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Voting Power in the U.S. Electoral College

The President of the United States is elected, not by a direct national popular vote, but by an Electoral College system in which separate state popular votes are aggregated by adding up electoral votes awarded on a winner-take-all basis to the plurality winner in each state. State electoral votes vary with population and at present range from 3 to 55. The Electoral College therefore generates the kind of weighted voting game susceptible to ***a priori* voting power** analysis using the various **power indices**; in particular, the question arises of whether and how much the power of voters varies from state to state.

The Constitution provides that states have electoral votes equal in number to their total representation in Congress. Each state has two Senators, while Representatives are apportioned among the states on the basis of population (though every state is guaranteed at least one Representative). The resulting apportionment of electoral votes gives a distinct advantage to small states, relative to straight apportionment by population. House size is now fixed at 435, there are 50 states, and the 23^{rd} Amendment gives three electoral votes to the District of Columbia, so the total number of electoral votes at present is 435 + 100 + 3 = 538, with 270 votes required for election. (If no candidate receives this required majority, other provisions in the Constitution come into effect.) The Constitution leaves the mode of selection of "Presidential electors" (the officials who actually cast "electoral votes") up to each state to decide. Since the mid-1830s, the almost universal state practice has been that each party nominates a slate of elector candidates, equal in number to the state's electoral votes and pledged to vote for the party's presidential candidate, between which voters choose. The slate that wins the most votes is elected and casts its bloc of electoral votes as pledged, producing the weighted voting game noted at the outset. This practice has been widely believed to give larger states an advantage in voting power that counteracts the small-state advantage in apportionment.

The development in the mid-1950s of the **Shapley-Shubik voting power index** (1954) provided a tool for evaluating *a priori* voting power in the Electoral College. While it is not possible to apply the Shapley-Shubik index directly to weighted voting games of the magnitude of the Electoral College, by the late 1950s Monte Carlo computer simulations (Mann and Shapley, 1964) provided good estimates of state voting power, which indicated that the expected bias in favor of larger states was quite modest. Since then other mathematical and computational techniques have been developed that can provide still more accurate estimates (Owen, 1975; Leech, 2005). Moreover, the rival **Banzahaf** (or **Penrose-Banzhaf**) **voting power measure** has since been proposed (Penrose, 1946; Banzhaf, 1965, 1968), which is (arguably) more appropriate than Shapley-Shubik for evaluating *a priori* voting power (Felsenthal and Machover, 1998, 2005). Table 1 shows both the Shapley-Shubik and Bazhaf voting power of states in the present Electoral College, calculated by the computer algorithms developed by Leech (2005), from which it is apparent that (i) the two indices provide very similar estimates of state voting power, and (ii) state voting power is approximately proportional to electoral votes, though (iii) the largest states — and especially the largest of all (California) — are somewhat advantaged relative to the apportionment of electoral votes. The last column shows the **absolute Banzhaf value** for each state, which has the following direct and useful interpretation. Suppose we know *nothing* about the workings of the Electoral College *other than its formal rules*, so our *a priori* expectation must be that states vote randomly, i.e., as if independently flipping fair coins (the *random voting* or *Bernoulli model*). On this assumption, the absolute Banzhaf value is the probability that a state casts a "decisive" bloc of electoral votes vote that determines the outcome of the election.

However, the 51-state weighted voting game presented in Table 1 is mostly a chimera. As noted above, a U.S. Presidential election really is a two-tier voting system, in which the casting of electoral votes is determined by popular vote majorities within each state. In such a two-tier system, individual *a priori* voting power is the probability that the voter casts a decisive vote within the state *and* that the state casts a decisive bloc of electoral votes, i.e., individual voting power within the state *times* state voting power. Clearly the first term is inversely related to the number of voters in the state. However, probability theory tells us that it is inversely *proportional*, not to the number of voters in the state, but (to a very good approximation) to the *square root* of this number. Accordingly individual *a priori* voting power in the two-tier system increases with the square root of the population of the state (in some degree counterbalanced by the small-state advantage in apportionment, in small degree reinforced by the large-state advantage in voting power shown in Table 1, and among small states largely hidden by the unavoidable crudeness in apportioning House seats). This effect, first noted with explicit reference to the Electoral College by Banzhaf (1968) — but noted in a more general context twenty years earlier by Penrose (1946) — may be dubbed the *Banzhaf effect*. (It is related to what Felsenthal and Machover, 1998, p. 66, call the *Penrose's square-root rule*; also see Riker and Shapley, 1968, and Owen, 1975.) Table 2 shows individual *a priori* voting power in selected states in both absolute and relative terms (scaled so that the voting power of the least favored voters is 1), while Chart 1 plots the relationship between state population and relative *a priori* voting power over all states. Voters in the most favored state (California) have almost three and half times the voting power of voters in the least favored state. (Without the small-state apportionment advantage, this ratio would be about ten to one.) The underlying square-root rule is indicated in the chart, which makes evident the small-state apportionment advantage, as well as the scattering that results as small states fall above or below House seat thresholds and California's advantage in state voting power. The chart also shows individual voting power under direct popular vote, which is substantially greater than mean voting power under the Electoral College (also indicated) — indeed, it is greater than voting power in every state except California.

Several critiques of the Banzhaf effect in the Electoral College (e.g., Margolis, 1983; Gelman et al., 2002, 2004; Katz et al., 2004) rest fundamentally on the (indisputable) observation that the random voting model is in no way representative of empirical voting patterns. But these critiques overlook the fact that the Banzhaf (and Shapley-Shubik) measures pertain to *a priori* voting power, measuring the power of states — and, in the two-tier version, of individual voters — in a way that takes account of the Electoral College voting rules *but nothing else*. *A priori*, a voter in California has three times the probability of casting a decisive vote than one in New Hampshire. But if we take account of recent voting patterns, poll results, and other information, a voter in New Hampshire may have a greater *empirical* (or *a posteriori*) probability of decisiveness in the upcoming election, and accordingly get more attention from the candidates and party organizations, than one in California.

If it is hardly related to empirical voting power in any particular election, the question arises of whether *a priori* voting power and the Banzhaf effect should be of concern to political science and practice. Constitution-makers arguably should — and to some extent must — design political institutions from behind a "veil of ignorance" concerning future political trends. Accordingly they should — and to some extent must — be concerned with how the institutions they are designing allocate *a priori*, rather than empirical, voting power. The framers of the U.S. Constitution did not require or expect electoral votes to be cast *en bloc* by states. (However, at least one delegate expected that state delegations in the House of Representatives would vote *en bloc*, which he thought would give large states a Banzhaf-like advantage; see Riker, 1986.) While party politicians within states initially manipulated the rules for selecting Presidential electors for immediate partisan advantage, in due course almost all states moved to the winner-take-all rule that gives rise to the Banzhaf effect. This dominant trend suggests that legislators in large states, operating behind a "veil of ignorance" concerning its long-term partisan implications, understood that doing so would enhance (in so far as other states had not done the same) or restore (in so far as other states had done the same) the influence of their state and its voters in Presidential elections. Legislators in small states then had little choice but do the same.

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EV	N	EV PROP	S-S INDEX	BANZHAF INDEX	ABSOLUTE BANZHAF
3	8	.005576	.005404	.005456	.022730
4	5	.007435	.007218	.007276	.030312
5	5	.009294	.009038	.009097	.037900
6	3	.011152	.010865	.010920	.045493
7	4	.013011	.012698	.012744	.053094
8	2	.014870	.014538	.014571	.060704
9	3	.016729	.016385	.016400	.068324
10	4	.018587	.018239	.018231	.075955
11	4	.020446	.020099	.020066	.083599
12	1	.022305	.021967	.021904	.091257
13	1	.024164	.023841	.023746	.098930
15	3	.027881	.027612	.027442	.114328
17	1	.031599	.031411	.031157	.129805
20	1	.037175	.037166	.036771	.153194
21	2	.039033	.039100	.038654	.161043
27	1	.050186	.050869	.050119	.208805
31	1	.057621	.058884	.057948	.241422
34	1	.063197	.064988	.063927	.266331
55	1	.102230	.110358	.114021	.475036
538	51	1.000000	1.000000	1.000000	4.166201

EV — Number of Electoral Votes
N — Number of States
EV PROP — Proportion of Electoral Votes
S-S INDEX — Shapley-Shubik Index Value
BANZHAF INDEX — (Relative) Banzhaf Index Vale
ABSOLUTE BANZHAF — Absolute Banzhaf Value
Shapley-Shubik and Banzhaf values calculated by *ssgenf* and *ipsgenf* at
http://www.warwick.ac/~ecaae/

TABLE 1

***A PRIORI* STATE VOTING POWER IN THE ELECTORAL COLLEGE**

STATE	ELECT SIZE	IND VP	EV	STATE VP	IND 2-T VP	REL VP
MT	392640	.00127334	3	.022730	.00002894	1.000000
UT	970074	.00081010	5	.037900	.00003070	1.060803
DE	340488	.00136738	3	.022730	.00003108	1.073857
NH	537107	.00108870	4	.030312	.00003300	1.140203
OK	1500107	.00065145	7	.053094	.00003459	1.195039
AK	272771	.00152771	3	.022730	.00003472	1.199770
WS	2329521	.00052277	10	.075955	.00003971	1.371895
CO	1870085	.00058346	9	.068324	.00003986	1.377338
MD	2302057	.00052587	10	.075955	.00003994	1.380054
MA	2756442	.00048058	12	.091257	.00004386	1.515269
NC	3498990	.00042655	15	.114328	.00004877	1.684919
MI	4317893	.00038398	17	.129805	.00004984	1.722080
OH	4933195	.00035923	20	.153194	.00005503	1.901409
IL	5394875	.00034352	21	.161043	.00005532	1.911389
PA	5334862	.00034544	21	.161043	.00005563	1.922110
FL	6951810	.00030262	27	.208805	.00006319	2.183181
NY	8242552	.00027791	31	.241422	.00006709	2.318163
TX	9066167	.00026499	34	.266331	.00007057	2.438416
CA	14715957	.00020799	55	.475036	.00009880	3.413738
US	122294000	.00007215	538	—	.00007215	2.492845

ELECT SIZE — Size of Electorate
[2000 Population x .4337, where .4377 = 2004 Total Presidential Vote/2000 US Population]
IND VP — Individual Absolute Banzhaf Voting Power within State
[By Stirling's approximation for n!, VP = $(2/\pi n)^{.5}$ where n = ELECT SIZE]
STATE VP — State Absolute Banzhaf Voting Power (from Table 1)
IND 2-T VP — Individual Banzhaf Voting Power in Two-Tier System [= IND VP x ST VP]
REL VP — Relative Individual 2-T Voting Power (scaled so that minimum [Montana] = 1)

TABLE 2

***A PRIORI* INDIVIDUAL VOTING POWER IN SELECTED STATES**

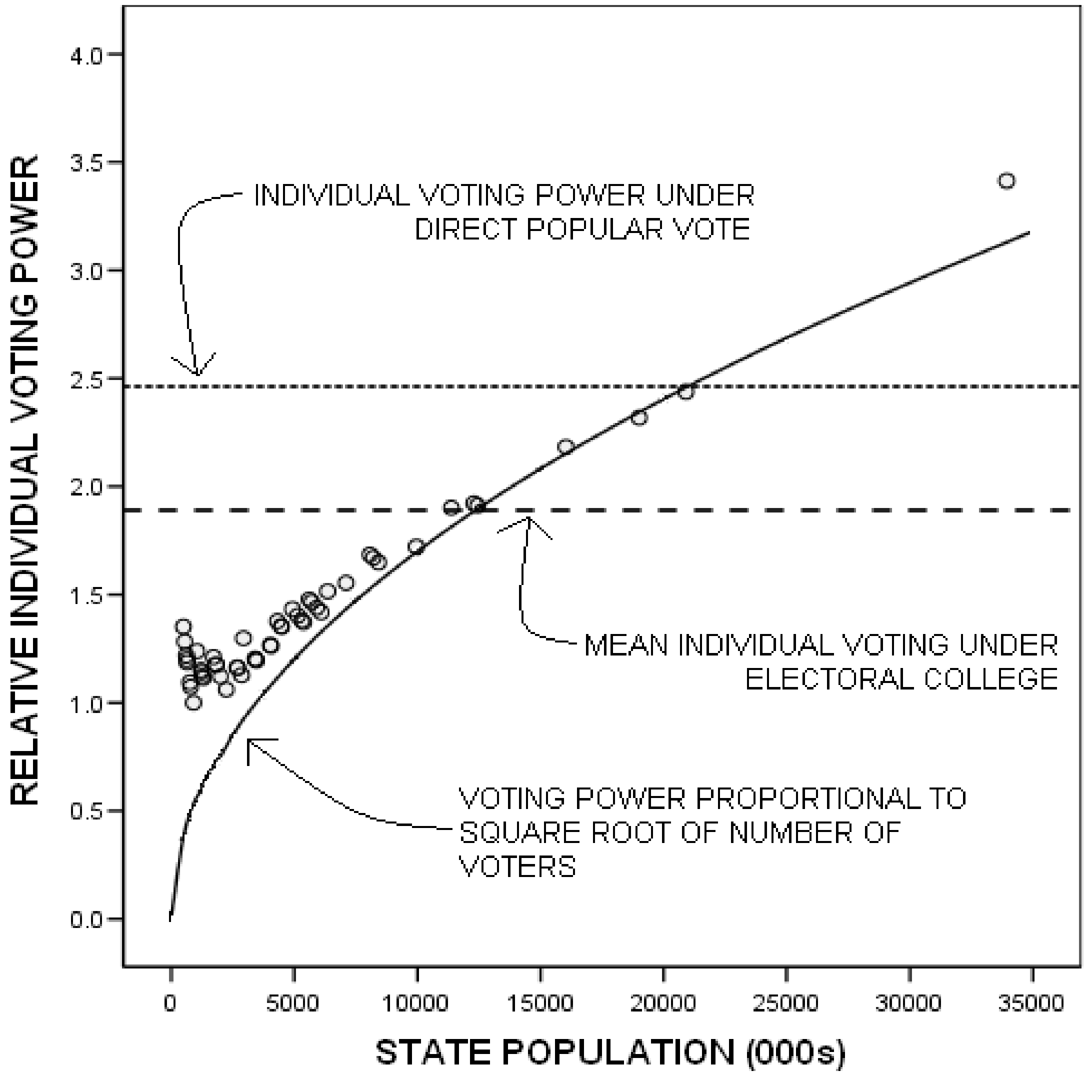

CHART 1

A PRIORI INDIVIDUAL VOTING BY STATE UNDER THE ELECTORAL COLLEGE (AND DIRECT POPULAR VOTE)