THE ALTERNATIVE VOTE AND COOMBS RULE VERSUS FIRST-PAST-THE-POST: A SOCIAL CHOICE ANALYSIS OF SIMULATED DATA BASED ON ENGLISH ELECTIONS, 1992-2010

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Abstract

This paper presents a social choice analysis, using simulated data based on English general elections from 1992 through 2010, of the properties of three voting rules: First-Past-the-Post, the Alternative Vote, and the Coombs Rule. More specifically, the paper examines (1) the plurality, anti-plurality, and Condorcet status of candidates in each election and the interrelationships among these statuses, (2) the effects of strict and partial single-peakedness of voter preferences, and (3) the identity of winners, Condorcet efficiency, and the relationship between votes and seats under the three voting rules. The analysis considers only the case of three candidates and, in the manner of basic social choice theory, the set of candidates and voter preferences over them are taken to be fixed.

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In May 2011, the United Kingdom held a referendum that proposed to replace its venerable First-Past-the-Post (FPTP or Simple Plurality) voting rule for general elections with the Alternative Vote (AV) system. Sometimes known as the Hare system and commonly referred to as Instant Runoff Voting (IRV) in the United States, AV has been used for many decades to elect the Australian House of Representatives. After a campaign that did not produce a particularly enlightening discussion of the properties and relative advantages and disadvantages of the two electoral systems, the FPTP status quo was preserved by a large margin.

While motivated in part by this political context, the principal aim of this paper is to contribute to applied social choice theory by using a number of social choice concepts and propositions in an analysis of several simulated data sets based on English elections from 1992 through 2010. In addition to the FPTP and AV social choice rules, the analysis takes seriously the titular injunction of Grofman and Feld (2004) — "if you like the Alternative Vote, then you ought to know about the Coombs Rule" — and includes this third rule. Thus the paper is in the spirit of Merrill's *Making Multicandidate Elections More Democratic* (1988), as well as related work by Fishburn and Gehrlein (1976, 1977), Chamberlin and Cohen (1978), and others. However, it uses data with first preferences based on actual election results, while the cited works were based on wholly simulated voting data. The analysis focuses on the plurality, anti-plurality, and Condorcet status of candidates, i.e., whether a candidate is the winner, runner-up, or loser by each criterion, and therefore is restricted to the case of three candidates. Therefore, the simulated data are based on English constituencies only, which typically feature elections with just three "serious" candidates. The analysis then turns to the identity of winners under each social choice rule, the Condorcet efficiency of the three rules, and the relationship between votes and seats under each rule. In the manner of basic social choice theory, the set of candidates and voter preferences over them are taken to be fixed. While the analysis might be expanded to cover manipulability and strategic voting, monotonicity, and candidate entry and exit, and other social choice concepts, these issues are not covered here.

Given the data used and the fact that the candidates are identified as Conservatives, Liberals, and Labourites, the analysis may provide some useful insights into contemporary British politics. But by its nature it cannot offer any overall assessment of the merits of the three voting rules nor any definitive projection of the likely effects of the alternative rules on British electoral politics. For the latter purpose, a suitable analysis would have to proceed in a less straightforward and more empirically grounded fashion, along the lines of Sanders et al. (2011).

1. Context

On an FPTP ballot, voters simply put an "X" beside the name of the candidate for whom they wish to vote, and the candidate with the most votes is elected. In contrast, on an AV ballot, voters rank the candidates in order of preference. If one candidate has a majority of first preferences, that candidate is elected; thus, if there are just two candidates, the two systems are equivalent. However, if there are three or more candidates none of whom is supported by a majority of first preferences,

under AV the candidate with the fewest first preferences is eliminated and his or her ballots are transferred to other candidates on the basis of second preferences. This process is repeated until one candidate is supported by a majority of (first preference and transferred) votes and is elected. The Coombs Rule is similar to AV except that it uses a different elimination rule: the (remaining) candidate with the most last preferences is eliminated, rather than the candidate with the fewest first preferences.[1]

Since we examine only three-candidate contests, AV and Coombs are limited to a single "instant runoff" in the event none of three candidates is supported by a majority of first preferences. We suppose that all voters rank all three candidates. Though AV may in practice allow voters to "truncate" their ballots by failing to rank all candidates (in the case of three candidates, by indicating only a first preference), Coombs really cannot do so, given the critical role it assigns to last preferences.[2] For purposes of definition and formal analysis, I also assume that there are no (plurality, anti-plurality, or pairwise) ties; in fact, this assumption is never contradicted in the four data sets based on English elections used here (each with about 40,000 voters per election).[3]

Since they were displaced by Labour as the main opposition to the Conservatives in the inter-war period, Liberals (Liberal Democrats in their current incarnation) have joined the Electoral Reform Society in advocating the Single Transferable Vote for general elections. STV is a multi-member district generalization of AV, is (quasi-) proportional in nature, and would clearly help the Liberals (and other minor parties) win more seats, even if their first preference support did not increase. More recently Liberals have supported AV as a less radical but more acceptable alternative to FPTP, and they demanded the AV referendum as a condition for joining the Conservatives in a coalition government after the May 2010 general election.

As a generally centrist party, the Liberals presumably would get substantial second preference support despite relatively little first preference support. Since AV asks voters to rank candidates and may count second preferences in determining winners, it appears at first blush that AV would help the Liberals considerably. However, while second-preference support under AV helps candidates once they get into a runoff, such support doesn't help candidates get into a runoff in the first place. As Aldrich et al. (2011) observe, if AV had been applied at the national level (e.g., in a hypothetical direct election of Prime Minister), the Liberals still would have lost the 2010 election, since they

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[1] This system was proposed by Coombs (1964: 397-399) as "an alternative to the Hare [AV] system" in a side discussion of "majority decision from the point of view of unfolding theory" in what was primarily a treatise on the analysis of psychological data.

[2] Thus Coombs, even more than AV, is vulnerable to what Australians call "donkey voting" — that is, casting ballots that rank candidates (beyond the first or several highest preferences) in an arbitrary order not reflecting preference, typically the order in which they are listed on the ballot.

[3] Moreover, so far as I know, there is no standard way to break ties under any of the three systems. Allowing for ties would modify (and complicate) several of the definitions and propositions in the next section.

would not have gotten into the runoff.[4] To say that AV applied at the national level would not help the Liberals does not imply that AV applied at the constituency level would not help the Liberals overall, since individual constituencies need not, and often do not, resemble (even approximately) the national electorate; a fair number of losing Liberal candidates place second under FPTP, qualifying them for runoffs under AV, many of which they might win. However, Coombs should be much more advantageous to Liberals than AV, because second preference support under Coombs not only helps candidates win a runoff in the event they get into one, but also help them get into a runoff in the first place, since more second (and first) preference support implies less last-preference support.

2. Social Choice Concepts and Propositions

A *ballot* (or *preference*) *profile* is a set of n rankings of candidates, where n is the number of voters. Given a profile with three candidates X, Y, and Z, the candidate with the most first preferences is the *Plurality Winner*, the candidate with the second most first preferences is the *Plurality Runner-Up*, and the candidate with the fewest first preferences is the *Plurality Loser*. Let n(PW), n(P2), and n(PL) be the number of ballots that rank the Plurality Winner, the Plurality Runner-Up, and Plurality Loser first. Given three candidates, it follows that $n(\text{PL}) < n/3 < n(\text{PW})$. A Plurality Winner who is the first preference of an absolute majority of voters, such that $n(\text{PW}) > n/2$, is a *Majority Winner*. Note that these concepts depend on the number of first preferences only. The FPTP winner is by definition the Plurality Winner, and the AV winner is the Majority Winner if one exists and otherwise is either the Plurality Winner or the Plurality Runner-Up, depending on the outcome of the instant runoff between them.

Given a ballot profile, the candidate with the fewest last preferences is the *Anti-Plurality Winner*, the candidate with the second fewest last preferences is the *Anti-Plurality Runner-Up*, and the candidate with the most last preferences is the *Anti-Plurality Loser*.[5] Let n(APW), n(AP2), and n(APL) be the number of ballots that rank the Anti-Plurality Winner, the Anti-Plurality Runner-Up, and Anti-Plurality Loser last. Given three candidates, it follows that $n(\text{APW}) < n/3 < n(\text{APL})$. The Coombs winner is the Majority Winner if one exists and otherwise is either the Anti-Plurality Winner or the Anti-Plurality Runner-Up, depending on the outcome of the instant runoff between them.

Given a ballot profile, let candidates X, Y, and Z have x, y, and z first preferences respectively, so $x + y + z = n$. Likewise let x_y be the number of voters who have a first preference for X and second preference for Y (and therefore a third preference for Z), let x_z be the number who have a first preference for X and a second preference for Z, so $x_y + x_z = x$; and likewise for other candidates.

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[4] Aldrich et al. examine four alternatives to FPTP (Condorcet, AV, Coombs, and Borda) and conclude that the Liberals would have won such a hypothetical national vote in 2010 under all four systems *except* the one they actually advocated.

[5] Under Anti-Plurality Voting, each voter casts a "negative" vote *against* one candidate, and the candidate with the fewest "negative" votes wins.

Given a ballot profile, if a majority of voters rank *X* over *Y*, i.e., if $x + z_x > n/2 > y + z_y$, we say that *X beats Y* in what the British call a "straight fight." This defines the pairwise *Condorcet relationship* between pairs of candidates. A *Condorcet Winner* is a candidate who beats both other candidates, a *Condorcet Runner-Up* is a candidate who beats one other candidate and is beaten by the other, and a *Condorcet Loser* is a candidate who is beaten by both other candidates. If *X* beats *Y*, *Y* beats *Z*, and *Z* beats *X*, or if *Y* beats *X*, *X* beats *Z*, and *Z* beats *Y*, a *Condorcet cycle* exists and there is neither a Condorcet Winner nor a Condorcet Loser.

Given three candidates, voter preferences are *single-peaked* in the event that there is one candidate who is not ranked last by any voter.[6] This candidate may be thought of as a (relatively) "centrist" candidate, who is a "compromise" between the two (relatively) "extreme" candidates and the second preferences of all voters who most prefer either of the other two.

We now gather together a number of elementary propositions relating these basic social choice concepts.

Proposition 1 A Majority Winner is also a Condorcet Winner.

A candidate who is the first preference of an absolute majority of voters beats each other candidate in a straight fight regardless of the distribution of second preferences.

Proposition 2 Given three candidates, if voter preferences are single-peaked:

(a) a Condorcet cycle cannot occur;
(b) an extreme candidate is the Condorcet Winner if and only if he is a Majority Winner;
(c) the centrist candidate is always the Anti-Plurality Winner;
(d) the centrist candidate is the Condorcet winner if there is no Majority Winner; and
(e) the centrist candidate may be the Plurality Loser but can be never be the Condorcet Loser.

Points (a) and (b) summarize more general results going back to Black (1948). The centrist candidate has no last preferences and, if not the Condorcet Winner, beats the extreme candidate who is not the Majority Winner.

Proposition 3 Given at least three candidates, plurality and anti-plurality status are completely independent; in particular,

(a) the Plurality Winner may be the Anti-Plurality Loser, and
(b) the Plurality Loser may be the Anti-Plurality Winner.

It is sufficient to give examples of each possibility. A candidate with a bare absolute majority of last preferences is certainly the Anti-Plurality Loser but may at the same time be the first preference of all other voters and, if the remaining first preferences are sufficiently equally split between the two other candidates, also has a plurality of first preferences. A candidate who is the second preference of every voter is the Plurality Loser but also has the fewest last preferences and so is the Anti-Plurality Winner.

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[6] Given more than three candidates, preferences are single-peaked if this condition holds for every triple of candidates (Sen 1970: 167-168).

Proposition 4 Given at least three candidates, plurality and Condorcet status are completely independent; in particular,

(a) the Plurality Winner may be the Condorcet Loser; and
(b) the Plurality Loser may be the Condorcet Winner.

It is again sufficient to give examples of each possibility. If the Plurality Winner is not a Majority Winner, the Plurality Winner may be the last preference of an absolute majority of voters and thus be beaten by each other candidate in a straight fight. The Plurality Loser may be the second preference of all voters who do not most prefer the Plurality Loser and thus (in the absence of a Majority Winner) beat each other candidate in a straight fight.

Proposition 5 Given at least three candidates, anti-plurality and Condorcet status are completely independent; in particular,

(a) the Anti-Plurality Winner may be the Condorcet Loser, and
(b) the Anti-Plurality Loser may be the Condorcet Winner.

In the following profile, *Z* is both the Anti-Plurality Winner and the Condorcet Loser and *X* is both the Anti-Plurality Loser and the Condorcet Winner.[7]

3	4	1	4	1	2
X	*X*	*Y*	*Y*	*Z*	*Z*
Y	*Z*	*X*	*Z*	*X*	*Y*
Z	*Y*	*Z*	*X*	*Y*	*X*

Proposition 6 Under FPTP,

(a) the Plurality Winner is (by definition) always elected; so
(b) the Condorcet Winner may fail to be elected; and in particular
(c) the Condorcet Loser may be elected.

Points (b) and (c) follow from Proposition 3.

Proposition 7 Under AV with three candidates,

(a) either the Plurality Winner or Plurality Runner-Up may be elected; so
(b) the Condorcet Winner may fail to be elected; but
(c) neither the Plurality Loser nor the Condorcet Loser can be elected.

Point (b) follows from Proposition 4. The Plurality Loser never gets into the runoff and, while the Condorcet Loser may do so, it is always beaten by the other candidate.

Proposition 8 Under AV with single-peaked preferences over three candidates,

(a) the AV winner is the Condorcet Winner unless the Condorcet Winner is the Plurality Loser, in which case
(b) the AV winner is the extreme candidate who beats the other extreme candidate.

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[7] The numbers at the head of each column indicates indicates the number of voters with that ranking. I am indebted to Dan Felsenthal for this example.

Proposition 9 Under Coombs with three candidates,
(a) either the Anti-Plurality Winner or Anti-Plurality Runner-Up may be elected;
(b) the Condorcet Winner may fail to be elected; but
(b) neither the Anti-Plurality Loser nor the Condorcet Loser can be elected.

Point (b) follows from Proposition 7. The Anti-Plurality Loser never gets into the runoff and, while the Condorcet Loser may do so, it is always beaten by the other candidate.

Proposition 10 Under Coombs with single-peaked preferences over three candidates,
(a) the centrist candidate is always the Anti-Plurality Winner, so
(b) the Condorcet Winner is always elected.

Point (a) follows because the centrist candidate never has any last preferences. Point (b) follows because either (i) the centrist candidate is the Condorcet Winner or (ii) an extreme candidate is a Majority (and Condorcet) Winner and wins without a runoff.[8]

While highly skewed second preferences are necessary to demonstrate the independence of plurality, anti-plurality, and Condorcet status, it will be helpful to state the following.

Proposition 11 If second preferences over three candidates are split equally, i.e., if $x_y = x_z = x/2$, and $y_x = y_z = y/2$, and $z_x = z_y = z/2$, plurality, anti-plurality, and Condorcet status are equivalent.

Suppose X is the Plurality Winner, Y is the Plurality Runner-Up, and Z is the Plurality Loser. Then $x > y > z$ and $y_z + z_y = y/2 + z/2 < x_z + z_x = x/2 + z/2 < x_y + y_x = x/2 + y/2$, so X is the Anti-Plurality Winner, Y is the Anti-Plurality Runner-Up, and Z is the Anti-Plurality Loser. Also $x + z_x = x + z/2 > y + z_y = y + z/2$, so X beats Y and likewise X beats Y and Y beats Z.

3. Data

The analysis that follows is based on simulated data mostly derived from constituency-level election returns from the five U.K. general elections from 1992 through 2010.[9] However, I use only data from English constituencies, because virtually all elections in England are essentially three-candidate (Conservative, Labour, Liberal Democrat) affairs, while those in Wales, Scotland, and Northern Ireland almost always include strong (often winning) candidates of "nationalist" parties as well. A few constituencies that do not fit the basic three-party pattern are also excluded.[10] Over the five general elections, this gives us a sample of 2642 three-candidate elections.

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[8] Point (b) generalizes to any number of candidates (Coombs 1964: 399; Grofman and Feld 2004: 650).

[9] This data comes from Pippa Norris's Shared Datasets website (http://www.hks.harvard.edu/fs/pnorris/Data/Data.htm). I am extremely grateful to Professor Norris for making this valuable data readily available.

[10] In 2010 these include the Speaker's constituency (since by tradition the Speaker is not opposed by major-party candidates) and one constituency won by a fourth-party (Green) candidate.

An obvious problem is that these were FPTP elections therefore the data provide us only with (what we take to be) the first preferences of voters, while analysis of AV and Coombs elections requires voters' preference rankings over all three candidates. In the principal dataset used here (with *Survey-Based Second Preferences*), second (and by default third) preferences in each constituency election are allocated in proportion to second preferences nationwide, as determined by surveys. Respondents who indicated "nationalist" or other fourth-party second preferences, or who did not indicate any second preference, were excluded from these calculations.[11] Table 1 reports the resulting second-preference distributions. As we would probably expect, voter preferences are "partially single-peaked" — that is, most (but not all) Labour ("left-of-center") voters have Liberals ("centrists") as their second preference and most (but not all) Conservative ("right-of-center") voters likewise have Liberals as their second preference, while Liberal voters have more equally divided second preferences (though the split varied considerably from election to election).[12]

The analysis also takes account of other ways of assigning second preferences that "bracket" the survey-based procedure. The second dataset (with *Strictly Single-Peaked Preferences*) assumes that preferences are fully ideologically structured in the sense that they are (strictly) single-peaked as defined earlier, with all Labour and Conservative voters having Liberal as their second preference. The third and fourth datasets assume that there is no ideological structuring of preferences at all and that all voters, regardless of their first preferences, are equally likely to have either other party as their second preference. In the third dataset (with *Random Second Preferences*), second preferences on average are split equally between the two other parties, but the split may be quite unequal in any individual ballot profile. In the fourth dataset (with *Impartial Second Preferences*), the second preference of each voter is determined (as if) by independently flipping fair coins, producing an almost equal split on every individual ballot profile. It will be useful at several points to compare these four variants of English data with data that has been simulated in a *wholly random* or *wholly impartial* manner, i.e., so that all preference rankings of all voters are equally likely — on average in the random case, in each individual election in the impartial case.

4. Plurality, Anti-Plurality, and Condorcet Status

Table 2 shows the distribution of Majority Winners, Plurality Winners, Plurality Runners-Up, and Plurality Losers by party affiliation each year and overall. The first point to observe is that Majority Winners occur in 60% of all English elections, so we know at the outset that the winners are the same under all three voting rules at least 60% of the time. Majority Winners are preponderantly candidates of whichever party won a majority of seats, i.e., Labour in 1997, 2001, and 2005 and (by smaller margins) Conservatives in 1992 and 2010.[13] Very few Liberal candidates are

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[11] For further details pertaining to this and the other datasets, see the Appendix.

[12] However, at the "New Labour" high tide of 1997 and 2001, Conservative second preferences for Labour exceeded Liberal second preferences for Conservatives.

[13] In 2010, the Conservatives won a majority of seats in England, though not nationally.

Plurality (let alone Majority) Winners, which accounts for their unhappiness with FPTP. Indeed, Liberal candidates are Plurality Losers about two-thirds of the time, though the proportion varies considerably from election to election. Perhaps unexpectedly, there is a distinct asymmetry between the two leading parties in that Labour candidates are consistently more likely to be Plurality Losers than Conservative candidates are.

Table 3, which shows the anti-plurality status of party candidates in each election based on survey-based second preferences, displays some striking features and a further asymmetry between Labour and Conservative candidates. Anti-Plurality Winners are almost always Liberals (literally always in 2010) and (in these five elections) literally never are Conservatives. Conversely, Anti-Plurality Losers literally never are Liberal candidates and are far more likely to be Conservatives than Labourites. Table 3 supports the expectation that Liberal candidates are likely to be especially helped by the Coombs rule.

Table 4 duplicates Table 3 for the other second preference assumptions but, to save space, does not break the data down by year. Thus the first column of Table 4 duplicates the last column of Table 3. Given strictly single-peaked preferences, Liberal candidates not only are never Anti-Plurality Losers but (consistent with Proposition 3) are always Anti-Plurality Winners, so the Coombs rule would work even more to the advantage of Liberals. On the other hand, given random or (especially) impartial second preferences, the Liberal advantage (and Conservative disadvantage) with respect to anti-plurality status disappears.

Table 5 shows Condorcet relationships and the Condorcet status of candidates by year given survey-based second preferences.[14] In good Labour years, Labour candidates typically beat both other candidates and Liberal candidates beat Conservative candidates while, in the best Conservative year of 1992, Conservative candidates typically beat both other candidates and Liberal candidates beat Labour candidates Therefore Labour candidates typically were Condorcet Winners in the first case and Conservative candidates in the second. Indeed, comparing Tables 2 and 5, we see that Labour (but not Conservative) candidates had Condorcet Winner status just about as frequently as they had Plurality Winner status in 1997, 2001, and 2005, and likewise Conservative (but not Labour) candidates had Condorcet Winner status just about as frequently as they had Plurality Winner status in 1992. Liberal candidates throughout were less likely than Conservative or Labour candidates to have Condorcet Winner status but, at the same time, they were consistently more likely to be Condorcet Winners than to be Plurality Winners. The unusual (national) "hung parliament" election of 2010 presents a somewhat different picture, as Liberal candidates beat Conservative candidates more often than not and beat Labour candidates much more often than not, but beat both in less than one-third of the constituencies. Even so, more Liberal than Labour candidates were Condorcet Winners. Finally, while Liberal candidates were rather more likely to be Condorcet Winners than Plurality Winners, they were far less likely to be Condorcet Losers than Plurality Losers. Even more strikingly, Liberal candidates were far less likely to be Condorcet Losers than either Labour or

[14] Remember that there are no ties, so when (for example) the first entry that says that Labour candidates beat Liberal candidates in hypothetical straight fights in 33.6% of the constituencies in 1992, Liberal candidates beat Labour candidates in the other 66.4%.

Conservative candidates were. Indeed, in 2010 not a single Liberal candidate was a Condorcet Loser. Finally, eight Condorcet cycles turn up among the 2642 profiles.

Table 6 duplicates Table 5 for the other second preference assumptions but, saving space in the manner of Table 4, does not break the data down by year. Strict single-peakedness strengthens the Condorcet (like the Anti-Plurality) status of Liberal Candidates — indeed, the Liberals become the dominant party, with 42.8% of it candidates being Condorcet Winners (and, consistent with Proposition 2(e), none being Condorcet Losers). On the other hand, random and impartial second preferences greatly weaken the Condorcet status of Liberal candidates; indeed, comparison of Tables 2 and 6 shows the Condorcet status of party candidates aligns very closely with plurality status — virtually exactly in the case of impartial second preferences. This to be expected, because (i) Proposition 11 tells us that if second preferences are always split equally, Condorcet relationships are determined by plurality status and (ii) if second preferences are determined impartially and the electorate is reasonably large, the equal split condition is very closely approximated. The same considerations imply that, given impartial second preferences, there will be (essentially) no Condorcet cycles, as is always true in the event of strict single-peaked preferences. However, random second preference profiles contain a slightly larger number of Condorcet cycles (11) than do the survey-based second preference profiles.

5. Plurality, Anti-Plurality, and Condorcet Interrelationships

Propositions 3, 4, and 5, which assert the logical independence of plurality, anti-plurality, and Condorcet status, were demonstrated by means of examples that entailed highly skewed second preferences. But in the absence of systematically skewed second preferences, our general expectation would be that candidates who are strong with respect to one status would tend to be strong with respect to the other two as well.

The first two rows of Table 7 support both Proposition 3 and this expectation with respect plurality and anti-plurality status. They show anti-plurality status for each category of plurality status given wholly random and wholly impartial ballot profiles.[15] Consistent with Proposition 3, every combination of plurality and anti-plurality status occurs with substantial frequency. At the same time and consistent with the general expectation, plurality and anti-plurality status are quite strongly associated. However, the third row, which pertains to the English data with survey-based second preferences, displays a quite different pattern, though it remains true that every combination occurs with substantial frequency. Plurality Winners tend to be Anti-Plurality Runners-Up, Plurality Runners-Up tend to be Anti-Plurality Losers, and Plurality Losers tend to be Anti-Plurality Winners. This might be puzzling until we look at the following rows (or recall earlier tables), which identify candidates by party. Plurality Winners are mostly Labour and Conservative candidates, and Labour and Conservative candidates are mostly the last preference of supporters of the other leading party and therefore are, even as Plurality Winners, rarely Anti-Plurality Winners. Liberal candidates, in contrast, are almost always Anti-Plurality Winners (as we saw in Table 3) regardless of their plurality

[15] Even with about 40,000 elections, ties occur in about 0.4% of the wholly impartial profiles. The percentages reported in the following tables exclude such profiles.

status. This pattern is even stronger given strictly single-peaked preferences, as shown in the next panel of Table 7. In contrast, given random second preferences, the Liberal advantage with respect to anti-plurality status is lost, and the overall pattern, as shown in the first row of the next panel of Table 7, more closely resembles that for wholly random (or impartial) preferences. Finally, given Proposition 11 and impartial second preferences, the alignment of plurality and anti-plurality status is almost perfect, and the most discrepant combinations (Plurality Winner and Anti-Plurality Loser and vice versa) do not occur at all.[16]

Table 8 shows Condorcet relationships with respect to plurality status for all preference assumptions. We would in general expect Plurality Winners to beat Plurality Runners-Up (substantially) more than half the time, Plurality Runners-Up to beat Plurality Losers with comparable frequency, and Plurality Winners to beat Plurality Losers with even greater frequency. We see that Condorcet relationships display this expected "transitive" pattern in wholly random and wholly impartial profiles.[17] However, in the profiles with survey-based second preferences, Plurality Winners beat Plurality Losers hardly more frequently than Plurality Runners-Up, and Plurality Losers beat Plurality Runners-Up more than half the time. In strictly single-peaked profiles, Plurality Winners actually beat Plurality Runners-Up more often than Plurality Losers, and Plurality Losers beat Plurality Runners-Up almost two thirds of the time. We can anticipate that this pattern follows from the fact the Liberal candidates are stronger (especially with strictly single-peaked preferences) with respect to Condorcet than plurality status, though Table 8 (unlike Table 9) does not directly confirm this, as it does not refer to party. (Table 9 that follows does.) Condorcet relationships with impartial second preferences are almost perfectly determined by plurality status. This again is a consequence of Proposition 11, in combination with the fact that impartial second preferences with many voters are almost always virtually equally split. Finally, profiles with random second preferences display roughly the same pattern in somewhat attenuated form.

Table 9 shows Condorcet status by plurality status, with candidates now identified by party. The first two rows are again consistent with both Proposition 4 and the general expectation of association between Condorcet and plurality status. The panel with survey-based second preferences shows that Plurality Winners are Condorcet Winners even more frequently than in wholly random or

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[16] In contrast, we see in the second row of Table 7 that discrepant combinations do appear with some frequency in wholly impartial profiles. This is because almost all such profiles are virtually tied with respect to first preferences (and therefore plurality status) as well as second and third preferences (and therefore anti-plurality status). The same consideration means that cycles occur in about 8.3% of the wholly impartial profiles, though they are absent from the profiles with impartial second preferences only.

[17] Given wholly impartial profiles, Plurality Winners beat Plurality Runners-Up, and Plurality runners-Up beat Plurality Losers, about 76% of the time, while Plurality Winners beat Plurality Losers about 90% of the time. These percentages (which, being based on simulations, are approximations) are fixed and intrinsic characteristics of an "impartial culture" (that, to the best of my knowledge, have never before been reported), comparable to the (likewise approximate) 8.8% of impartial culture profiles with three alternatives that produce Condorcet cycles (Sen 1970: 164) and the 20.4% of impartial culture profiles with two candidates that produce "election inversions" given a sufficiently large number of uniform districts (Feix et al. 2004).

impartial profiles. This is not surprising, because 60% of Plurality Winners are also Majority (and therefore Condorcet) Winners, whereas only 8% of wholly random profiles (and none of the wholly impartial profiles) have Majority Winners. However, Plurality Runners-Up are most likely to be Condorcet Losers, while Plurality Losers are most likely to be Condorcet Runners-Up. The latter point is due almost entirely to the fact that Liberals constitute the great majority of Plurality Losers but rarely are Condorcet Losers, while the former point is due largely to the fact that Conservatives constitute the great majority of Plurality Runners-Up but mostly are Condorcet Losers. With strictly single-peaked preferences, Liberals are even more likely to be Condorcet Winners when not Plurality Winners, making Conservative and Labour candidates less likely to be Condorcet Winners when they are Plurality Winners. With random second preferences, the pattern of association is similar to that with wholly random or impartial preferences, except that there is a much larger overlap between Plurality and Condorcet Winner status (at the expense of overlap between Plurality Runner-Up and Condorcet Winner status) due to the much greater prevalence of Majority Winners. Finally, profiles with impartial second preferences exhibit almost perfect alignment of plurality and Condorcet status.

Table 10 shows Condorcet relationships by anti-plurality status. In wholly random data, the pattern is similar to that with plurality status and, in wholly impartial data, it is essentially identical. But given survey-based second preferences, Anti-Plurality Winners rarely beat the Anti-Plurality Runners-Up. We can anticipate, and Table 11 confirms, that this is because Anti-Plurality Winners are almost always Liberals, who rarely beat the stronger leading party candidate but usually beat the weaker one — that is, Liberals are usually Condorcet Runners-Up. Strictly single-peaked preferences mitigate but do not reverse this pattern and imply that the Anti-Plurality Loser is the weaker of the two "extreme" (leading party) candidates, who must be the Condorcet Loser. Finally, profiles with random second preferences display roughly the same pattern as wholly random impartial preferences, and Condorcet relationships with impartial second preferences are almost perfectly determined by anti-plurality status.

The first two rows of Table 11 are consistent with Proposition 5 and the general expectation of association between Condorcet and anti-plurality status. But the third row shows that with survey-based second preferences, Anti-Plurality Winners are mostly Condorcet Runners-Up and Anti-Plurality Runners-Up are mostly Condorcet Winners. This is because the former category is composed almost entirely of Liberals, who are rarely Condorcet Winners but usually Condorcet Runners-Up, while the latter is composed almost entirely of Labor and Conservative candidates, who are almost always either Condorcet Winners or Condorcet Losers. Strict single-peakedness actually mitigates this discrepant pattern because all Anti-Plurality Winners are Liberals and many more of them are Condorcet Winners as well, and only "extreme" (Labour or Conservative) candidates are Anti-Plurality or Condorcet Losers. The random and impartial second preference profiles exhibit the characteristic patterns previously found in Tables 7 and 9.

6. Winners under FPTP, AV, and Coombs

We now directly compare winners under the three voting rules. Having anticipated that Liberal candidates would be helped by a switch to the AV system and helped even more by a switch to the Coombs system, we can now confirm and quantify these expectations.

Table 12 contains six panels, each of which crosstabulates winners under AV and FPTP given survey-based second preferences, one for each election plus one showing the average for all elections. Table 12, and the similar Table 13, display case counts rather than percentages, since it seems more natural to refer to the actual number of seats won by each party, or that shift from one party to another, than to percentages.

The marginal frequencies in each panel show the overall distribution of (English) seats under each electoral system, given survey-based second preferences. Liberals would be consistently helped by a switch to AV, gaining seats in every election. Conservatives would be consistently hurt by a switch to AV, losing seats in every election and barely placing ahead of the Liberals in 1997. Labour would be inconsistently affected by a switch to AV, gaining a substantial number of seats in good Labour years, losing a few seats in 1992, and gaining a few in 2010. While the Conservatives would retain their seat majority in 1992 under AV (in England at least), the 2010 election would produce a "hung parliament" within England (as well as Britain as a whole), from which they won a comfortable majority of seats under FPTP, with Labour as well as the Liberals gaining seats at their expense.[18]

The sum of the off-diagonal cells in each panel of Table 12 indicates the number of constituencies in which AV and FPTP winners differ, which ranges from a low of 22 in 1992 to a high of 69 in 1997 and averages 43.8 over the five elections. This constitutes about 8% of all constituencies but more than one-fifth of the elections without Majority Winners (in which AV and FPTP winners could possibly differ). It is striking that there is not a single instance in which a Liberal candidate who won under FPTP would lose under AV. On the other hand, an average of 23.6 Liberal candidates who lost under FPTP each year would win under AV, with the number of Liberal pickups ranging from 11 in 2005 to 36 in both 1997 and 2010. Of these Liberal gains, more than three-quarters would come at the expense of Conservatives, a proportion that ranges from 50% in 2010 to 97% in 1997. A switch from FPTP to AV would also cause some seats to transfer between the two leading parties, but not in a symmetric fashion — almost all would switch from Conservatives to Labour. Labour would gain an average 19 of seats from the Conservatives each year, ranging from none in 1992 to 33 in 1997. In contrast, Conservatives would gain only six seats in total from Labour, all in the best Conservative year of 1992.

Table 13 crosstabulates winners under Coombs and FPTP in the same manner as Table 12. The basic seat shift patterns are similar to those in the previous table but considerably more pronounced. The marginal frequencies show that the Liberals would be consistently and dramatically helped by a switch to Coombs, gaining a large number of seats in every election. Conservatives would actually gain 10 seats in their best year of 1992, but they would be largely wiped out in 1997 (being pushed far below the Liberals in seat totals and standing only slightly better than the Liberals actually did under FPTP) and would place third in seats in 2001 as well. Labour on average wins just

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[18] The more detailed analysis of Sanders et al. (2011) estimates the 2010 distribution English seats under AV as Conservatives 277, Labour 184, and Liberals 69. The greater success of the Conservatives, and lesser success of Labour and Liberals, in these estimates probably reflects the role of the UK Independence Party. The BES survey data reported in their Table 3 show that most UKIP supporters in England indicated a Conservative second preference, so many of their votes would transfer to Conservatives under AV.

about the same number of seats as under FPTP, better in their best years, about the same in 2005, and doing worse in their bad years — indeed, being pushed into third place in 2010.

The interior cells of Table 13 show that the number of constituencies in which the Coombs winner differs from the FPTP winner is just about double the number in which the AV winner differs from the FPTP winner, both on average and in each election except 2005 (for which the number is still considerably larger). No Liberal candidate who won under FPTP would lose under Coombs (as was true with AV) but the number of Liberal gains under Coombs each year greatly exceeds those under AV, averaging 65 as opposed to 23.6 and ranging from a low of 28 in 1992 to an extraordinary 143 in 2010. As before, the Liberal gains come mainly at the expense of Conservatives but not quite so consistently or disproportionally as in the case of AV (necessarily because the Liberal gains are so much greater). As with AV, Conservatives gain seats from Labour only in 1992 (when a residual tinge of "old Labour" relegated Labour candidates to Anti-Plurality Loser status in over two-thirds of all constituencies). Labour again gains seats from Conservatives in the good Labour years of 1997, 2001, and 2005, and in roughly comparable numbers.

Table 14 crosstabulates AV and FPTP winners given strictly single-peaked preferences, random second preferences, and impartial second preferences. These tables display (total) percentages, rather than case counts, and the comparable table is shown for survey-based second preferences as well. Unsurprisingly, strict single-peakedness enhances Liberal gains from a switch to AV from FPTP, almost entirely at the (further) expense of Conservatives. Also unsurprisingly, given random or impartial second preferences, which eliminate rather than enhance the Liberal second preference advantage, the shift in electoral systems has very little effect on seats won — virtually none in the impartial case.

Table 15 likewise crosstabulates Coombs and FPTP winners given strictly single-peaked preferences, random second preferences, and impartial second preferences. Again unsurprisingly, strict single-peakedness further enhances the already dramatic Liberal gains from a switch to Coombs from FPTP, to the extent that the Liberals are now the leading party with respect to seats won, though they fall short of a majority. Necessarily, these large Liberal gains come at the expense of Labour as well as Conservatives. Once again, given random or impartial second preferences, the shift in electoral systems has very little effect.

7. Condorcet Efficiency

FPTP, AV, and Coombs sometimes elect different candidates, but who is the "best" candidate that ought to be elected? In one sense, each voting rule provides an implicit definition of the best candidate and faithfully elects that candidate. In another sense, our three sets of social choice concepts offer three implicit definitions of the best candidate: the Plurality Winner, the Anti-Plurality Winner, and the Condorcet Winner. FPTP faithfully elects the Plurality Winner, Anti-Plurality Voting (see footnote 4) faithfully elects the Anti-Plurality Winner, and several voting systems (but not AV or Coombs) faithfully elect the Condorcet Winner (if one exists).

The Condorcet Winner probably has the greatest support among social choice theorists as the candidate who ought to be elected (see, for example, Merrill 1988). In any case, it is a common exercise to evaluate voting rules in terms of their *Condorcet efficiency*, i.e., the rate at which they

elect the Condorcet Winner, when one exists. Table 16 reports the Condorcet efficiency of FPTP, AV, and Coombs in all the data we have examined.[19]

With respect to the English data with survey-based second preferences, even FPTP has a high rate of Condorcet efficiency at 89%. But recall that 60% of these elections have Majority Winners and, in these elections, every voting rule is 100% efficient. In the 40% of elections without Majority Winners, Condorcet efficiency falls to just below 72.5%. Table 16 also reports the party affiliation of non-elected Condorcet Winners. As we might expect, almost all failures by FPTP to elect Condorcet Winners occur when Condorcet Winners are Liberals.

We know before examining any data that (given only three candidates) AV cannot be less Condorcet efficient than FPTP. AV fails to elect the Condorcet Winner only when the Condorcet Winner is the Plurality Loser, in which case FPTP also fails. Given the English data with survey-based second preferences, AV has a Condorcet efficiency of 95.8%, which falls to 89.5% in elections without Majority Winners. All Labour and Conservative Condorcet Winners who fail to be elected by FPTP are elected by AV, because they are all Plurality Runners-Up. About half of the Liberal Condorcet Winners who fail to be elected by FPTP are also elected by AV but about half are not, because they are Plurality Losers.

With respect to the English data with survey-based second preferences, Coombs and AV have very similar rates of Condorcet efficiency, but they achieve this in quite different ways: AV by electing all Labour and Conservative candidates who are Condorcet Winners, Coombs by electing all Liberal candidates who are Condorcet Winners. This is because, as shown in Table 4, no Liberal candidates are Anti-Plurality Losers, so every Liberal candidate who is a Condorcet Winner gets into (and wins) a Coombs runoff. On the other hand, as shown in Table 11, some Labour and Conservative candidates are both Condorcet Winners and Anti-Plurality Losers and therefore fail to be elected under Coombs, even though they may be Plurality Winners elected by FPTP.

Strict single-peakedness greatly reduces the Condorcet efficiency of FPTP and AV but increases the Condorcet efficiency of Coombs to 100%. The latter effect is a logical necessity, given Proposition 10. The former effect results because strict single-peakedness makes many more Liberal candidates Condorcet Winners without making them Plurality Winners. Given strict single-peakedness, the few Labour and Conservative candidates who are Condorcet Winners not elected by FPTP given survey-based second preferences no longer fall into the "non-elected Condorcet Winners" category; this is not because they are now elected by FPTP but because they are no longer Condorcet Winners (having been replaced in this status by Liberals). Considering only elections in which there is no Majority Winner, we see that the Condorcet efficiency of FPTP (especially) and AV is (greatly) further reduced. This is because the Liberal is the Condorcet Winner in every such profile but is still very likely not to be the Plurality Winner and quite likely to be the Plurality Loser.

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[19] The percentages reported in Table 16 exclude ballot profiles that produce Condorcet cycles (so there is no Condorcet Winner), as well as profiles that entail ties in Condorcet relationships (found only in the wholly impartial profiles).

Given random and (especially) impartial second preferences, the Condorcet efficiency of all three systems approaches or reaches 100% because, as implied by Proposition 11, plurality, anti-plurality, and Condorcet statuses become virtually indistinguishable. Finally, we also display the Condorcet efficiency of the three voting rules given wholly random and wholly impartial profiles.

8. Proportionality of Votes and Seats

Since FPTP, AV, and Coombs are all methods of electing one candidate in a single-member district (and perhaps entail defensible criteria as to which candidate should be elected given the ballot profile in that district), their performance should be evaluated primarily in terms of voter preferences over the candidates for this single office. But given that FPTP is used to fill parliamentary seats in the U.K. (and elsewhere), critical attention often falls on the (lack of) proportionality between votes and seats in the nation and parliament as a whole. Thus it is in order to compare the performance of AV and Coombs with that of FPTP in this respect.

Table 17 summarizes and compares the proportionality between votes and seats (with respect to England only) under FPTP, AV, and Coombs given survey-based second preferences, with one panel for each of the five years and one showing averages. The top row in each panel reports the percentage of votes won by candidates of each party (not been previously presented or discussed). Below the vote percentages are the seat percentages for each party under each voting rule (previously reported, but not in percentage form, in Tables 12 and 13). The three columns to the right of these seat percentages show the deviations between "% of seats" and "% of votes" for each party and system. Under a proportional electoral system, these deviations would be close to zero.

Clearly all three systems typically produce large deviations from proportionality of votes and seats. It is useful to have some summary measure for comparison between years and voting rules. The most standard one may be the Gallagher (1991) Index of Disproportionality, defined as follows:

$$D = \sqrt{\sum (V_i - S_i)^2 \,/\, 2} \ ,$$

where V_i is the percentage of votes and S_i is the percentage of seats for party i. The last column of Table 17 shows the value of the Gallagher Index for each year and rule.[20]

The Gallagher Index values show that all three systems yield distinctly disproportional results in every election, though AV and (especially) Coombs happen to do notably better in 2010 than in other elections or than FPTP in any election. In contrast, in 1997 and 2001, AV and (especially) Coombs yield particularly disproportional results. Overall, the three systems have very similar index values, FPTP actually being marginally less disproportional than AV and Coombs. However, the

[20] It should be noted that, in the bottom panel showing averages over all elections, the Gallagher Index values are indeed the averages of index values over the five elections, not the (smaller) index values that result when the formula is applied to the average deviations in that panel. (An electoral system that gives highly disproportional results in every individual election might favor different parties in different elections so that, when the index formula is applied to average vote and seat percentages, it appears to be quite proportional.)

three systems produce their similar levels of disproportionality in different ways. FPTP, in typical "majoritarian" fashion, generally favors the leading parties at the expense of the trailing Liberal party and, when one leading party wins a decisive victory, favors that party at the expense of its principal opponent. AV somewhat mitigates the seat penalties imposed on Liberals by FPTP but it also enhances the seat advantage of the winning party (at least when that party is Labour). Coombs has the same effects as AV but in a still more pronounced fashion. Overall, Labour gets the most favorable treatment under all systems.

Finally Table 18 replicates Table 17 with strictly single-peaked preferences. This entails no change for FPTP, since FPTP counts only first preferences, and AV is not much affected either. However, Coombs vastly overcorrects the penalty imposed on the Liberals by FPTP and to a lesser extent by AV and, as a result, becomes even more disproportional. Similar tables are not presented for random or impartial second preferences, since Tables 14 and 15 show that in these circumstances AV and Coombs produce essentially the same seat distributions as FPTP.

9. Concluding Remarks

This essay has presented a comprehensive social choice analysis of datasets derived from recent English general elections in terms of the plurality, anti-plurality, and Condorcet status of Labour, Liberal, and Conservatives candidates in each election, and it has traced out their implications for the operations of the First-Past-the-Post, Alternative Vote, and Coombs voting rules.

Information on second preferences derived from surveys indicates that Liberal candidates have advantages with respect to second preferences that do not translate into electoral success under FPTP. A switch to AV, as advocated by the Liberals, would translate into a modestly larger number of seats for the party. A switch to the Coombs system would translate into a substantially larger number of seats for the Liberals. If, contrary to the survey data, preferences were strictly single-peaked, the effect of a switch to either AV or Coombs would be greater yet — indeed, a switch to the Coombs system would make the Liberals the leading party with respect to parliamentary seat shares, even as it remains in a relatively distant third place with respect to first preferences. On the other hand, if, also contrary to the survey data, the Liberals had no advantage over the other parties with respect to second preferences, a switch from FPTP to either AV or Coombs would have little or no effect on the distribution of seats among the three parties.

This conclusion seems to raise the political question of why the Liberals advocate AV (at least if full STV remains off the table) rather than Coombs. The fundamental answer is probably that Coombs is a social choice rule that has attracted at best modest attention even in the academic and theoretical literature but is almost entirely unknown to politicians and political activists. Moreover, it has serious problems as a practical voting rule, especially in elections with many candidates.[21] And of course, being even more favorable to Liberals than AV, Coombs would engender even more determined opposition from Conservative and Labour politicians.

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[21] The principal problem was identified in footnote 2.

The social choice analysis of these data sets can be extended in several ways. One extension is to examine the extent of vulnerability of the AV and Coombs rules to "monotonicity failure" in these data. Another is to examine the effects of candidate entry and exit and resulting "spoiler problems." I will report on these matters in other papers. A third extension is to consider the related problems of strategy-proofness and manipulation. Of course, no voting rule is strategy-proof given three or more candidates, but FPTP is particularly vulnerable, as it is relatively easy for ordinary voters to recognize when "tactical voting" is advantageous and how to do it. Contrary to the original claims of its enthusiastic advocates, AV (like Coombs) is also vulnerable to strategic manipulation, but the strategic calculations are considerably more complex. It is worth noting that the quantities used to determine Plurality Winner, Runner-Up, and Loser status presumably incorporate whatever tactical voting occurs under FPTP, and that they therefore if anything underestimate true first preference support for Liberal candidates (or, more generally, first preference support for the Plurality Loser, regardless of the partisan identity of that candidate), which would be more accurately revealed under AV and Coombs.[22] From, this point of view, the advantage that Liberals — or, more generally, that a small centrist party — would gain under the AV and Coombs rules may be understated in the analysis presented here.

Appendix: Description of Data Sets

The first four datasets take as first preferences the actual votes cast for the Conservative, Liberal Democrat, and Labour candidates; they differ in how second (and third) preferences are assigned. They include all English constituencies from 1992 through 2010 in which (i) all three major parties ran candidates and (ii) one of these three candidates was elected. The last two datasets are wholly simulated.

Dataset 1: Survey-Based Second Preferences. Second preferences for 1992 through 2005 is taken from Curtice (2009), whose data in turn came from the British Election Studies for 1992 and 1997 and from ICM/BBC (pre-election) surveys for 2001 and 2005. Data for 2010 comes from Table 3 in Sanders et al. (2011), which in turn came from the 2010 British Election Study that (in anticipation of a possible referendum on AV) asked respondents to rank their local candidates on an AV-style ballot. Responses indicating no second preference or a second preference for a party other than Conservative, Liberal or Labour were excluded from calculations displayed in Table 1. For example, Table 1 shows Liberal respondents in 2010 splitting their second preferences 60.9% for Labour and 39.1% for Conservatives; these percentages pertain to all survey respondents who (i) indicated a first preference for the Liberals and (ii) a second preference for either other major party. The data from Curtice are for Britain as a whole, while Sanders et al. provide data broken down by "country" (England, Scotland, and Wales).[23] Since the distribution of second preferences is just about

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[22] This conjecture appears to be supported by the survey data reported in Table 2 of Sanders et al. (2011).

[23] Undoubtedly, second preferences do differ considerably across individual constituencies, but no reliable survey data are available at the constituency level. For a possible way around this problem, see Herrmann

the same in England as in Britain as a whole I used British estimates of second preferences for 2010 as well as the earlier elections.

Dataset 2: Strictly Single-Peaked Preferences. All Conservative and Labour voters are assigned a second preference of Liberal. Liberal voters are assigned the same survey-based second preferences as in Data Set 1.

Dataset 3: Random Second Preferences. In each constituency election, the number of Liberal second preferences assigned to Labour voters was determined by a random draw from a normal distribution with a mean of $L/2$ and a standard deviation of $L/6$, where L is the number of Labour voters, subject to the constraint that the result is non-negative and no greater than less than L, and then rounded to the nearest integer. The remaining Labour voters were assigned Conservative second preferences. The second preferences of Conservative and Liberal voters were determined in like manner.

Dataset 4: Impartial Second Preferences. In each constituency election, the number of Liberal second preferences assigned to Labour voters was determined by a random draw from a normal distribution with a mean of $L/2$ and a standard deviation of $\sqrt{L/8}$, i.e., the normal approximation the binomial distribution with $n = L/2$ and $p = 0.5$ (as if voters were independently flipping fair coins), and rounded to the nearest integer. The remaining Labour voters were assigned Conservative second preferences. The second preferences of Conservative and Liberal voters were determined in like manner.

Dataset 5: Wholly Random Preferences. The wholly random dataset is based on 16,000 simulated elections with an average 40,000 voters each (about the same as the actual constituency elections). Each wholly random ballot profile was generated by drawing the number L of first preferences for the Labour candidate from a normal distribution with a mean of $m = 40{,}000/3$ and a standard deviation of 4000, subject to the constraint that the result is non-negative, and then rounded to the nearest integer. Then the number of such ballots ranking the Conservative candidate second was drawn from a normal distribution with a mean of $L/2$ and a standard deviation of $L/6$, subject to the constraint that the result is non-negative and not greater than L, and then rounded to the nearest integer, with the Liberal candidate ranked second on the remaining ballots. The numbers for the other rankings were determined in like manner.

Dataset 6: Wholly Impartial Preferences. The wholly impartial dataset is based on 16,000 simulated elections with an average 40,000 voters each. Each wholly impartial profile was generated by drawing the numbers for each of the six ballot rankings from a normal distribution with a mean of $m = 40{,}000/6$ and a standard deviation equal to $\sqrt{m/4}$, i.e., the normal approximation of the binomial distribution with $n = m$ and $p = 0.5$, and then rounded to the nearest integer. This simulation procedure essentially implements the "impartial culture" condition commonly used in social choice theory.

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et al. (2012).

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Survey Second Preferences		Year				
First Preference	Second Preference	1992	1997	2001	2005	2010
Labour	Liberal	80.6%	81.2%	79.2%	72.8%	88.0%
	Conservative	19.4%	18.8%	20.8%	27.2%	9.3%
Liberal	Labour	46.2%	74.4%	73.6%	67.5%	60.9%
	Conservative	53.8%	25.6%	26.4%	32.5%	39.1%
Conservative	Labour	10.4%	31.6%	30.9%	28.0%	11.8%
	Liberal	89.6%	68.4%	69.1%	72.0%	88.2%

Table 1 Survey second preferences of Labour, Liberal, and Conservative voters by year

All Second Preferences		1992	1997	2001	2005	2010	All
Majority Winner	Lab	23.1%	50.5%	48.3%	30.6%	16.9%	33.8%
	Lib	0.6%	2.5%	3.2%	3.2%	4.1%	2.7%
	Con	43.7%	7.2%	11.6%	19.2%	35.2%	23.4%
	None	32.7%	39.8%	36.9%	47.1%	43.7%	40.0%
Plurality Winner	Lab	37.1%	62.2%	61.2%	54.5%	36.0%	50.2%
	Lib	1.7%	6.5%	7.6%	8.9%	8.3%	6.6%
	Con	61.2%	31.3%	31.3%	36.3%	55.7%	43.3%
Plurality Runner-Up	Lab	32.9%	17.8%	20.5%	23.0%	26.6%	24.1%
	Lib	30.4%	18.2%	19.1%	31.7%	43.1%	28.5%
	Con	36.7%	63.9%	60.4%	45.4%	30.3%	47.3%
Plurality Loser	Lab	30.1%	19.9%	18.4%	22.6%	37.5%	25.7%
	Lib	67.9%	75.3%	73.3%	59.4%	48.6%	64.9%
	Con	2.1%	4.7%	8.3%	18.0%	13.8%	9.4%
Total		529	527	528	527	531	2642

Table 2 Majority and plurality status of party candidates by year

Survey Second Preferences		1992	1997	2001	2005	2010	All
Anti-Plurality Winner	Lab	0.2%	20.5%	17.8%	12.9%	0.0%	10.3%
	Lib	99.8%	79.5%	82.2%	87.1%	100.0%	89.7%
	Con	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Anti-Plurality Runner-Up	Lab	31.4%	70.6%	65.9%	51.4%	45.4%	52.9%
	Lib	0.2%	20.5%	17.8%	12.9%	0.0%	10.3%
	Con	68.4%	8.9%	16.3%	35.7%	54.6%	36.8%
Anti-Plurality Loser	Lab	68.4%	8.9%	16.3%	35.7%	54.8%	36.9%
	Lib	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	Con	31.6%	91.1%	83.7%	64.3%	45.2%	63.1%
Total		529	527	528	527	531	2642

Table 3 Anti-plurality status of party candidates by year with survey-based second preferences

England 1992-2010		Second Preference Type			
		Survey	Strict SP	Random	Impartial
Anti-Plurality Winner	Labour	10.3%	0.0%	44.2%	50.0%
	Liberal	89.7%	100.0%	16.0%	6.5%
	Conservative	0.0%	0.0%	39.8%	43.4%
Anti-Plurality Runner-Up	Labour	52.9%	55.9%	30.8%	24.4%
	Liberal	10.3%	0.0%	30.8%	28.6%
	Conservative	36.8%	44.1%	38.4%	47.0%
Anti-Plurality Loser	Labour	36.9%	44.1%	24.9%	25.6%
	Liberal	0.0%	0.0%	53.2%	64.9%
	Conservative	63.1%	55.9%	21.8%	9.5%

Table 4 Anti-plurality status of party candidates by second preference assumption

Survey-Based Second Preferences	1992	1997	2001	2005	2010	All
Labour beats Liberal	33.6%	70.0%	69.3%	57.7%	24.1%	50.9%
Labour beats Cons.	36.3%	72.5%	68.4%	61.5%	42.9%	56.3%
Liberal beats Cons.	34.4%	74.2%	66.9%	54.1%	59.3%	57.8%
Labour is CW	33.3%	66.4%	63.8%	55.6%	27.7%	49.3%
Liberal is CW	6.8%	15.4%	11.6%	12.1%	29.8%	15.1%
Conservative is CW	59.2%	18.2%	24.4%	31.7%	42.6%	35.2%
Labour is CL	63.3%	23.9%	26.1%	36.4%	59.1%	41.8%
Liberal is CL	6.0%	11.2%	14.0%	15.7%	0.0%	9.4%
Conservative is CL	29.9%	64.9%	59.7%	47.2%	40.9%	48.5%
Condorcet cycle	0.8%	0.0%	0.2%	0.6%	0.0%	0.3%

Table 5 Condorcet relationships by year with survey-based second preferences

All Preferences	Second Preference Assumption			
	Survey	Strict S-P	Random	Impartial
Labour beats Liberal	50.9%	33.8%	73.6%	73.2%
Labour beats Cons.	56.3%	55.9%	51.5%	51.2%
Liberal beats Cons.	57.8%	76.6%	21.5%	15.0%
Labour is CW	49.3%	33.8%	49.5%	50.1%
Liberal is CW	15.1%	42.8%	6.7%	6.6%
Conservative is CW	35.2%	23.4%	42.7%	43.3%
Labour is CL	41.8%	44.1%	24.8%	25.7%
Liberal is CL	9.4%	0.0%	58.9%	64.8%
Conservative is CL	48.5%	55.9%	15.6%	9.5%
Condorcet cycle	0.3%	0.0%	1.0%	0.0%

Table 6 Condorcet relationships by second preference assumptions

Anti-Plurality by Plurality	PW is			P2 is			PL is		
	APW	AP2	APL	APW	AP2	APL	APW	AP2	APL
Wholly Random Preferences									
All	53.9%	34.5%	11.6%	29.4%	38.1%	32.5%	16.7%	27.4%	55.9%
Wholly Impartial Preferences									
All	56.4%	30.2%	13.5%	30.5%	39.5%	29.9%	13.2%	30.3%	56.6%
Survey-Based Second Preferences									
All	16.9%	72.7%	10.4%	25.5%	14.7%	59.8%	57.6%	12.7%	29.8%
Labour	10.3%	38.8%	1.0%	0.0%	8.2%	15.8%	0.0%	5.5%	20.3%
Liberal	6.6%	0.0%	0.0%	25.5%	3.1%	0.0%	57.6%	0.0%	0.0%
Cons.	0.0%	33.9%	9.4%	0.0%	3.3%	44.0%	0.0%	7.2%	9.5%
Strictly Single-Peaked Preferences									
All	6.6%	89.8%	3.6%	28.5%	8.6%	62.9%	64.9%	1.7%	33.4%
Labour	0.0%	49.9%	0.2%	0.0%	4.2%	19.9%	0.0%	1.7%	24.0%
Liberal	6.6%	0.0%	0.0%	28.5%	0.0%	0.0%	64.9%	0.0%	0.0%
Cons.	0.0%	39.9%	3.4%	0.0%	4.3%	43.0%	0.0%	0.0%	9.4%
Random Second Preferences									
All	70.0%	26.4%	3.6%	20.2%	47.3%	32.5%	9.7%	26.4%	63.9%
Labour	36.8%	12.4%	1.2%	5.4%	11.2%	7.3%	2.3%	7.2%	16.4%
Liberal	3.9%	2.3%	0.5%	5.0%	12.6%	10.9%	6.7%	15.9%	42.2%
Cons.	29.4%	11.7%	2.0%	9.8%	23.5%	14.3%	0.8%	3.3%	5.3%
Impartial Second Preferences									
All	99.5%	0.5%	0.0%	0.5%	98.6%	0.9%	0.0%	1.0%	99.0%
Labour	50.0%	0.2%	0.0%	0.1%	23.9%	0.2%	0.0%	0.3%	25.4%
Liberal	6.4%	0.2%	0.0%	0.1%	28.0%	0.5%	0.0%	0.5%	64.4%
Cons.	43.1%	0.2%	0.0%	0.3%	46.7%	0.3%	0.0%	0.2%	9.2%

Table 7 Anti-plurality by plurality status for all preference assumptions

Condorcet Relation-ships	Wholly Random Preferences	Wholly Impartial Preferences	Survey-Based Second Preferences	Strictly Single-Peaked Preferences	Random Second Preferences	Impartial Second Preferences
PW beats P2	78.7%	75.6%	91.7%	88.2%	95.4%	99.8%
P2 beats PL	70.6%	76.0%	43.5%	33.6%	78.4%	99.5%
PW beats PL	88.3%	90.3%	94.1%	72.0%	99.1%	100.0%

Table 8 Condorcet relationships by plurality status and preference assumption

Condorcet by Plurality	PW is			P2 is			PL is		
	CW	C2	CL	CW	C2	CL	CW	C2	CL
Wholly Random Preferences									
All	78.9%	16.5%	4.6%	16.8%	57.4%	25.8%	4.3%	26.1%	69.9%
Wholly Impartial Preferences									
All	75.1%	21.3%	3.6%	20.5%	59.3%	20.1%	3.5%	21.0%	75.4%
Survey-Based Second Preferences									
All	89.0%	9.5%	1.6%	6.7%	37.9%	55.5%	4.3%	52.8%	42.9%
Labour	47.2%	2.7%	0.2%	2.2%	4.1%	17.7%	0.0%	1.7%	24.0%
Liberal	6.6%	0.0%	0.0%	4.3%	24.3%	0.0%	4.3%	51.1%	9.4%
Cons.	35.2%	6.8%	1.4%	0.2%	9.5%	37.8%	0.0%	0.0%	9.5%
Strictly Single-Peaked Preferences									
All	63.8%	32.6%	3.6%	8.4%	28.6%	62.9%	27.8%	38.8%	33.4%
Labour	33.8%	16.1%	0.2%	0.0%	4.2%	19.9%	0.0%	1.7%	24.0%
Liberal	6.6%	0.0%	0.0%	8.4%	20.1%	0.0%	27.8%	37.1%	0.0%
Cons.	23.4%	16.5%	3.4%	0.0%	4.3%	43.0%	0.0%	0.0%	9.4%
Random Second Preferences									
All	95.6%	4.3%	0.2%	4.3%	74.2%	21.5%	0.2%	21.4%	78.4%
Labour	48.6%	1.7%	0.1%	1.5%	17.8%	4.6%	0.0%	5.7%	20.0%
Liberal	6.0%	0.7%	0.0%	0.7%	20.0%	7.8%	0.2%	13.0%	51.7%
Cons.	41.0%	1.9%	0.1%	2.1%	36.4%	9.1%	0.0%	2.7%	6.7%
Impartial Second Preferences									
All	99.8%	0.2%	0.0%	0.1%	99.4%	0.5%	0.0%	0.5%	99.5%
Labour	50.1%	0.1%	0.0%	0.0%	24.0%	0.1%	0.0%	0.1%	25.6%
Liberal	6.5%	0.0%	0.0%	0.0%	28.3%	0.2%	0.0%	0.3%	64.6%
Cons.	43.2%	0.0%	0.0%	0.1%	47.1%	0.2%	0.0%	0.1%	9.3%

Table 9 Condorcet by plurality status for all preference assumptions

Condorcet Relationships	Wholly Random Preferences	Wholly Impartial Preferences	Survey-Based Second Preferences	Strictly Single-Peaked Preferences	Random Second Preferences	Impartial Second Preferences
APW beats AP2	68.9%	76.2%	29.1%	42.8%	73.2%	99.7%
AP2 beats APL	80.3%	75.7%	91.3%	100.0%	86.0%	99.5%
APW beats APL	90.2%	90.3%	88.0%	100.0%	93.9%	100.0%

Table 10 Condorcet relationships by anti-plurality status and preference assumption

Condorcet by Anti-Plurality	APW is			AP2 is			APL is		
	CW	C2	CL	CW	C2	CL	CW	C2	CL
Wholly Random Preferences									
All	69.3%	26.9%	3.8%	27.4%	47.8%	24.8%	3.4%	15.2%	81.4%
Wholly Impartial Preferences									
All	75.9%	20.5%	3.6%	20.3%	59.0%	20.7%	3.8%	20.5%	75.7%
Survey-Based Second Preferences									
All	25.5%	65.1%	9.4%	69.7%	23.9%	6.4%	4.8%	11.0%	84.2%
Labour	10.3%	0.0%	0.0%	38.5%	7.8%	6.2%	0.7%	0.8%	35.7%
Liberal	15.2%	65.1%	9.4%	0.0%	10.3%	0.0%	-	-	-
Cons.	0.0%	0.0%	0.0%	31.2%	5.8%	0.2%	4.1%	10.2%	48.5%
Strictly Single-Peaked Preferences									
All	42.8%	57.2%	0.0%	57.2%	42.7%	0.0%	0.0%	0.0%	100.0%
Labour	0.0%	0.0%	0.0%	33.8%	22.0%	0.0%	0.0%	0.0%	44.1%
Liberal	42.8%	57.2%	0.0%	0.0%	0.0%	0.0%	-	-	-
Cons.	0.0%	0.0%	0.0%	23.4%	20.7%	0.0%	0.0%	0.0%	55.9%
Random Second Preferences									
All	71.7%	24.6%	3.8%	25.5%	62.0%	12.5%	2.9%	13.4%	83.7%
Labour	37.0%	6.6%	0.9%	12.1%	15.1%	3.5%	1.0%	3.6%	20.4%
Liberal	4.6%	8.4%	2.7%	2.0%	21.3%	7.5%	0.3%	4.0%	49.3%
Cons.	30.1%	9.6%	0.2%	11.4%	25.6%	1.5%	1.7%	5.8%	14.1%
Impartial Second Preferences									
All	99.7%	0.3%	0.0%	0.3%	99.2%	0.5%	0.0%	0.5%	99.5%
Labour	50.0%	0.0%	0.0%	0.1%	24.1%	0.2%	0.0%	0.0%	25.5%
Liberal	6.5%	0.1%	0.0%	0.1%	28.2%	0.2%	0.0%	0.3%	64.6%
Cons.	43.2%	0.2%	0.0%	0.1%	46.8%	0.1%	0.0%	0.2%	9.3%

Table 11 Condorcet by anti-plurality status for all preference assumptions

1992		FPTP Winner			Total
		Lab.	Lib.	Cons.	
AV Winner	Lab.	186	0	0	186
	Lib.	4	9	12	25
	Cons.	6	0	312	318
Total		196	9	324	529

1997		FPTP Winner			Total
		Lab.	Lib.	Cons.	
AV Winner	Lab.	327	0	33	360
	Lib.	1	34	35	70
	Cons.	0	0	97	97
Total		328	34	165	527

2001		FPTP Winner			Total
		Lab.	Lib.	Cons.	
AV Winner	Lab.	321	0	19	340
	Lib.	2	40	17	59
	Cons.	0	0	129	129
Total		323	40	165	528

2005		FPTP Winner			Total
		Lab.	Lib.	Cons.	
AV Winner	Lab.	283	0	19	302
	Lib.	4	47	7	58
	Cons.	0	0	167	167
Total		287	47	193	527

2010		FPTP Winner			Total
		Lab.	Lib.	Cons.	
AV Winner	Lab.	173	0	24	197
	Lib.	18	44	18	80
	Cons.	0	0	254	254
Total		191	44	296	531

Average		FPTP Winner			Total
		Lab.	Lib.	Cons.	
AV Winner	Lab.	258.0	0.0	19.0	277.0
	Lib.	5.8	34.8	17.8	58.4
	Cons.	1.2	0.0	191.8	193.0
Total		265.0	34.8	228.6	528.4

Table 12 AV winners by FPTP winners by year with survey-based second preferences

1992		FPTP Winner			Total
		Lab.	Lib.	Cons.	
Coombs Winner	Lab.	158	0	0	158
	Lib.	13	9	15	37
	Cons.	25	0	309	334
Total		196	9	324	529

1997		FPTP Winner			Total
		Lab.	Lib.	Cons.	
Coombs Winner	Lab.	323	0	45	368
	Lib.	5	34	76	115
	Cons.	0	0	44	44
Total		328	34	165	527

2001		FPTP Winner			Total
		Lab.	Lib.	Cons.	
Coombs Winner	Lab.	321	0	39	360
	Lib.	2	40	48	90
	Cons.	0	0	78	78
Total		323	40	165	528

2005		FPTP Winner			Total
		Lab.	Lib.	Cons.	
Coombs Winner	Lab.	280	0	16	296
	Lib.	7	47	16	70
	Cons.	0	0	161	161
Total		287	47	193	527

2010		FPTP Winner			Total
		Lab.	Lib.	Cons.	
Coombs Winner	Lab.	128	0	0	128
	Lib.	63	44	80	187
	Cons.	0	0	216	216
Total		191	44	296	531

Average		FPTP Winner			Total
		Lab.	Lib.	Cons.	
Coombs Winner	Lab.	242.0	0.0	20.0	262.0
	Lib.	18.0	34.8	47.0	99.8
	Cons.	5.0	0.0	161.6	166.6
Total		265.0	34.8	228.6	528.4

Table 13 Coombs winners by FPTP winners by year with survey-based second preferences

Survey-Based Second Preferences		FPTP Winner			Total
		Lab.	Lib.	Cons.	
AV Winner	Lab.	48.8%	0.0%	3.6%	52.4%
	Lib.	1.1%	6.6%	3.4%	11.1%
	Cons.	0.2%	0.0%	36.3%	36.5%
Total		50.2%	6.6%	43.3%	100.0%

Strictly Single-Peaked		FPTP Winner			Total
		Lab.	Lib.	Cons.	
AV Winner	Lab.	48.0%	0.0%	3.2%	51.2%
	Lib.	1.9%	6.6%	6.5%	15.0%
	Cons.	0.2%	0.0%	33.8%	33.8%
Total		50.2%	6.6%	43.3%	100.0%

Random Second Preferences		FPTP Winner			Total
		Lab.	Lib.	Cons.	
AV Winner	Lab.	48.4%	0.1%	1.6%	50.0%
	Lib.	0.2%	5.9%	0.7%	6.8%
	Cons.	1.6%	0.6%	41.0%	43.2%
Total		50.2%	6.6%	43.3%	100.0%

Impartial Second Preferences		FPTP Winner			Total
		Lab.	Lib.	Cons.	
AV Winner	Lab.	50.1%	0.0%	0.0%	50.1%
	Lib.	0.0%	6.6%	0.0%	6.6%
	Cons.	0.0%	0.0%	43.3%	43.3%
Total		50.1%	6.6%	43.2%	100.0%

Note: there are 4 off-diagonal cases, no more than 1 in any cell

Table 14 AV winners by FPTP winners by second preference assumption

Survey-Based Second Preferences		FPTP Winner			Total
		Lab.	Lib.	Cons.	
Coombs Winner	Lab.	45.8%	0.0%	3.8%	49.6%
	Lib.	3.4%	6.6%	8.9%	18.9%
	Cons.	0.9%	0.0%	30.6%	31.5%
Total		50.2%	6.6%	43.3%	100.0%

Strictly Single-Peaked		FPTP Winner			Total
		Lab.	Lib.	Cons.	
Coombs Winner	Lab.	33.3%	0.0%	0.0%	33.8%
	Lib.	16.3%	6.6%	19.9%	42.8%
	Cons.	0.0%	0.0%	23.4%	23.4%
Total		50.2%	6.6%	43.3%	100.0%

Random Second Preferences		FPTP Winner			Total
		Lab.	Lib.	Cons.	
Coombs Winner	Lab.	47.5%	0.2%	2.3%	50.0%
	Lib.	0.5%	5.7%	1.4%	7.5%
	Cons.	2.1%	0.8%	39.6%	42.4%
Total		50.2%	6.6%	43.3%	100.0%

Impartial Second Preferences		FPTP Winner			Total
		Lab.	Lib.	Cons.	
Coombs Winner	Lab.	50.1%	0.0%	0.0%	50.1%
	Lib.	0.0%	6.6%	0.0%	6.6%
	Cons.	0.0%	0.0%	43.3%	43.3%
Total		50.2%	6.6%	43.3%	100.0%

Note: there are 4 off-diagonal cases, no more than 1 in any cell

Table 15 Coombs winners by FPTP winners by second preference assumption

Survey-Based Second Preferences		FPTP	AV	Coombs
Overall Condorcet Efficiency		89.0%	95.8%	95.2%
Non-elected Condorcet Winner is	Labour	2.2%	0.0%	0.7%
	Liberal	8.6%	4.2%	0.0%
	Conservative	0.2%	0.0%	4.1%
Condorcet Efficiency if no MW		72.5%	89.3%	88.5%
Strictly Single-Peaked Preferences		FPTP	AV	Coombs
Overall Condorcet Efficiency		63.8%	72.2%	100.0%
Non-elected Condorcet Winner is	Labour	0.0%	0.0%	-
	Liberal	36.2%	27.8%	-
	Conservative	0.0%	0.0%	-
Condorcet Efficiency if no MW		9.6%	30.6%	100.0%
Random Second Preferences		FPTP	AV	Coombs
Overall Condorcet Efficiency		95.6%	99.8%	97.0%
Non-elected Condorcet Winner is	Labour	1.5%	0.0%	1.0%
	Liberal	0.8%	0.2%	0.3%
	Conservative	2.1%	0.0%	1.7%
Condorcet Efficiency if no MW		88.9%	99.6%	94.3%
Impartial Second Preferences		FPTP	AV	Coombs
Overall Condorcet Efficiency		99.8%	100.0%	100.0%
Non-elected Condorcet Winner is	Labour	0.1%	-	-
	Liberal	0.1%	-	-
	Conservative	0.1%	-	-
Condorcet Efficiency if no MW		99.6%	100.0%	100.0%
Wholly Random Preferences		FPTP	AV	Coombs
Overall Condorcet Efficiency		78.9%	95.6%	96.6%
Condorcet Efficiency if no MW		76.9%	95.3%	96.6%
Wholly Impartial Preferences		FPTP	AV	Coombs
Overall Condorcet Efficiency		75.8%	95.6%	96.3%
Condorcet Efficiency if no MW		75.8%	95.6%	96.3%

Note: excludes profiles with cycles and pairwise ties.

Table 16 Condorcet efficiency of FPTP, AV, and Coombs for all preference assumptions

Survey-Based Second Preferences			Lab.	Lib.	Cons.	Lab.	Lib.	Cons.	Gal. Index
1992	% of votes		34.4%	19.5%	46.1%	Deviations			
	% of seats	FPTP	37.1%	1.7%	61.2%	+ 2.7%	−17.8%	+ 15.1%	16.6%
		AV	35.2%	4.7%	60.1%	+ 0.8%	−14.8%	+ 14.0%	14.4%
		Coombs	29.9%	7.0%	63.1%	−4.5%	−12.5%	+ 17.0%	15.3%
1997	% of votes		45.8%	18.8%	35.4%	Deviations			Index
	% of seats	FPTP	62.2%	6.5%	31.3%	0.164	−12.3%	−4.1%	14.8%
		AV	68.3%	13.3%	18.4%	+ 22.5%	−5.5%	−17.0%	20.3%
		Coombs	69.8%	21.8%	8.3%	+ 24.0%	+ 3.0%	−27.0%	25.6%
2001	% of votes		43.1%	20.2%	36.6%	Deviations			Index
	% of seats	FPTP	61.2%	7.6%	31.3%	+ 18.1%	−12.6%	−5.3%	16.0%
		AV	64.4%	11.2%	24.4%	+ 21.3%	−9.0%	−12.2%	21.7%
		Coombs	69.8%	17.0%	14.8%	+ 25.1%	−3.2%	−21.8%	23.6%
2005	% of votes		37.6%	24.4%	37.9%	Deviations			Index
	% of seats	FPTP	54.5%	8.9%	36.6%	+ 16.9%	−15.5%	−1.3%	16.2%
		AV	57.3%	11.0%	31.7%	+ 19.7%	−13.4%	−6.2%	17.4%
		Coombs	56.2%	13.3%	30.6%	+ 18.6%	−11.1%	−7.3%	16.2%
2010	% of votes		30.5%	26.4%	43.0%	Deviations			Index
	% of seats	FPTP	36.0%	8.3%	55.7%	+ 5.5%	−18.1%	+ 12.7%	16.1%
		AV	37.1%	15.1%	47.8%	+ 6.6%	−11.3%	+ 4.8%	9.9%
		Coombs	24.1%	35.2%	40.7%	−6.4%	+ 8.8%	−2.3%	7.9%
All	% of votes		38.2%	21.7%	40.1%	Deviations			Index
	% of seats	FPTP	50.2%	6.6%	43.3%	+ 12.0%	−15.1%	+ 3.2%	15.9%
		AV	52.1%	10.9%	37.0%	+ 13.9%	−10.8%	−3.1%	16.7%
		Coombs	50.3%	17.8%	39.1%	+ 12.1%	−3.9%	−1.0%	17.2%

Table 17 Proportionality of FPTP, AV, and Coombs by year with survey-based second preferences

Strictly Single-Peaked Preferences			Lab.	Lib.	Cons.	Lab.	Lib.	Cons.	Gal. Index
1992	% of votes		34.4%	19.5%	46.1%	Deviations			
	% of seats	FPTP	37.1%	1.7%	61.2%	+ 2.7%	−17.8%	+ 15.1%	16.6%
		AV	35.2%	7.2%	57.2%	+ 0.8%	−12.3%	+ 11.1%	9.2%
		Coombs	23.1%	33.3%	43.7%	−11.3%	+ 13.8%	−2.4%	12.7%
1997	% of votes		45.8%	18.8%	35.4%	Deviations			Index
	% of seats	FPTP	62.2%	6.5%	31.3%	+ 16.4%	−12.3%	−4.1%	14.8%
		AV	68.1%	17.1%	14.8%	+ 22.6%	−1.7%	−20.6%	21.5%
		Coombs	50.5%	42.3%	7.2%	+ 4.7%	+ 23.5%	−28.2%	26.2%
2001	% of votes		43.1%	20.2%	36.6%	Deviations			Index
	% of seats	FPTP	61.2%	7.6%	31.3%	+ 18.1%	−12.6%	−5.3%	16.0%
		AV	64.0%	15.2%	20.8%	+ 20.9%	−5.0%	−15.8%	18.9%
		Coombs	48.3%	40.2%	11.6%	+5.2%	+ 20.0%	−25.0%	22.9%
2005	% of votes		37.6%	24.4%	37.9%	Deviations			Index
	% of seats	FPTP	54.5%	8.9%	36.6%	+ 16.9%	−15.5%	−1.3%	16.2%
		AV	55.4%	17.8%	26.8%	+ 17.8%	−6.6%	−11.1%	15.6%
		Coombs	30.6%	50.3%	19.2%	−7.0%	+ 25.9%	−18.7%	23.1%
2010	% of votes		30.5%	26.4%	43.0%	Deviations			Index
	% of seats	FPTP	36.0%	8.3%	55.7%	+ 5.5%	−18.1%	+ 12.7%	16.1%
		AV	33.6%	17.7%	48.8%	+ 3.1%	−8.7%	+ 5.8%	7.7%
		Coombs	16.9%	47.8%	35.2%	−13.6%	+ 21.4%	−7.8%	18.8%
All	% of votes		38.2%	21.7%	40.1%	Deviations			Index
	% of seats	FPTP	50.2%	6.6%	43.3%	+ 12.0%	−15.1%	+ 3.2%	13.8%
		AV	51.2%	15.0%	33.8%	+ 13.0%	−6.7%	−6.3%	11.3%
		Coombs	33.8%	42.8%	23.4%	−4.4%	+ 21.1%	−16.7%	19.3%

Table 18 Proportionality of FPTP, AV, and Coombs by year with strictly single-peaked preferences