppose h = k. Since x and y are distinct packages, $S_x \neq S_y$ and there is some $i \in S_x \cap \bar{S}_y$ and some $j \in \bar{S}_x \cap S_y$. From the Preference Table, $y \; P_j \; x$ always and $x \; P_i \; y$ in this case since $B > 0 = \frac{k-h}{n}C$. So neither x nor y Pareto dominates the other. So there can be no Pareto domination between two equally expansive packages (intuitively because this entails "zero-sum" redistribution). Can a more expansive package Pareto dominate a less expansive one? Examination of the Preference Table shows that when $S_x \neq S_y$ (i.e., $h \neq k$), more expansive x can Pareto dominate less expansive y only if $S_x \cap \bar{S}_y = V$, i.e., when $S_x = V$, which implies $x = x^n$ and $\bar{S}_y = V$, which implies $y = x^0$.

LEMMA 13. For any $x \neq x^0$, y PD x only if x^0 PD x.

Let $y \in V^h$ and $x \in V^k$. By Lemma 12, $h < k$. From the Preference Table, y PD x only if: (i) $S_x \cap \bar{S}_y = \emptyset$; or (ii) $B \leq \frac{k-h}{n}C$. But if (i), $S_x \subseteq S_y$ so $k \leq h$, which by Lemma 12 contradicts the supposition that y PD x. So we must have (ii). But then $B \leq \frac{k-h}{n}C < kC/n$ so by Lemma 8 x^0 PD y.

The following theorem in effect summarizes the preceeding results:

THEOREM 6. In distributive politics

(a) if $B = \lambda C/n$, $P(V) = \bigcup_{h=0}^{k} V^h$ where $k \leq n-1$ is the largest integer such that $k < \lambda$;

(b) if $B = C$, $P(V) = V$; and

(c) if $B > C$, $P(V) = V - \{x^0\}$.

(a) By Lemma 13 x is Pareto dominated only if x^0 PD x. By Lemma 8, x^0 PD x if and only if $B \leq kC/n$ where $x \in V^k$. Let $B = \lambda C/n$. Then x^0 PD x if and only if $\lambda \leq k$ and $x \in P(V)$ if and only if $k < \lambda$.

(b) If $B = C$, by Lemma 8 x^0 Pareto dominates no $x \in V^k$ such that $k < n$. Thus $V - \{x^0, x^n\} \subseteq P(V)$ by Lemma 13. Moreover, x^0 I x^n, so $P(V) = V$.

(c) If $B > C$, by the same reasoning as in (b) above, again $V - \{x^0, x^n\} \subseteq P(V)$. However, x^n PD x^0 by Lemma 9, so $P(V) = V - \{x^0\}$.

The concept of the"uncovered set"is introduced in Miller (1980; cf. McKelvey and Ordeshook, 1976). Let $D(x) = \{y\colon x \; P \; y\}$, i.e., the set of outcomes dominated by x. Where majority preference is strict (i.e., there are no "ties"), x <u>covers</u> y if and only if $D(y) \subset D(x)$ (which implies $F(x) \subset F(y)$). Let V** designate the set of <u>uncovered</u> outcomes. It follows that x belongs to V** if and only if there is a path of majority domination of one or two steps from x to every other outcome, i.e., for any $y \in V$, either $x\ P\ y$ or there is some $z \in V$ such that $x\ P\ z\ P\ y$. It is shown that $V^{**} \subseteq V^* \cap P(V)$ and that several important voting processes --viz, sophisticated voting under amendment procedure, cooperative voting, and electoral competition--lead to decisions in V^{**}. Moreover, further research shows that, given a multidimensional issue space, V^{**} is generally a small compact centrally located subset of the issue space (even when there is no "majority rule equilibrium," i.e., $V^{***} = \emptyset$ and $V^* = V$) (McKelvey, 1981; also see Shepsle and Weingast, 1981b). The question thus arise of the size and location of the uncovered set V^{**} in distributive politics.

Since distributive politics entails some majority preference ties, the discussion in Miller (1980) is insufficiently general. In the more general case, x <u>covers</u> y if and only if $D(y) \subseteq D(x)$ and $F(x) \subset F(y)$ <u>or</u> $D(y) \subset D(x)$ and $F(x) \subseteq F(y)$. It further follows that x belongs to V^{**} so defined if and only if, for all $y \in V$, either $x\ P\ y$ or there is some $z \in V$ such that $x\ P\ z\ R\ y$ or $x\ R\ z\ P\ y$. (We write this $y \in Q_2(x)$.) It still follows that $V^{**} \subseteq P(V)$ and that the outcome of electoral competition belongs to V^{**} (Miller, 1979 and in progress).

Does the concept of the uncovered set V^{**} bring any greater order and stability to distributive politics--by providing theoretical justification for the empirical prevalence of universalism, for example, or--pointing in the opposite direction--by sustaning the well-known (minimum) "size principle" (Riker, 1962) as applied to distributive politics? The answer to these questions is negative. Instead we have the following:

THEOREM 7. In distributive politics $V^{**} = V^* \cap P(V)$.

That is, in distributive politics, the uncovered set is of maximum possible size.

We consider four classes of cases separately: Class I, $B \leq \frac{n+1}{2n}C$; Class II, $\frac{n+1}{2n} < B < C$; Class III, $B = C$; and Class IV, $B > C$.

<u>Class I</u>. In this class, $V^* = \{x^0\}$, i.e., $x^0\ P\ x$ all $x \in V - \{x^0\}$. Thus $x^0 \in V^{**}$. By the same token, there is no z such that $z\ R\ x^0$,

so $x^0 \in Q_2(x)$ for any x. Thus $x^0 = V^{**} = V^* \subseteq P(V)$.

Class II. In this class $V^* = V$ and $P(V) = \bigcup_{h=0}^{k} V^k$, where k is the largest integer such that $B > kC/n$. It is sufficient to show that, for all $x \in V^k$, either $x \in P(V)$ or $Q_2(x) = V - \{x\}$. If $x \in P(V)$, we will in fact always show that, for all $y \in V^h$, either x P y or there is some $z \in V$ such that x P z P y.

We consider separately the following cases within this class:

	k < h	k = h	h < k
(1) < n/2	Case 1	Case 5a	Case 3
(2) < n/2 <	Case 2	---	Case 6
(3) n/2 <	Case 1	Case 5b	Case 4

(1) n/2 greater than both k and h
(2) n/2 intermediate between k and h
(3) n/2 less than both k and h

Case 1. By Lemma 3', x P y.

Case 2a: $k < n/2 < h \leq n-1$. Pick some $z \in V^{\frac{n-1}{2}}$ such that $S_z \subset S_y$. Then x P z (Lemma 3') and z P y (Lemma 5).

Case 2b: $1 \leq k < n/2 < h$. Pick some $z \in V^{\frac{n+1}{2}}$ such that $S_x \subset S_z$. Then x P z (Lemma 5) and z P y (Lemma 3').

Case 2c: $k = 0$ and $h = n$. Then x P y since $B < C$.

Case 3. Pick some $z \in V^g$ where $g = k + \frac{n-1}{2}$ and $S_x \subset S_z$; therefore $g-k = \frac{n-1}{2}$ and $g-h \geq \frac{n+1}{2}$. Thus x P z (Lemma 5) and z P y (Lemma 6).

Case 4. Pick some $z \in V^g$ where $g = k - \frac{n-1}{2}$ and $S_z \subset S_y$. Therefore $h-g = \frac{n-1}{2}$ and $k-g \geq \frac{n+1}{2}$. Thus x P z (Lemma 6) and z P y (Lemma 5).

Case 5a. Pick some $z \in V^g$ such that $g = \frac{n+1}{2}$ and $S_x \cap \bar{S}_y \subseteq S_z \subseteq \bar{S}_y$. In words, pick some z that provides for $\frac{n+1}{2}$ projects, only projects not provided for by y, and including all such projects (at most k-1 in number) also provided for by x. Then $g-k = g-h = \frac{n+1}{2} - k \leq \frac{n-1}{2}$. Since $S_x \not\subseteq S_y$, $S_x \cap \bar{S}_y \neq \emptyset$, so $n_{zx} \geq 1$ and $n_{z\bar{x}} \leq \frac{n-1}{2}$. So x P z (Lemma 2). Since $S_z \subseteq \bar{S}_y$, $S_z \cap S_y = \emptyset$, so $n_{zy} = 0$ and $n_{z\bar{y}} = \frac{n+1}{2}$. Also $B > \frac{n+1}{2n}C > \frac{g-k}{n}C$. So z P y (Lemma 2(a)).

Case 5b. Pick some $z \in V^g$ such that $g = \frac{n-1}{2}$ and $\bar{S}_x \subset S_z$, i.e., z provides for everyone not provided for by x. Thus $\bar{S}_x \cap \bar{S}_z = \emptyset$ so $n_{\overline{xz}} = 0$ and $n_{x\bar{z}} = \frac{n+1}{2}$. Also $k-g = k - \frac{n-1}{2} \leqslant (n-1) - (\frac{n-1}{2}) = \frac{n-1}{2}$, so $k-g \;\; \frac{n-1}{2}$. Thus x P z (Lemma 2(a)). Since x and y are both majority packages of equal expansiveness $S_y \cap \bar{S}_x \neq \emptyset$, say $v \in S_y \cap \bar{S}_x$. Since $\bar{S}_x \subset S_z$, also $\bar{S}_z \subset S_x$; thus $v \in \bar{S}_z$. But also $v \in S_y$, so $\bar{S}_z \not\subset S_y$ and $\bar{S}_z \cap \bar{S}_y \neq \emptyset$. Thus $n_{\overline{yz}} > 0$ and $n_{y\bar{z}} \leqslant \frac{n-1}{2}$. So z P y (Lemma 2).

<u>Case</u> <u>6</u>. If $B \geqslant kC/n$ $(k \leqslant n-1)$, then x P x^0 P y (Theorem 4). If $B < kC/n$, then x^0 PD x (Lemma 8) and $x \notin P(V)$.

<u>Class III</u>. The arguments made in Class II hold here as well except in the following cases.

<u>Case</u> <u>2c</u>: $k = 0$ and $h = n$. Now x^0 I x^n; further x^0 and x^n are <u>equivalent</u>, i.e., $D(x^0) = D(x^n)$ and $F(x^0) = F(x^n)$. Since $Q_2(x^0) = V - \{x^n\}$ and x^0 and x^n are equivalent, x^0 is uncovered (cf. Miller, in progress).

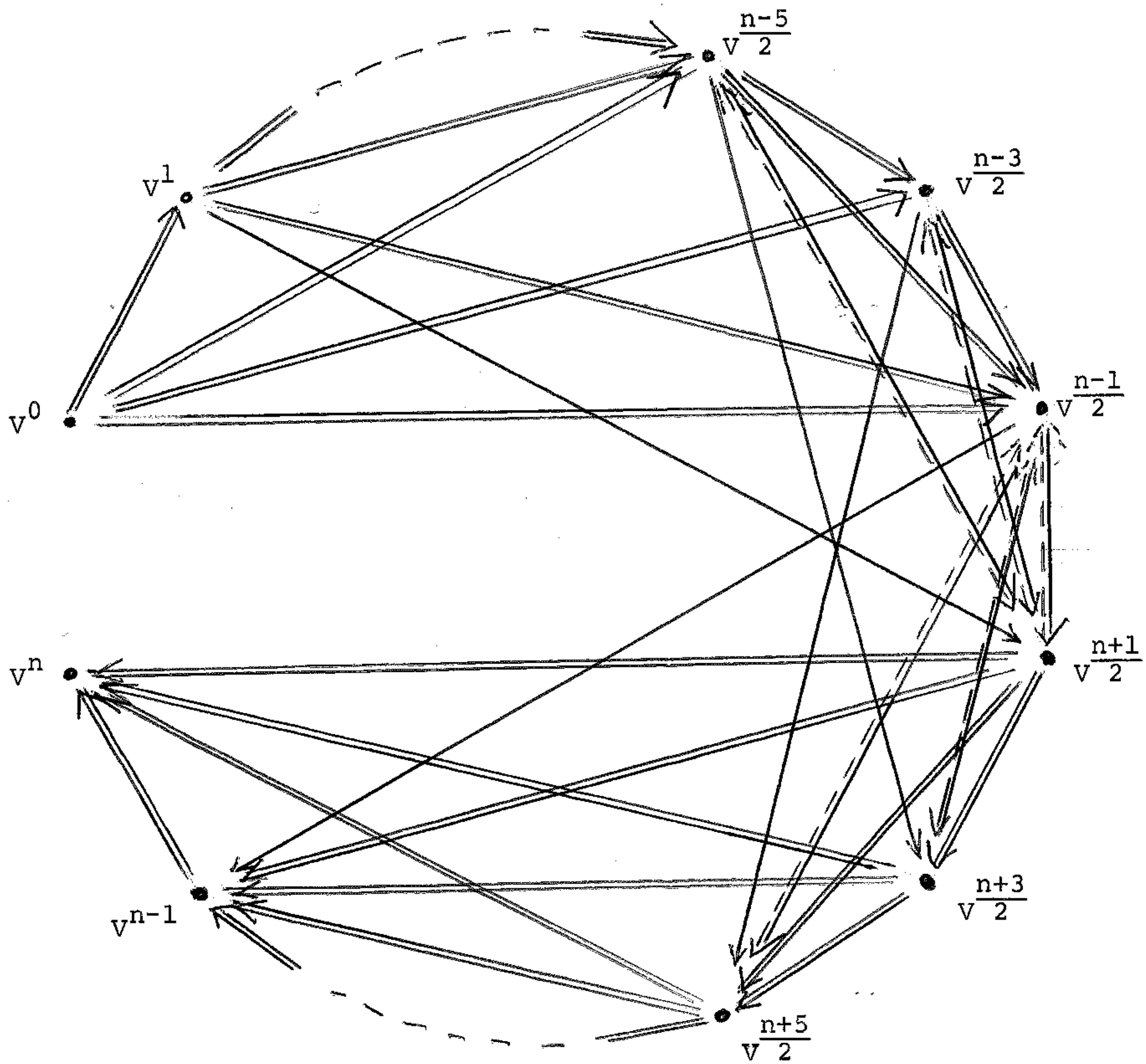
<u>Case</u> <u>6</u>. If $B = kC/n$ and $k = n$, then x^0 I x^n and the above considerations apply.

<u>Class IV</u>. The arguments made in Class II hold here as well except in the following cases.

<u>Case</u> <u>2c</u>. Now x^n PD x^0, so $x \notin P(V)$.

<u>Case</u> <u>6</u>. Now x^n PD x^0, so $x \notin P(V)$.

Thus for every $x \in V^* \cap P(V)$ and every $y \in V$, $y \in Q_2(x)$ or x and y are equivalent, so $V^* \cap P(V) = V^{**}$.


$v^{\frac{n-5}{2}}$
v^{1}
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$v^{\frac{n+1}{2}}$
v^{n-1}
$v^{\frac{n+3}{2}}$
$v^{\frac{n+5}{2}}$

Figure 1.

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