A vote counting rule is *responsive to voter* i if there are two vote configurations V and V' such that (i) $V'_i \neq V_i$, (ii) $V'_j = V_j$ for all j other than i, and (iii) $F(V') \neq F(V)$. In words, a vote counting rule is responsive to voter i if there is some configuration of other votes in which i's vote 'makes a difference.' A vote counting rule is *fully responsive* if it is responsive to every eligible voter. Every variant of majority rule is fully responsive.

A voting counting rule is *non-negatively responsive* if, for every pair of vote configurations V and V' such that $V'_i \geq V_i$ (or $V'_i \leq V_i$) for every voter i, it follows that $F(V') \geq F(V)$ (or $F(V') \leq F(V)$, respectively) — that is, in so far as the vote counting rule is responsive, it responds in the 'right' way. Every variant of majority rule is non-negatively responsive.

A vote counting rule is *neutral* if $F(V_1, \ldots, V_n) = -F(-V_1, \ldots, -V_n)$. Put otherwise, a neutral vote counting rule treats the two options m and ϕ symmetrically. Every variant of majority rule (as defined above) is neutral.

A vote counting rule F' is *more resolute* than another rule F if $F'(V) = 0$ implies $F(V) = 0$ but not conversely. If $d < d'$, d-majority rule is more resolute than d'-majority rule, given a sufficently large number n votes. Likewise the relative version of any rule is more resolute than the absolute version of the same rule.

A vote counting rule F is *almost resolute* if it is true that if (i) $F(V) = 0$, (ii) $V'_i \neq V_i$ for any voter i, and (iii) $V'_j = V_j$ for all voters j other i, then $F(V) \neq 0$. This says that the tie result is a 'knife-edge' condition, in that a tie is broken if any voter changes his vote in any way. Simple relative majority rule is almost resolute and is the only variant of majority rule to be so.

A vote counting rule is *positively responsive* if it is both non-negatively responsive and almost resolute. *May's (1952) Majority Rule Theorem* says this: *a vote counting rule is anonymous, neutral, and positively responsive if and only if it is simple relative majority rule*. Henceforth, any unqualified reference to 'majority rule' should be understood as referring to simple relative majority rule.

A vote counting rule is *resolute* if it is never true that $F(V) = 0$. Simple majority rule is resolute in the special case in which (i) no voter abstains *and* (ii) the number n of voters is odd. But, in general, a resolute vote counting rule must violate either anonymity or neutrality. Simple majority rule can be made resolute rule in either of two ways. Anonymity can be (slightly) violated, so that there is a distinguished voter (e.g., a 'chairman') whose vote breaks any tie. (Even this assures resoluteness only if the 'chairman' never abstains.) Or neutrality can be (slightly) violated so that a tie is broken in favor the favored option. The latter is the tie-breaking device most commonly used in parliamentary bodies, with ϕ (rejection of the motion) being the favored alternative. Thus any variant of majority rule can be rendered resolute as follows:

$$F(V) = +1 \quad \text{if and only if } n(+1) \geq d \times n' (\text{ or } d \times n); \text{ and}$$
$$F(V) = -1 \quad \text{otherwise.}$$

The *standard parliamentary vote counting rule* is the resolute variant of simple relative majority rule.

2. VOTING AGENDAS

Under sequential binary procedure of the sort used by committees, a voting *agenda* specifies the 'motions' or 'questions' that are put to a vote and the order in which these votes occur. This is equivalent to saying that the agenda specifies the alternatives that are possible voting outcomes and the particular sequence of votes by which this set is winnowed down to a final outcome.

2.1. Agendas, Alternatives, and Proposals

The *agenda set* A is a finite subset of alternatives that are put before the committee for voting. Often it is natural to use the word 'agenda' to refer simply to this set of alternatives (as was done in the introductory section). But in its more precise sense, the agenda also specifies the *particular sequence of 'questions'* put before the committee for voting, according to which it will arrive at the final outcome. The nature of this sequence of votes, or *agenda structure*, depends in turn on two further matters: the character of the *voting procedure* in use and the *voting order* in which questions are brought up for consideration under this procedure.

The agenda set A is generated by *proposals* made by committee members. These proposals may be designated 'motions,' 'bills,' 'amendments,' 'substitutes,' etc., according to the order in which they are proposed and relevant parliamentary usage. These designations influence the structure of the agenda and, in particular, the order in which alternatives are voted on.

It is crucial to recognize that the set of *alternatives* and the set of *proposals* generally do not coincide. Almost always the set of alternatives is larger than the set of proposals. First, virtually always a committee has available the alternative of 'doing nothing,' which inaction has the effect of maintaining the *status quo* or effecting a reversion to some pre-established alternative (e.g., some level of taxation and/or expenditure). This alternative is always available to the committee as a possible outcome, though typically it is not explicitly proposed. We refer to this distinguished element of A as the *status quo alternative*, which in this section we designate ϕ. Thus, even if just one motion m is proposed, a committee faces a choice, i.e., whether to accept or reject this motion, and two alternatives are on the agenda: (1) to accept the motion (m) and to reject the motion and maintain the status quo (ϕ).

The set of alternatives further exceeds the set of proposals whenever two or more *compatible* proposals are offered — that is, proposals that are literally not *alternatives* to one another, because they are not mutually exclusive. Suppose two compatible bills or amendments to a bill are proposed. These two proposals generate four alternatives: (1) adoption of both, (2) adoption of the first and rejection of the second, (3) rejection of the first and adoption of the second, and (4) rejection of both.

Under parliamentary voting of the sequential binary type, the *number of votes taken* is typically equal to the *number of proposals made*, as each proposal generates a 'question' and there is a vote on each question. The number of votes thus typically falls short of the number of alternatives.

Each sequential binary vote is typically a yes/no vote on the question of accepting some proposal, for example: 'Is the substitute amendment accepted?', 'Is the amendment to the motion accepted?', 'Is the bill as amended adopted?' These yes/no votes implicitly entail paired comparisons between alternatives or sets of alternatives. To demonstrate this and to fix intuitively the notion of a voting agenda, let us consider a variety of examples.

2.2. Agenda Examples

Agenda Example 1 Consider the simplest non-trivial case of parliamentary voting: a motion m is introduced and a single amendment a is proposed. These two proposals generate an agenda set of three alternatives, as follows:

m : adoption of the motion (unamended);
$m + a$: adoption of the motion perfected by the amendment; and
ϕ : rejection of the motion.

In Anglo-American legislatures, yes/no votes would be taken in turn on these questions.

Question 1. Is the amendment accepted?
Question 2. Is the motion (as amended or not) accepted?

If the amendment is accepted, the committee then chooses between $m + a$ and ϕ; if the amendment is rejected, the committee then chooses between m and ϕ. In either event, ϕ remains as a possible outcome after the first vote. Thus, at the first vote, the committee is effectively choosing between m and $m + a$, and the agenda may be summarized by the following implicit pairings of alternatives, where each number refers to a vote:

1. $m + a$ vs. m;
2. winner of 1 vs. ϕ.

Agenda Example 2 We might regard the labelling of the alternatives as m, $m + a$, and ϕ to be somewhat arbitrary. For example, amendment a may add some language to motion m; but it could just as well be that the original motion includes the language and an amendment is offered to strike it. Labelling the status quo alternative is less arbitrary, yet in the long run the nature of the prevailing status quo depends on previous acts of the committee or on external events. Thus Agenda Example 1 (with the three alternatives generically labelled x, y, and z) has three variants according to which alternative has which parliamentary status (original motion, motion as amended, status quo) and, as a result, according to the *order* in which proposals are made and alternatives are voted on.

Agenda Example 2a.
1. x vs. y;
2. winner of 1 vs. z

Agenda Example 2b.
1. x vs. z;
2. winner of 1 vs. y

Agenda Example 2c.
1. y vs. z;
2. winner of 1 vs. x

Agenda Example 3 To complicate matters a bit, suppose that a motion m is proposed, then an amendment a to the motion, and finally an amendment a' to the amendment. (Proposal a' is a *second-order* amendment, as it pertains to the previous amendment, not to the original motion.) These three proposals generate an agenda set of four alternatives, as follows:

- m : adoption of the motion (unamended);
- $m + a$: adoption of the motion perfected by the (unamended) amendment;
- $m + a'$: adoption of the motion perfected by the amended amendment; and
- ϕ : rejection of the motion.

In Anglo-American legislatures, voting would take place as follows.

Question 1. Is the amendment to the amendment accepted?
Question 2. Is the amendment (as amended or not) accepted?
Question 3. Is the motion (as amended or not) accepted?

As before, the agenda may be summarized by the following implicit pairings:

1. $m + a'$ vs. $m + a$;
2. winner of 1 vs. m;
3. winner of 2 vs. ϕ.

Agenda Example 4 Generalizing from these examples suggests the following stylized structure for any agenda set A with generically labelled alternatives $x_0, x_1, \ldots, x_m$, where the subscripts indicate the *order of voting*:

1. x_0 vs. x_1;
2. winner of 1 vs. x_2;

......

k. winner of $k - 1$ vs. x_k;

......

m. winner of $m - 1$ vs. x_m.[3]

Black (1948) (1958) originally considered such a generalized sequential binary procedure, calling it *ordinary committee procedure*. Farquharson

[3] We begin with the subscript 0 because, when we consider the parliamentary status of alternatives (as in Agenda Examples 4a and 4b), it is convenient to designate the status quo x_0.

(1969a) showed that voting on amendments of successively higher orders resulted in an agenda of this type, which he accordingly called *amendment procedure*. Given an agenda with k alternatives, amendment procedure generates $k!/2$ distinct agenda variants, each entailing $k - 1$ votes, according to the order in which the k alternatives are voted on.

Agenda Example 4a If we take account of the parliamentary status of alternatives, we shall adopt the convention that the subscripts designate the order in which alternatives were generated by proposals, so x_0 is the status quo, x_1 is the initial motion, x_2 is the motion with the initial amendment, and so forth. Under standard Anglo-American parliamentary procedure (exemplified by Agenda Examples 1 and 3), amendment agendas are — to use the terminology of Shepsle and Weingast (1984) — *backwards-built*, in that the voting order is the *reverse* of the order in which alternatives were generated and accordingly the reverse of the order of the subscripts. We call such an agenda — particularly one that put the status quo last in the voting order — a *standard amendment agenda*. The generic standard amendment agenda is therefore:

1. x_m vs. x_{m-1};
2. winner of 1 vs. x_{m-2};

......

k. winner of $k - 1$ vs. x_{m-k};

......

m. winner of $m - 1$ vs. x_0.

More generally, we call any agenda in which some alternative is invariably paired with the status quo at the final vote a *standard agenda*.

Agenda Example 4b In less formal settings, amendment agendas may be *forward-built* — that is, the voting order is the same as the order which alternatives were generated. Voting would take place as follows:

Question 1. Shall the status quo be replaced by the first proposed alternative?
Question 2. Shall the (perhaps revised) status quo (i.e. the winner of the first vote) be replaced by the second proposed alternative?
Question 3. And so forth.[4]

[4] Such an agenda resembles what Wilson (1986) calls a *forward-moving agenda*, except that in the latter each vote is taken before the next proposal is made.

The resulting agenda is just that shown for Agenda Example 4, if we now specify that the alternatives are subscripted to reflect the order in which they were generated. Note that Agenda Examples 4a and 4b have identical *structures* but opposite *voting orders*.

Many results in the theory of voting pertain specifically to amendment agendas. But, taking note of an influential article by Ordeshook and Schwartz (1987) which points out that much actual parliamentary voting does not take place under such agendas, we should also consider agendas that do not fit the amendment pattern.

Agenda Example 5 Suppose that a bill b is introduced, an amendment a to the bill is proposed, then a substitute bill s, and finally an amendment a' to the substitute bill. These are the alternatives:

b :	passage of the original bill (unamended);
$b+a$:	passage of the original bill perfected by its amendment;
s :	passage of the substitute bill (unamended);
$s+a'$:	passage of the substitute bill perfected by its amendment; and
ϕ :	rejection of any bill.

In the U.S. Congress and most Anglo-American legislatures, voting would proceed as follows.

Question 1.	Is the amendment to the original bill accepted?
Question 2.	Is the amendment to the substitute bill accepted?
Question 3.	Is the substitute bill (as amended or not) accepted (in place of the original bill, as amended or not)?
Question 4 .	Is the surviving bill (original or substitute, as amended or not) accepted?

The agenda implicitly pairs alternatives in this fashion:

1. b vs. $b+a$;
2. s vs. $s+a'$;
3. winner of 1 vs. winner of 2;
4. winner of 3 vs. ϕ.

This agenda departs from the standard amendment pattern by being *discontinuous*, in that the winner at vote 1 does not enter the following

vote but is set aside for later consideration (at vote 3). This is an example of what Banks (1989) calls *two-stage amendment procedure*, under which an original bill is perfected under amendment procedure, a substitute bill perfected under amendment procedure, a choice is made between the two perfected bills, and finally a choice is made between the surviving bill and the status quo. (There might be additional amendments pertaining to the original and substitute bills. Also a substitute amendment might be proposed, generating a *three-stage amendment agenda* or one two-stage agenda nested within another.)

Agenda Example 6 Suppose a motion m is introduced, an amendment a_1 is moved, and then another amendment a_2 is moved that is *compatible* with the first — that is, both are *first-order* amendments in that they both pertain to the original motion, they are not mutually exclusive, and both may be incorporated into the surviving bill. The alternatives are as follows:

m :	adoption of the original motion (unamended);
$m+a_1$:	adoption of the motion perfected by the first amendment only;
$m+a_2$:	adoption of the motion perfected by the second amendment only;
$m+a_1+a_2$:	adoption of the motion perfected by both amendments; and
ϕ :	rejection of the motion.

These three proposals generate five alternatives, because the two amendments are compatible. (By the same token, a single amendment corresponding to a_1+a_2 might be offered. Then, under standard parliamentary procedure, any committee member could request *division of the question* into its component parts, so that separate votes would be taken on a_1 and a_2.) In Congress and most Anglo-American legislatures, three votes would be taken.[5]

[5] Standard parliamentary procedure would require that this be what Ordeshook and Schwartz (1987) call a *multi-period agenda* — that is, amendment a_1 must be voted up or down before amendment a_2 is formally introduced. It may be reasonable to suppose, however, that it is generally known at the time a_1 is voted on that a_2 will be proposed. Note that a forward moving agenda is also multi-period. In any event, we assume that the whole voting agenda is known before any voting takes place.

Question 1. Is the first amendment accepted?
Question 2. Is the second amendment accepted?
Question 3. Is the motion (as it may be amended) accepted?

In this case, it is not really true that two alternatives are paired at the first vote, but we might summarize the agenda as follows:

1. m vs. $m + a_1$;
2. $\left\{\begin{array}{ll} m \text{ vs. } m + a_2, & \text{if } m \text{ wins at 1} \\ m + a_1 \text{ vs. } m + a_1 + a_2, & \text{if } m + a_1 \text{ wins at 1;} \end{array}\right\}$;
3. winner of 2 vs. ϕ.

The way in which this agenda departs from an amendment agenda is indicated by the way in which the specification of *both* alternatives paired at the second vote is contingent upon the result of the first vote (though the agenda is *continuous* — in either contingency, the winner of the first vote enters the second vote). Directly related is the fact that this agenda is *incomplete* — that is, it is possible (in this case, certain) that some alternative never enters the voting (i.e., $m+a_2$ if $m+a_1$ wins the first vote, and $m+a_1+a_2$ if m wins the first vote). For this reason, the number of votes falls *two* short of the number of alternatives. Precisely because this agenda is incomplete, the informal specification of the agenda in terms of pairwise votes is not entirely appropriate. We shall return to this point in the next subsection, where we present an analytical device that both identifies and sidesteps this ambiguity.

Agenda Example 7 Incompleteness arises more profoundly if we consider a type of agenda commonly used in informal voting bodies, which was employed in Plott and Levine's (1976) pioneering experimental study of agenda influence on voting outcomes. To use their introductory example, suppose that a group must decide what kind of banquet to give and that two questions have been raised.

Question 1 : Shall the dress be formal or informal?
Question 2 : Shall the cuisine be French or Mexican?

These two questions generate four alternatives:

x : formal dress, French cuisine;
y : informal dress, French cuisine;
z : formal dress, Mexican cuisine; and
v : informal dress, Mexican cuisine.

Each question, though not yes/no, is binary, and a sequential binary agenda results. Given two questions, the agenda has two variants, according to which question is put to a vote first. In general, the raising of k dichotomous questions generates $k!$ variants of an incomplete agenda with 2^k alternatives but only k votes.

What Ferejohn (1975; also see Kramer, 1972) calls a *bill-by-bill* agenda can arise in a formal parliamentary setting and is structurally equivalent to an agenda of the Plott–Levine type. Consider a set of bills, each of which may be independently passed or defeated. An alternative is complete specification of bills defeated and passed.[6] (Alternatively, an 'omnibus' bill corresponding to a particular alternative might be proposed at the outset, followed by a request for division of the question.) As illustrated by the dress/cuisine example, such agendas can be specified by listing the 'questions' in order but, because they are profoundly incomplete, they cannot be specified by listing pairwise votes between alternatives.

A generalization of a bill-by-bill agenda is what is commonly called *issue-by-issue* voting, in which each bill (or issue) may have a multiplicity (potentially even an infinite number) of positions. A position is chosen (perhaps under amendment procedure) on the first issue, then on the second, and so forth.

Agenda Example 8 All agendas we have considered thus far are *uniform*, in that the same number of votes is taken regardless of the results of earlier votes. But agendas need not be uniform. The most prominent example results from what Farquharson (1966 and 1969a) calls *successive procedure*. For example, suppose a motion m, an amendment a, and a second-order amendment a' are proposed, as in Agenda Example 3. Instead of following amendment procedure, the first vote might be on the 'question of principle' of whether to change the status quo; if this question fails, no further votes are taken. Only if the question of principle is decided favorably are more specific amendment considered. Thus the committee might vote on these questions.

Question 1 : Should the status quo ϕ be modified?
Question 2 : If so, should the original motion m be amended?

[6] This is certainly a multi-period agenda. Note that compatible first-order amendments (as in Agenda Example 6) generate an 'amendment-by-amendment' agenda 'standardized' by a final vote against the status quo.

Question 3 : If so, should the original amendment a be modified by the second-order amendment a'?

Agenda Example 9 Alternatively, the committee might vote on these questions.

Question 1 : Should the amended motion modified by the second-order amendment a' be accepted?
Question 2 : If not, should the amended motion a be adopted?
Question 3 : If not, should the original motion m be accepted?

We call this a *sequential* agenda, though the term 'successive' is often applied to this kind of agenda as well. This is because Agenda Examples 8 and 9 are *structurally equivalent*; in both, alternatives are voted up or down in a fixed order, and voting terminates once an alternative is voted up. The difference is that, in Agenda Example 8, the voting order follows the order in which alternatives were generated by proposals, while in Agenda Example 9, the voting order is the reverse of this. Put otherwise, Agenda Example 8 is forward-built, while Agenda Example 9 is backward-built.[7] Agendas of the latter type are commonly used in continental European legislatures (Bjurulf and Niemi, 1982; Rasch, 1987).

2.3. Agenda Structures

It is useful to have some general method for representing the structure of sequential binary voting agendas. Farquharson (1956a, 1969a) devised a convenient tool for describing a wide class of voting procedures, including all those of the sequential binary type, which we call the *agenda tree*.[8]

[7] Ordeshook and Schwartz (1987: p. 182) observe that agendas similar to Agenda Example 9 may occur even in an Anglo-American legislature, when it is 'obliged to take some action (to adopt a budget, say) and so must keep voting on proposed actions until one passes.' Farquharson (1966) first introduced the term 'successive procedure,' which he interpreted in the manner of Agenda Example 8. In his book (1969a) and in much voting theory, the voting order is taken as given and is not related to the parliamentary status of alternatives, so the distinction between Agenda Examples 8 and 9 disappears; the term 'successive,' may then be employed to cover the agenda structure common to both.

[8] The following generally follows the formalization presented by McKelvey and Niemi (1978); similar but less formal descriptions are given in Miller (1977b), McKelvey (1981), Dummett (1984: Chapter 3), and elsewhere, as well as by Farquharson. Much of the terminology is adapted from Ordeshook and Schwartz (1987), but their method of representing an agenda structure differs from the more standard one presented here (see footnote 11).

A sequential binary voting process has a number of possible *outcomes*, each corresponding to selection of a particular alternative from the agenda. At the outset, the whole agenda set A constitutes the set of possible outcomes. As a result of a sequence of binary votes, alternatives are eliminated as possible outcomes, until but one remains — the actual voting outcome. The agenda tree graphically depicts all possible ways in which the initial set of alternatives may be narrowed down to a single outcome.

Intuitively, a 'tree' is a branching structure, with a single starting point and many end points. More formally a tree is a type of *directed graph*, i.e., a finite set V of *nodes* together with *directed lines* (arrows) between certain *ordered pairs* of nodes. A directed graph is *asymmetric* if, for any pair of nodes v and v', an arrow from v to v' precludes an arrow from v' to v. A directed graph is *complete* if, for any pair of nodes v and v', there is an arrow from v to v' or from v' to v; otherwise it is *incomplete*. A *path* from node v to node v' is a sequence of two or more distinct nodes, beginning with v and ending with v' such that there is an arrow from each node to the following node; v' is *reachable* from v if there is a path from v to v'. A *tree* is an asymmetric directed graph such that: (1) there is a unique *initial node* with no incoming arrows; and (2) there is at most one path from one node to another. It follows that there is exactly one path from the initial node to each other node. The *order* of node v is the number of steps in this path. If there is an arrow from v to v', we say v' *follows* v and call the arrow a *branch*.

The following points derive straightforwardly (Harary, Norman and Cartwright, 1965: pp. 283–286). In a tree: (1) every other node is reachable from the initial node and this is true of no other node; (2) every node, other than the initial node, follows exactly one other node; and (3) there is a non-empty subset of nodes, called *terminal nodes*, from which no other node is reachable.

A *binary tree* is a tree such that exactly *two* nodes follow each non-terminal node. In a binary tree, the two nodes following a non-terminal node v can be denoted v_0 (following via the 'left-hand' branch) and v_1 (following via the 'right-hand' branch).

A *binary agenda tree* is defined by an agenda set A of alternatives, a binary tree with at least as many terminal nodes as there are alternatives in A, and a function Γ which assigns a one-element subset of A to each terminal node so that (i) every one-element subset of A is assigned to at least one terminal node and (ii), for every pair of terminal nodes v_0 and v_1 following the same node v, $\Gamma(v_0)$ and $\Gamma(v_1)$ are distinct.

In an agenda tree, each non-terminal node represents the occasion for a vote and is called a *decision node*. Each terminal node represents the final outcome of a particular sequence of votes and is called an *outcome node*. A decision node such that *both* following nodes are outcome nodes is called a *final* decision node.

Consider Agenda Example 1, the agenda tree for which is depicted in Figure 1. The initial node v represents the first vote (on the question accepting the amendment). The two nodes of order 1 represent the second vote on the question of adopting the motion, in the event the amendment fails (v_0) or succeeds (v_1). (Throughout, we follow the convention that the left-hand branch of an agenda tree is followed in the event the question put to a vote fails and the right-hand branch is followed in the event the question succeeds.) The four terminal nodes represent the ends of the four distinct *voting paths* and each entails a particular outcome. Following the indexing convention just noted, the assignment function Γ is as follows:

$$\Gamma(v_{00}) = \{\phi\}; \qquad \Gamma(v_{10}) = \{\phi\}; \text{ and}$$
$$\Gamma(v_{01}) = \{m\}; \qquad \Gamma(v_{11}) = \{m+a\}.$$

Note that two votes are taken in any case, so all paths from the initial node to a terminal node are of the same length. Though there are only three alternatives, the tree has four outcome nodes. This occurs because the outcome $\{\phi\}$ can be reached by two distinct paths: rejection of the amendment followed by rejection of the motion, and acceptance of the amendment followed by rejection of the motion. Figures 2–8 show agenda trees for other Agenda Examples. (While in Figure 1 the nodes are labelled generically, in Figures 2–8 nodes are labelled by their 'reachable sets,' as defined just below.)

Associated with each node v of an agenda tree over A is some subset $\Gamma(v)$ of *outcomes reachable from* v. At any outcome node v, $\Gamma(v)$ is the one-element set given by the assignment function Γ discussed above. At any decision node v, $\Gamma(v)$ is a multi-element set defined as follows: an alternative x belongs to $\Gamma(v)$ if and only if there is some outcome node v' such that v' is reachable from v and $\Gamma(v') = \{x\}$. Two consequences follow immediately from this definition: (1) for the initial node v^*, $\Gamma(v^*) = A$; and (2) at for any pair of nodes v_0 and v_1 following node v, the union of $\Gamma(v_0)$ and $\Gamma(v_1)$ coincides with $\Gamma(v)$. For the decision nodes in Figure 1,

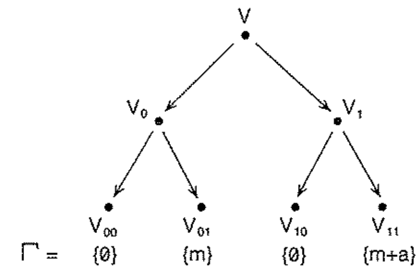


FIGURE 1 An Amendment Agenda Tree with a Single Amendment (Agenda Example 1)

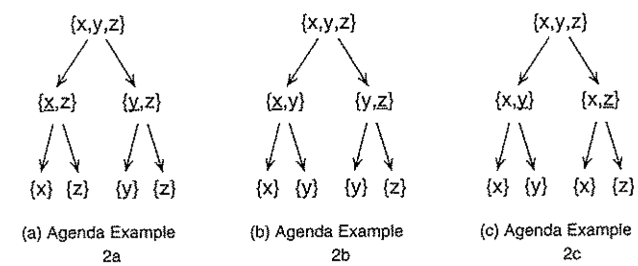


FIGURE 2 Amendment Agenda Tree Variants with Three Alternatives (Agenda Example 2)

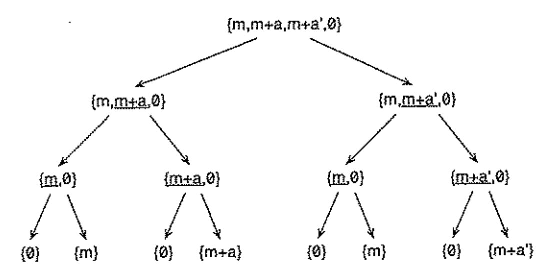


FIGURE 3 An Amendment Agenda Tree with a Second-order Amendment (Agenda Example 3)

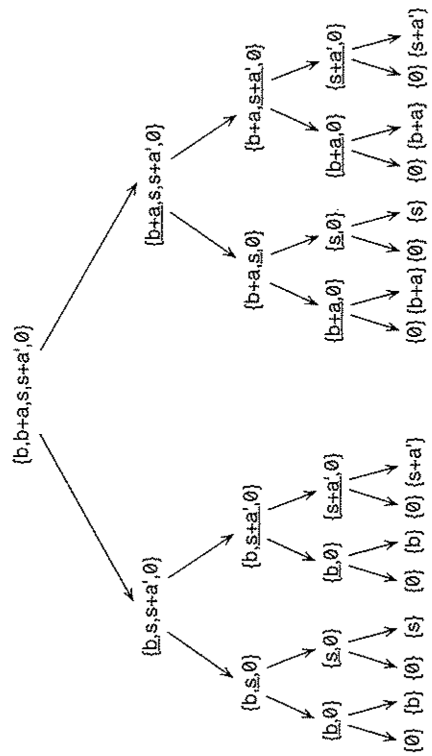


FIGURE 4 An Agenda Tree with a Substitute Bill (Agenda Example 5)

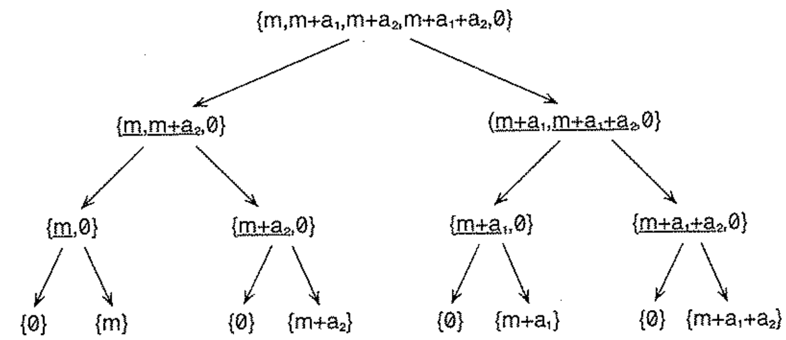
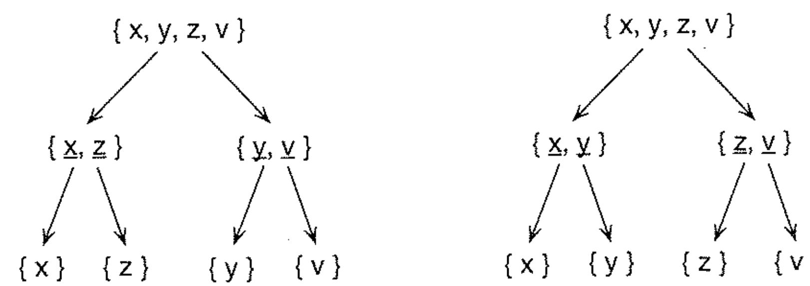


FIGURE 5 An Agenda Tree with Two First-order Amendments (Agenda Example 6)



(a) Dress Voted On First

(b) Cuisine Voted On First

FIGURE 6 Uniform Partition (Plott-Levine/Bill-by-Bill) Agenda Tree Variants (Agenda Example 7)

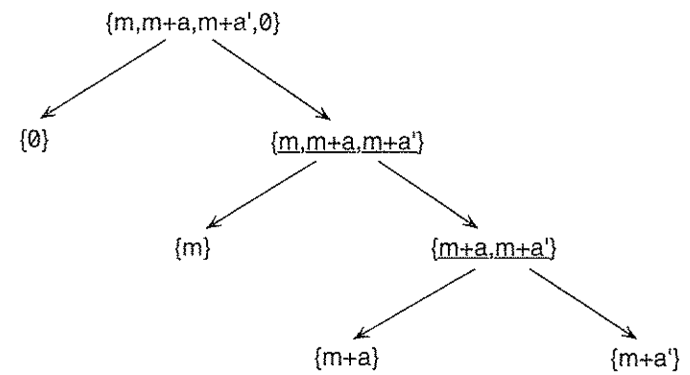


FIGURE 7 A Successive Agenda Tree (Agenda Example 8)

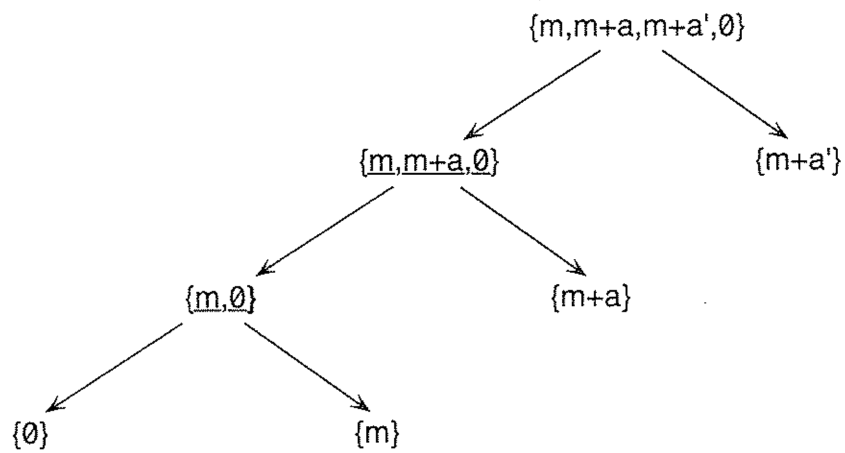


FIGURE 8 A Sequential Agenda Tree (Agenda Example 9)

these *reachable sets* are: $\Gamma(v) = \{m, m+a, \phi\}$, $\Gamma(v_0) = \{m, \phi\}$, and $\Gamma(v_1) = \{m+a, \phi\}$. Reachable sets identify nodes in Figures 2–8 and subsequent agenda diagrams.

Consequence (2) above implies that each of $\Gamma(v_0)$ and $\Gamma(v_1)$ is a subset of $\Gamma(v)$. Here (and following Farquharson, 1969a, p. 11) we make the additional assumption that each is a *proper* subset. Substantively, this means we restrict our attention to *non-repetitive* agendas, i.e., those under which

the effect of every vote is to eliminate *at least one* alternative as a possible outcome.[9]

For any decision node v, let $\Gamma^*(v)$ be the intersection of $\Gamma(v_0)$ and $\Gamma(v_1)$. Substantively, $\Gamma^*(v)$ is the set of possible outcomes before vote v is taken that will remain as possible outcomes after vote v is taken, *regardless of the result of the vote*. We may thus characterize $\Gamma^*(v)$ as the set of alternatives that are *unchallenged* at v. All alternatives in $\Gamma(v)$ and not in $\Gamma^*(v)$ are *challenged* at v. The set of alternatives challenged at v is partitioned into two disjoint *surviving* sets $\Gamma'(v_0)$ and $\Gamma'(v_1)$ at the following nodes. Put otherwise, $\Gamma'(v_0)$ is the subset of alternatives in $\Gamma(v_0)$ that do not also belong to $\Gamma(v_1)$, and likewise for $\Gamma'(v_1)$. ($\Gamma'(v)$ is not defined at the initial node v.) Substantively, $\Gamma'(v_0)$ is the set of alternatives that were challenged at vote v but still survive as possible outcomes at v_0, and likewise for $\Gamma'(v_1)$. For the non-initial decision nodes in Figure 1, the surviving sets are $\Gamma'(v_0) = \{m\}$ and $\Gamma'(v_1) = \{m+a\}$. Figures 2–8 show the sets $\Gamma'(v)$ at each non-initial decision node v by underlining the elements of $\Gamma(v)$ that belong to $\Gamma'(v)$.

Now we can more formally define various properties of sequential binary agendas. An agenda is *uniform* if all of its outcome nodes are of the same order; otherwise, an agenda is *non-uniform*. Substantively, under a uniform agenda a fixed number of votes is taken, whereas under a non-uniform agenda the result of an earlier vote may determine whether a later vote is taken. Of the agendas we have considered, only the successive and sequential types (Agenda Examples 8 and 9) are non-uniform.

An agenda is *complete* if, for any node v, the number of elements in its reachable set $\Gamma(v)$ is no less than $k - h$, where k is the size of the agenda set and h is he order of node v; otherwise, an agenda is *incomplete*. Substantively, under a complete agenda no more than one alternative can be eliminated as a possible outcome at any vote, whereas under an incomplete agenda two or more alternatives may be eliminated. In an agenda that is both complete and non-repetitive, the number of elements in $\Gamma(v)$ is precisely $k - h$; such an agenda is necessarily uniform, every outcome node being of order $k - 1$, and exactly one alternative is eliminated as a possible outcome at every vote. Among the agendas we have considered, amendment agendas (Agenda Examples 1–4) and their two-stage (or multi-stage) variants (Agenda Example 5) are complete, and the others are incomplete.

[9] This means we do not consider agendas that include preliminary votes (the results of which can be reversed by subsequent votes) or procedural votes.

An agenda is *continuous* if, at every decision node v, whenever x belongs to $\Gamma'(v)$, x also belongs to either $\Gamma'(v_0)$ or $\Gamma'(v_1)$; otherwise, an agenda is *discontinuous*. Substantively, under a continuous agenda any alternative that is once challenged continues to be challenged as long as it survives or until it becomes the voting outcome. Of the agendas we have considered, only two-stage (or multi-stage) amendment agendas (Agenda Example 5) are discontinuous.

An agenda is *symmetric* if the assignment function Γ works in a symmetric fashion in this sense: at every decision node v, (i) $|\Gamma(v_0)| = |\Gamma(v_1)|$ (where $|S|$ is the number of elements in the set S) and (ii) the subagenda beginning at v_0 is equivalent to the subagenda beginning at node v_1 if the elements of $\Gamma'(v_1)$ replace those of $\Gamma'(v_0)$ or *vice versa*. Put more substantively, under a symmetric agenda the result of a given vote determines only *what* alternatives survive as possible outcomes, not the *structure* of the remaining agenda. A symmetric agenda is necessarily uniform. All uniform agendas we have considered are symmetric. An example of a non-symmetric agenda is provided by what Banks (1989) calls a *two-stage conditional amendment agenda*. In such an agenda, the voting order on the substitute and its variants is conditioned on which variant of the original bill has survived.

A *partition agenda* is an agenda such that, at every decision node v, the sets $\Gamma(v_0)$ and $\Gamma(v_1)$ constitute a partition in $\Gamma(v)$. Given a partition agenda, $\Gamma^*(v)$ is empty and $\Gamma'(v) = \Gamma(v)$ for every non-initial node v. In words, every reachable alternative is challenged at every vote. Given more than two alternatives, partition agendas are necessarily incomplete. Successive (or sequential) and bill-by-bill (or Plott–Levine) agendas are of this type, as are agendas that arise out of a request to divide the question. A *uniform partition agenda* is necessarily symmetric, and any such agenda can represent a bill-by-bill (or Plott–Levine) agenda, given an appropriate labelling of alternatives. However, once alternatives are labelled, only a subset of such agendas are admissible bill-by-bill agendas.[10]

More or less the polar opposite of a partition agenda is a *pairwise agenda*, i.e, a binary agenda such that, for every non-initial node v, the surviving set $\Gamma'(v)$ is a one-element set. At each decision node, therefore, two challenged alternatives are paired for a vote. The informal specification of agendas

[10]For example, given the labelling of alternatives in Agenda Example 7, the two admissible variants are shown in Figure 6; the inadmissible variant has $\Gamma(v_0)=\{x,v\}$ and $\Gamma(v_1)=\{y,z\}$. Given three or more questions, 'conditional' bill-by-bill agendas can also be devised, such that the order in which later questions are taken up depends on the result of earlier votes.

presented in the previous subsection (in terms of 'x vs. y' or 'winner at vote 2 vs. z') is therefore fully adequate for pairwise agendas but entailed problems (noted in connection with Agenda Examples 6 and 7) for non-pairwise agendas.[11] Every pairwise agenda is complete and every complete non-repetitive agenda is pairwise. Therefore every pairwise agenda is uniform, invariably requiring exactly $k-1$ votes. Amendment agendas are pairwise, as are their discontinuous (two-stage or multi-stage, conditional or unconditional) variants.

Finally, a *standard* agenda is one in which at every outcome node follows a final decision node v such that $\Gamma(v)$ is a two-element set $\{x, x_0\}$, where x_0 is the status quo. Put substantively, a standard agenda invariably entails a final vote against the status quo. Backward-built amendment agendas are standard but forward-built ones are not. Successive, sequential, and bill-by-bill agendas are not standard. An agenda with compatible amendments (such as Agenda Example 6) is in effect a bill-by-bill agenda 'standardized' by a final vote against the status quo.

An agenda tree does not fully specify a voting procedure, for it does not indicate how votes are cast and counted at each node to determine which following node the voting process will reach. A complete specification of a sequential binary voting procedure, therefore, is a binary agenda tree in conjunction with a vote counting rule assigned to each decision node of the tree. Unless otherwise noted, we assume that the standard parliamentary vote counting rule (the resolute variant of relative majority rule) is assigned to every decision node.[12]

3. VOTER PREFERENCES

Having considered various types of voting agendas, we now turn to the preferences of committee members. In this section we briefly review standard notation and assumptions, introduce several rather natural restrictions

[11]Ordeshook and Schwartz (1987) in effect define an agenda as a binary agenda tree together with a function that assigns a unique alternative to each non-initial node. The substantive supposition behind their definition is that all binary agendas are effectively pairwise, but this supposition seems hard to justify.

[12]In general, different vote counting rules can be assigned to different decision nodes of an agenda tree. An example is provided by voting on treaties, constitutional amendments, or other special legislation in the U.S. Congress, where a two-thirds majority is required on final passage but earlier votes on proposed amendments are taken on the basis of simple majority rule.