THE 2016 ELECTION INVERSION (AND THE 2020 NEAR-INVERSION) IN HISTORICAL AND THEORETICAL PERSPECTIVE

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Abstract

- This presentation examines the 2016 Electoral College inversion (and 2020 near-inversion) in light of
  - the history of presidential elections and
  - various theoretical and methodological approaches for analyzing the causes and expected frequency and direction of inversions.
- In so doing, it
  - summarizes, updates, and extends my 2012 paper (based on historical data) “Election Inversions by the U.S. Electoral College” (in Dan S. Felsenthal and Moshé Machover, eds., *Electoral Systems: Paradoxes, Assumptions, and Procedures*, Springer), and
  - summarizes and extends my 2015 PCS presentation (based on simulations) “Election Inversions by Variants of the U.S. Electoral College”.
- These are available at:
  - **https://userpages.umbc.edu/~nmiller/RESEARCH/218103_1_En_4.pdf**
  - **https://userpages.umbc.edu/~nmiller/ELECTINVBYVARIANTEC.PCS2015.pptx**

The 2016 Presidential Election

- The 2016 U.S. presidential election was surprising [shocking?] in many ways.
- One way was that the Electoral College produced
  - the second 'election inversion' within five elections, but
  - only the fourth such inversion in U.S. electoral history.
- The Electoral College produces an *election inversion* when
  - the presidential candidate who wins the most popular votes nationwide nevertheless
  - fails to win the most electoral votes, and therefore loses the election.
- Other terms such as 'reversal of winners,' 'wrong winner,' 'divided verdict,' and 'misfire'' are also used to describe this phenomenon.
  - The terms 'compound majority paradox' and 'referendum paradox' are commonly used in European political science and social choice theory.
- The same phenomenon occurs in FPTP parliamentary systems such as
  - U.K. (e.g., 1951) and
  - Canada (e.g., 2019),

  when the party whose candidates poll the most votes nationwide fails to win the greatest number of parliamentary seats.
  - Also in elections for the U.S. House or Representatives (e.g., 2012) and state legislatures.

The 2016 Inversion

2016	*Popular Vote Percent*	*Two-Party Popular Vote Percent*	*Electoral Votes*
Clinton (D)	48.02	51.11	233 (43.31%)
Trump (R)	45.93	48.89	305 (56.69%)
Other	6.05	—	—
Total	100.00	100.00	538

- What was distinctive about the 2016 inversion is that
  - Clinton won the popular vote by more than a bare margin (>2%), while
  - at the same time Trump won a substantial electoral vote majority.
- Such a combination has not previously occurred.

Note: Electoral vote totals shown here and throughout are based on the "standard operation" of the Electoral College (in particular, winner-take-all in every state and no faithless electors).

The 1876 Inversion

1876	*Popular Vote Percent*	*Two-Party Popular Vote Percent*	*Electoral Votes*
Tiden (D)	50.92	51.53	184 (49.86%)
Hayes (R)	47.92	48.47	185 (50.14%)
Other	1.16	—	—
Total	100.00	100.00	369

- In 1876 Tilden's relative popular vote margin (~3%) was larger than Clinton's but the electoral vote split was as close as possible.
- Moreover, 1876 is the only inversion in which the electoral vote loser won an absolute majority of the total popular vote.
- All this takes official state-by-state vote counts at face value despite the fact that
  - fraud and intimidation were clearly rampant in several states, and
  - vote counts were bitterly disputed in the same states, and
  - a special Electoral Commission was appointed to resolve the disputes and it awarded every contested electoral vote to Hayes. (Had it done anything else, there would have been no inversion.)

The 1888 Inversion

1888	*Popular Vote Percent*	*Two-Party Popular Vote Percent*	*Electoral Votes*
Cleveland (D)	48.63	50.41	168 (41.90%)
Harrison (R)	47.80	49.59	233 (58.10%)
Other	3.57	—	—
Total	100.00	100.00	401

- In 1888
  - Harrison's relative electoral vote margin was somewhat greater than Trump's in 2016 but
  - the popular vote margin was substantially smaller than Clinton's in 2016 (< 1%).

The 2000 Inversion

2000	*Popular Vote Percent*	*Two-Party Popular Vote Percent*	*Electoral Votes*
Gore (D)	48.38	50.27	267 (49.63%)
Bush (R)	47.87	49.73	271 (50.37%)
Other	3.75	—	—
Total	100.00	100.00	538

- In 2000, both the popular vote and electoral vote margins were very small.
- Moreover, there is (presumably) about a 50% chance that the inversion would not have occurred if there had been a state-wide recount in Florida.

An Inversion in 1824?

1824	*Popular Vote Percent*	*Electoral Vote*	*House "Runoff"*	
			Representatives	*Delegations*
Jackson	41.3	99	71	7
J. Q. Adams	30.9	84	87	13
Clay	13.0	37	—	—
Crawford	11.2	41	54	4
Other	3.6	0	—	—
Total	100.0	261	212	24

- 1824 is sometimes counted as an inversion.
- But this is a misclassification because Jackson won a plurality of both popular and electoral votes.
- But since Jackson won less than a majority of electoral votes, there was a "runoff" election in the House Representatives, which elected Adams.
  - While voting was winner-take-all within state delegations, there was no inversion in the House either.
- *Note*: in six states the legislature appointed electors (so there was no popular vote for president).
  - Also, in six states electors were elected by district, not at-large.

An Inversion in 1960?

1960	*Popular Vote Percent*	*Two-Party Popular Vote Percent*	*Electoral Vote*
Kennedy (D)	49.72	50.09	303
Nixon (R)	49.55	49.91	219
Independent Electors	0.17	—	15
Others	0.56	—	0
Total	100.00	100.00	537

- A better case for a fifth inversion could be made for 1960, since the popular vote in Alabama (and therefore also nationally) was indeterminate.
- There is no good way of establishing the "popular vote for president" in Alabama in 1960, because
  - electors ran as individuals on an at-large basis (not on general tickets), and
  - while all 11 Democratic elector candidates won,
    - they had somewhat different individual vote totals, and
    - only 5 were pledged to (and did vote for) the national ticket, while
    - 6 were unpledged (and ultimately cast electoral votes for Byrd).
- In any event, a national recount might have made Nixon the popular vote winner.

A Possible Inversion in 1880

1880	*Popular Vote Percent*	*Two-Party Popular Vote Percent*	*Electoral Votes*
Garfield (R)	48.30	50.047	213 (57.72%)
Hancock (D)	48.21	49.953	156 (42.28%)
Other	3.49	—	—
Total	100.00	100.000	369

- The popular vote was so closely split that a national recount might well have given Hancock a popular plurality (without changing the electoral vote count).

1860: A Massive but Counterfactual Inversion

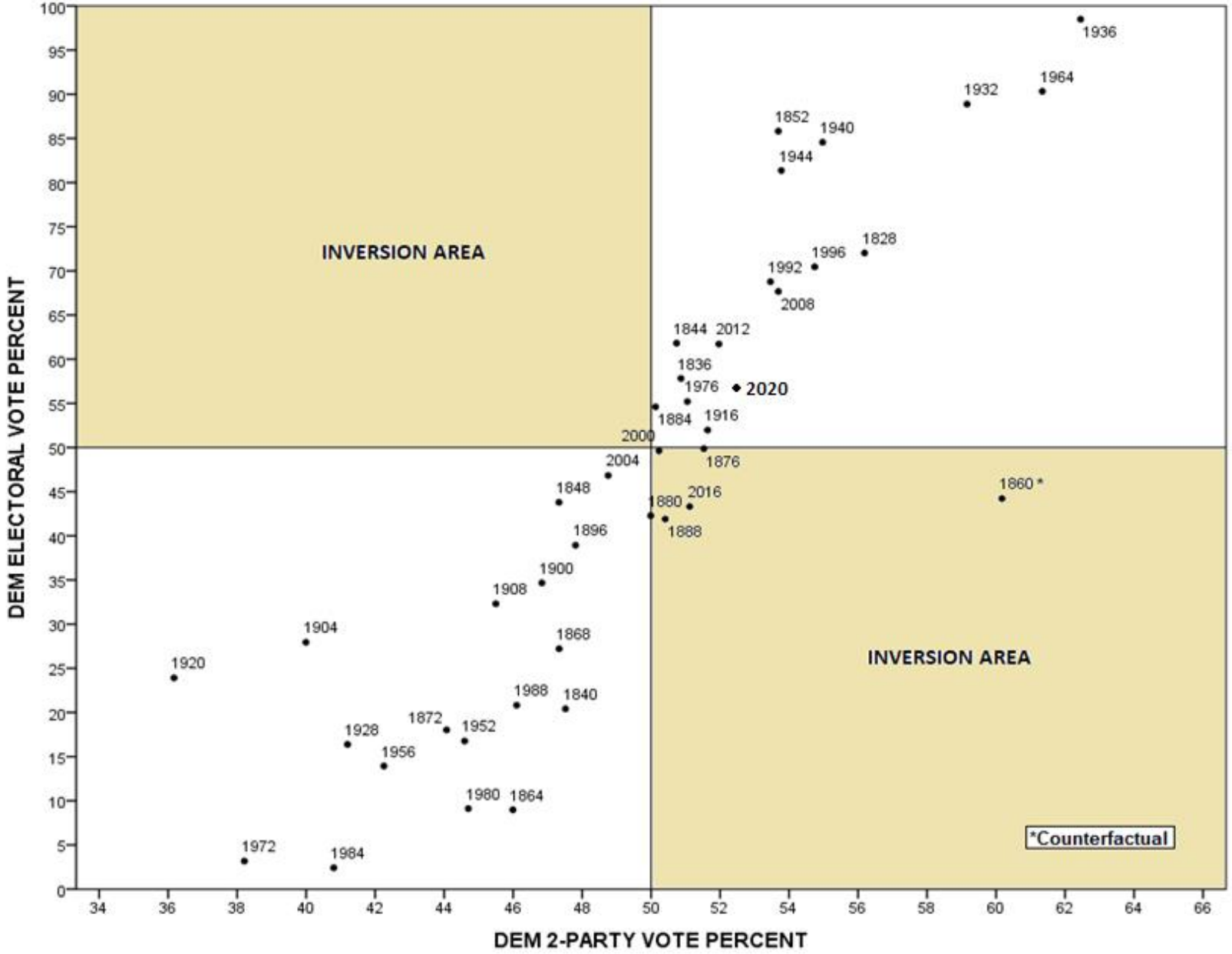
1860	*Popular Vote Percent*	*Electoral Vote*	*Unified Opposition*	
			Popular Vote Percent	*Electoral Vote*
Lincoln (R)	39.82	180	39.82	169
Douglas (ND)	29.46	12	60.16	134
Breckinridge (SD)	18.09	72		
Bell (CU)	12.61	39		
Others	0.02	0		
Total	100.00	303	100.00	303

- Had the Northern and Southern wings of the Democratic Party remained united and even if they had also captured the (mostly southern and border) Constitutional Union vote, Lincoln would still have won an electoral vote majority.
  - Lincoln won almost no votes in slave states (none at all in any future Confederate state other than VA),
  - while winning almost all free states by absolute but typically modest majorities.

Historical Analysis

- Miller (2012) begins with 1828, i.e., the first election in which electors
  - in all states except SC were popularly elected (so that there was a "popular vote" in each state and nationally), and
  - were almost always elected at-large, so that states typically cast electoral votes on a winner-take-all basis.
- Throughout everything is done on a strictly two–party basis.
  - In particular, all elections in which a third candidate carried one or more states and thereby won electoral votes are excluded,
  - namely 1832, 1856, 1860, 1892, 1912, 1924, 1948, 1960, 1968.
    - However, 1948, 1960, and 1968 are included in the "inversion interval" chart below.
- Moreover, throughout it is assumed that the Electoral College operates in its now "standard" manner, in particular that
  - electoral votes are cast on a winner-take-all basis (even in ME and NE),
  - electoral votes cast by "faithless electors" are counted as if they had been faithful, and
  - DC is treated as if it were a state.

Historical Overview: Standard Electoral College



The Probability of Election Inversions: Historical Estimates

- Number of Inversions/Number of elections (since 1828)

  4/49 = .0816

- Clearly an important determinant of the probability of an election inversion is the probability of a close division of the popular vote.
  - Note that the 2000-2020 and 1876-1888 periods both are characterized by unusually close elections.
- Considering only elections in which the winner's popular vote margin was no greater than 3 percentage points, the frequency of inversions has been considerably higher, namely

  4/13 = .3077

- All historical inversions have favored the Republican party.
  - More generally, the historical scattergram suggests a bit of of a Republican bias in the long-term popular vote-electoral vote relationship.
- However, in each of the periods 2000-2020 and 1876-1888, Republicans won the popular vote only once:
  - 2004 (by 2.5%)
  - 1880 (by less than 0.1%)
  - making it unlikely that inversions could favor Democrats.

Extracting More information From the Historical Record

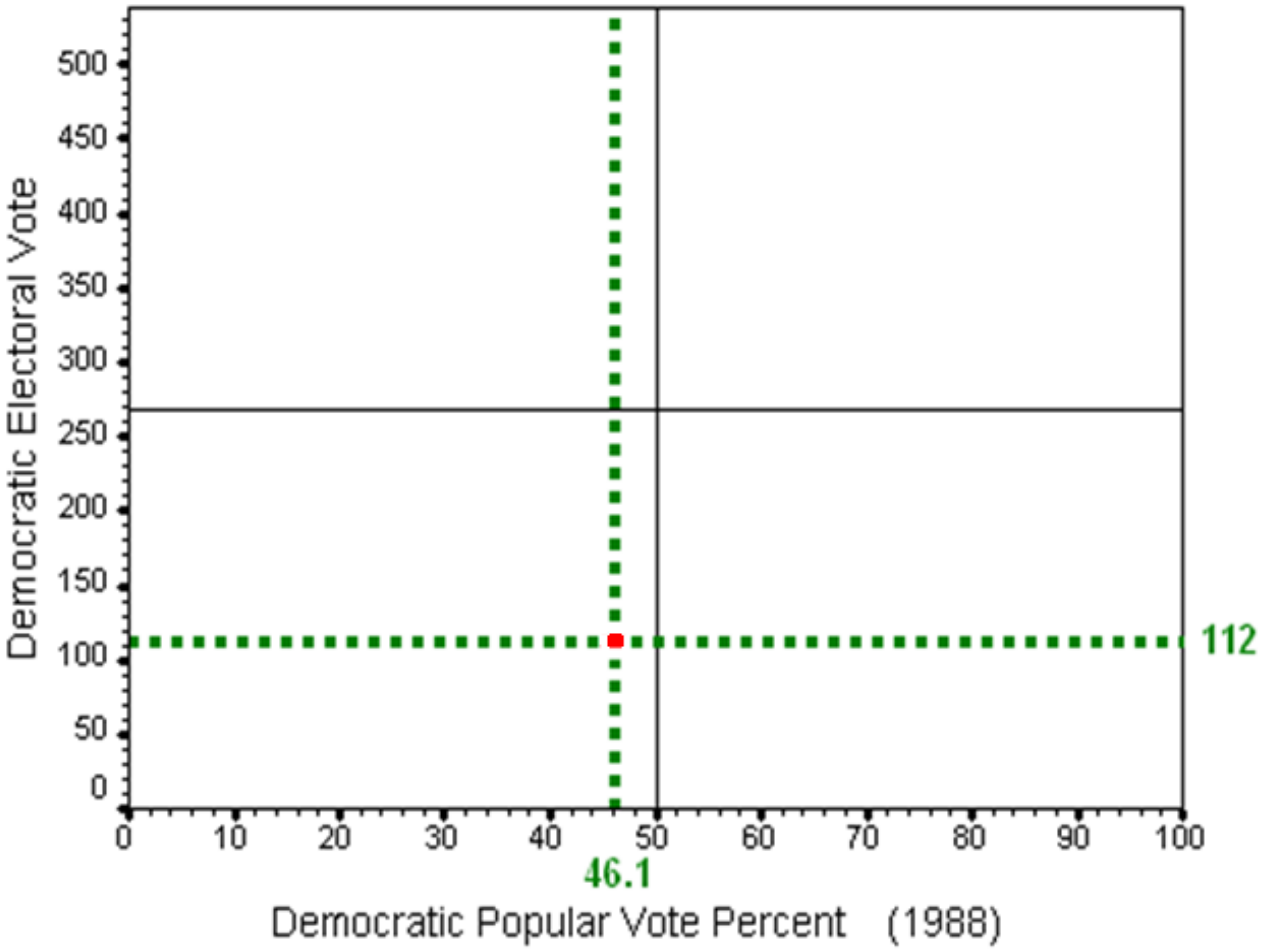
- Looking only at the national popular and electoral votes, we can classify elections only as inversions or non-inversions.
- By using *state-by-state popular vote percentages* (together with the apportionment of electoral votes), we can produce a more informative historical analysis of the propensity of the EC to produce inversions.
- Every presidential election generates an “inversion interval”.
- These intervals
  - vary magnitude
  - and are either pro-Rep or pro-Dem in direction.
- In each election, if the popular vote had fallen (or actually did fall) within this interval, an election inversion would (or actually did) occur.

The PVEV Step Function

- The first step is to produce the Popular Vote-Electoral Vote (*PVEV*) *step function* (essentially an unsmoothed votes-seats curve) for each election,
  - which shows the number of electoral votes a candidate would have won as a function of varying popular vote percentages, given the "electoral landscape/alignment" that characterized that election.
  - The *electoral landscape/alignment* is specified by the cardinal ranking of the states in terms of the differences among them with respect to their (Democratic) two-party popular vote percentages.
- The PVEV employs the kind of *uniform swing analysis* pioneered by Butler (1951) and Brookes (1960), which has also be called
  - *hypothetical (single-year) swing analysis* (Niemi and Fett, 1986),
  - the *Bischoff method* (Peirce and Longley, 1981),
  - and has also been employed by Nelson (1974) Sterling (1974 and 1981), Garand and Parent (1991), and many others.
- The PVEV function
  - is a *cumulative distribution function* and is therefore (weakly) monotonic, and
  - is a *step function* because, while the independent variable (PV) is essentially continuous, the dependent variable (EV) is discrete (taking on only whole number values and jumping up in relatively large discrete steps).

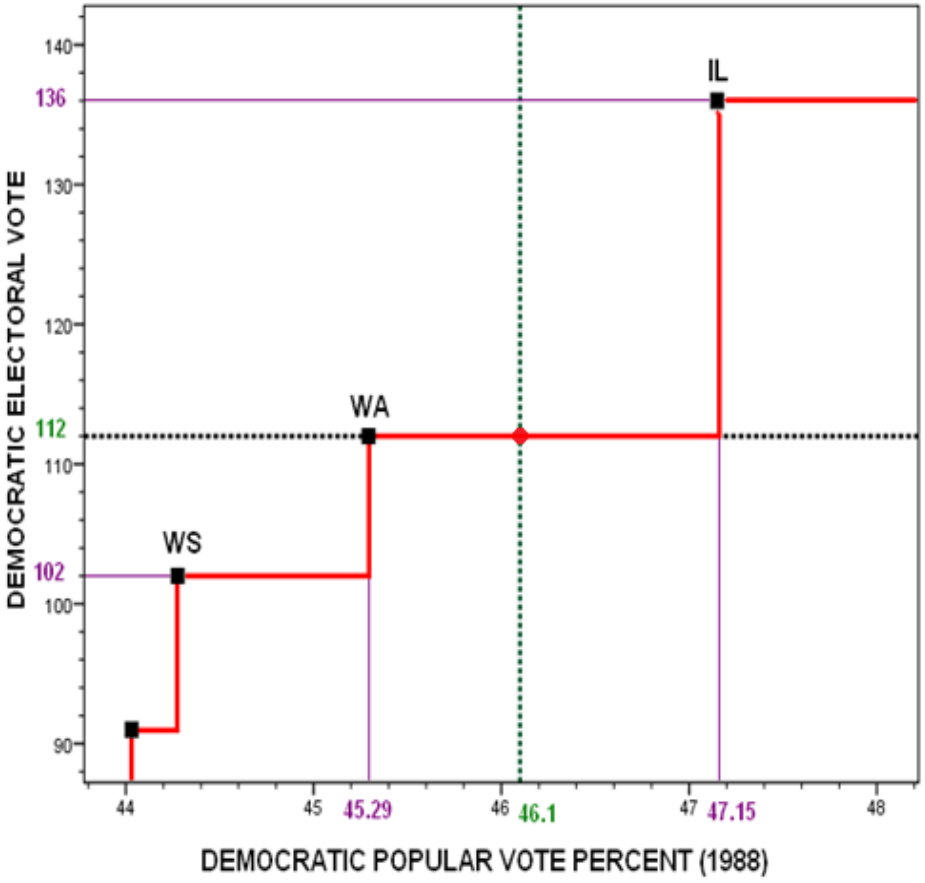
The PVEV Step-Function: 1988 as an Example

- In 1988, Dukakis received 46.10% of the two-party national popular vote and won 112 electoral votes (though one was lost to a “faithless elector”).

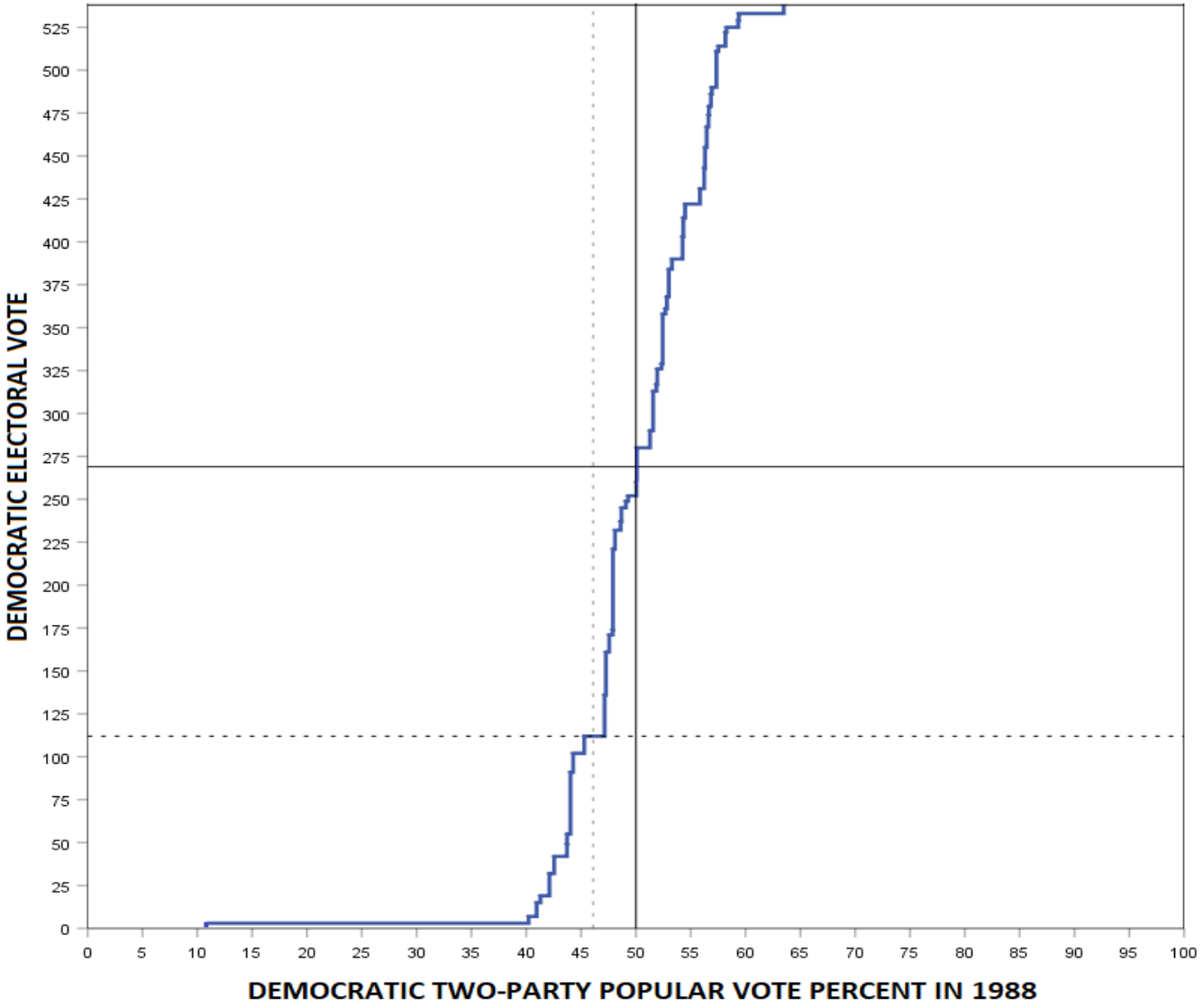


1988 Example (cont.)

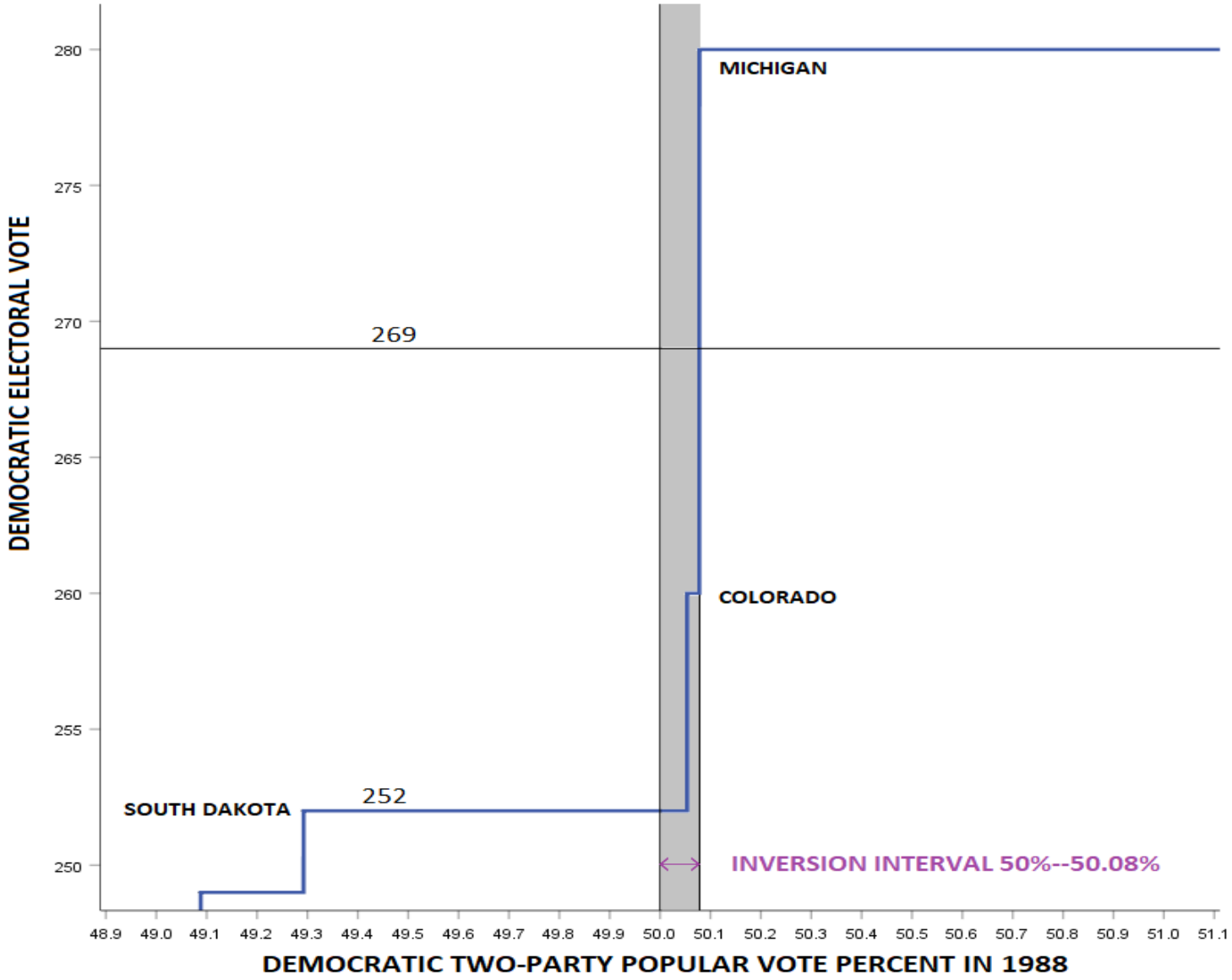
- Of all the states that Dukakis carried, he carried Washington (10 EV) by the smallest margin (50.81%).
  - If the Dukakis national popular vote of 46.10% were *to decline uniformly across all states*, his EV total would remain at 112 until it falls by 0.81 percentage points to 45.29% when WA would tip out of his column (reducing his EV to 102).
- Of all the states that Dukakis failed to carry, he failed to carry Illinois (24 EV) by the smallest margin (48.95%).
  - If the Dukakis popular vote of 46.10% were to *increase uniformly* across all states, his EV total would remain 112 until increases by 1.05 percentage points to 47.15% when IL would tip into his column (increasing his EV to 136).



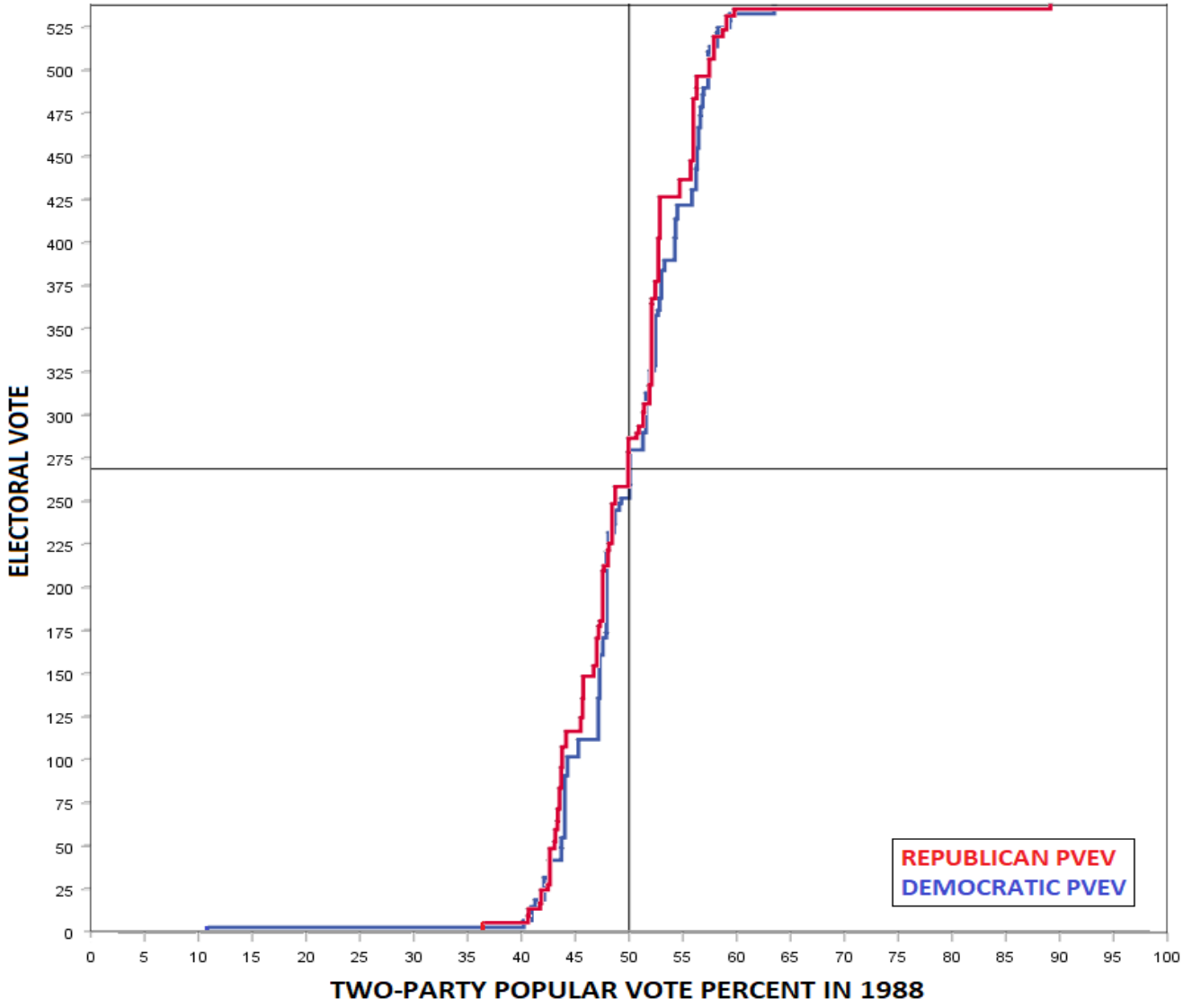
The Full PVEV for 1988 Appears To Go Through the Perfect-Tie Point



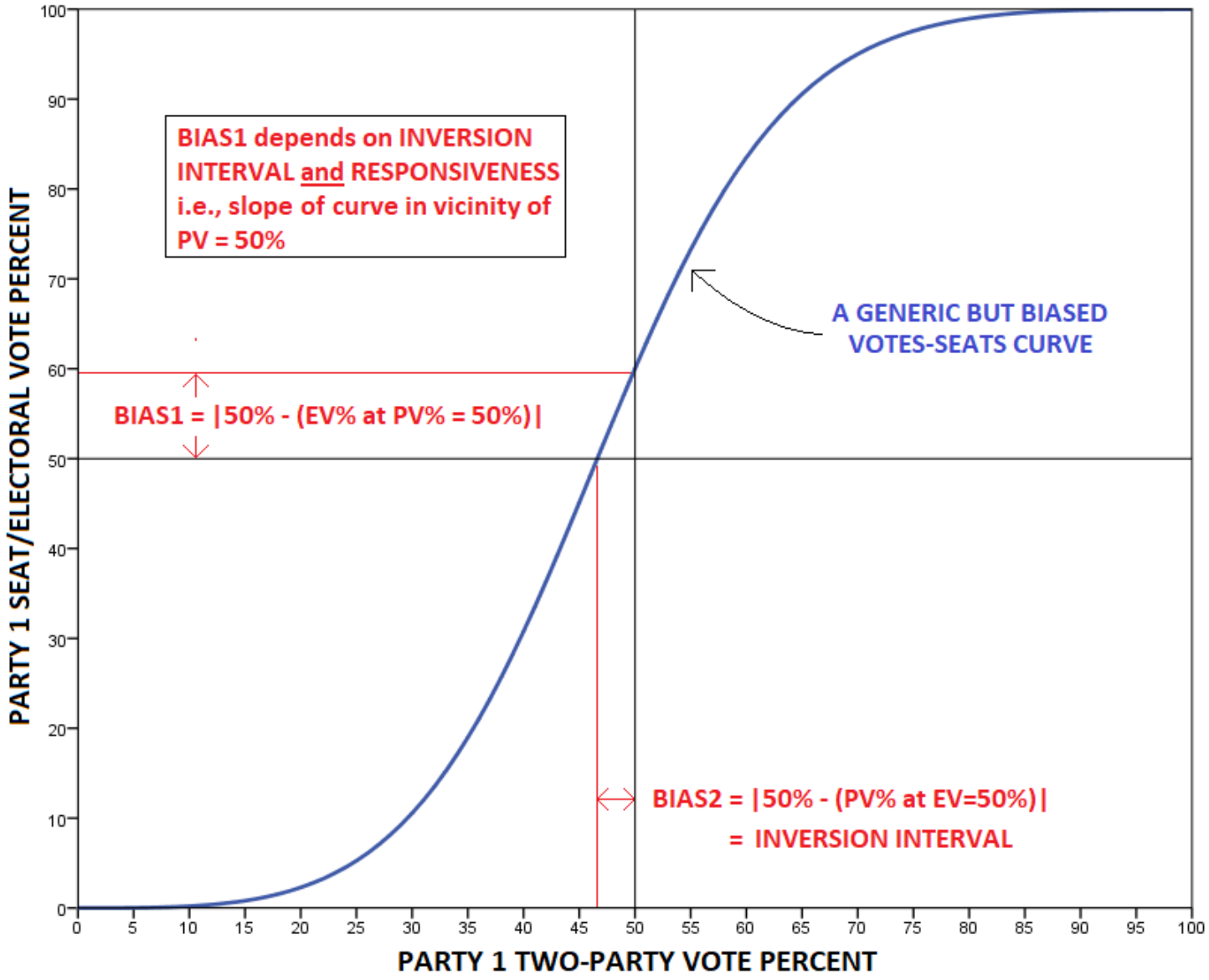
But If We Zoom in on PV ≈ 50%, We Find a Small Pro-Republican *Inversion Interval* 0.08% Wide



The 1988 PVEV Exhibits a High Degree of *Partisan Symmetry* (and of *Responsiveness*)

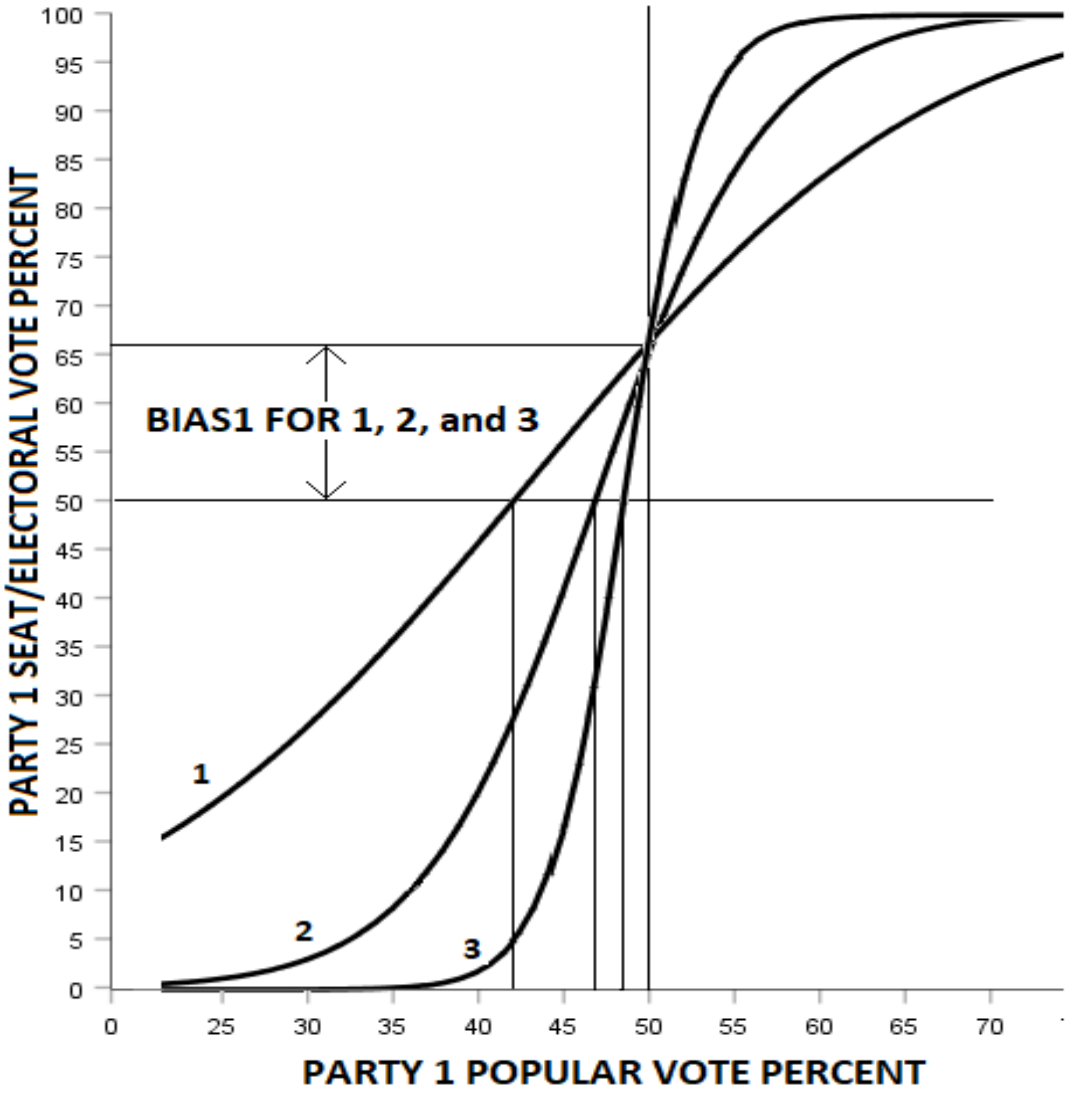


The Inversion Interval and "Partisan Bias"



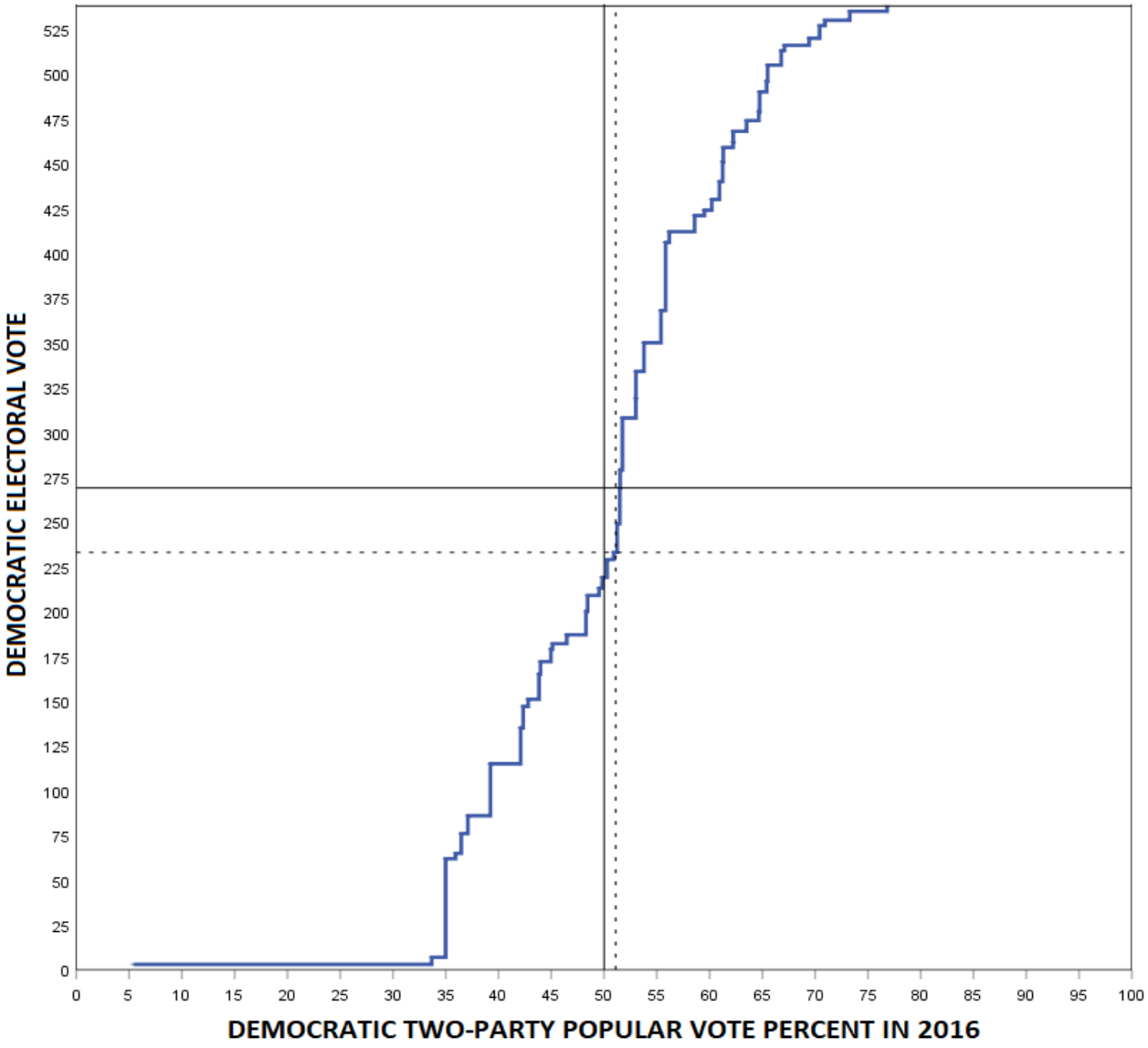
A Note on “Partisan Bias” (cont.)

- Vote-Seat (PVEV) curves 1, 2, and 3 are all clearly biased in favor of Party 1.
- While they all have the same Seat/EV share at PV=50% (“Bias 1”),
- they have very different *inversion intervals* (“Bias 2”).
- This is because they have very different swing ratios (degrees of responsiveness).
- *Especially for presidential elections*, Bias 2 matters much more than Bias 1.



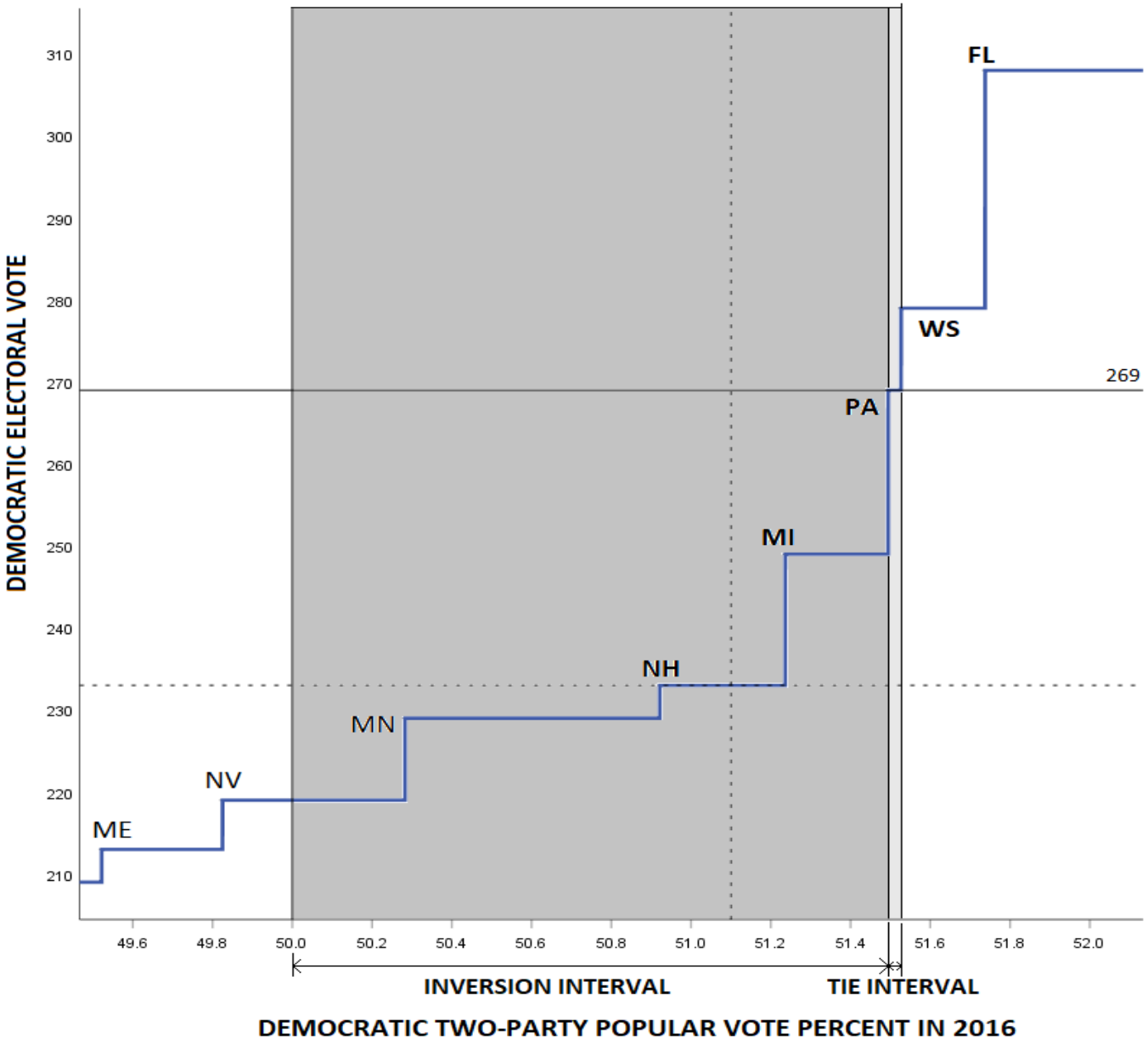
The Democratic PVEV in 2016

- It is less responsive than in 1988 (reflecting increased polarization of “red” vs. ‘blue’ states).
- Even without zooming in, we can see that there is a substantial inversion interval within which the actual PV falls
- The Dem PVEV “sags” below its general trend in the vicinity of PV = 50%.

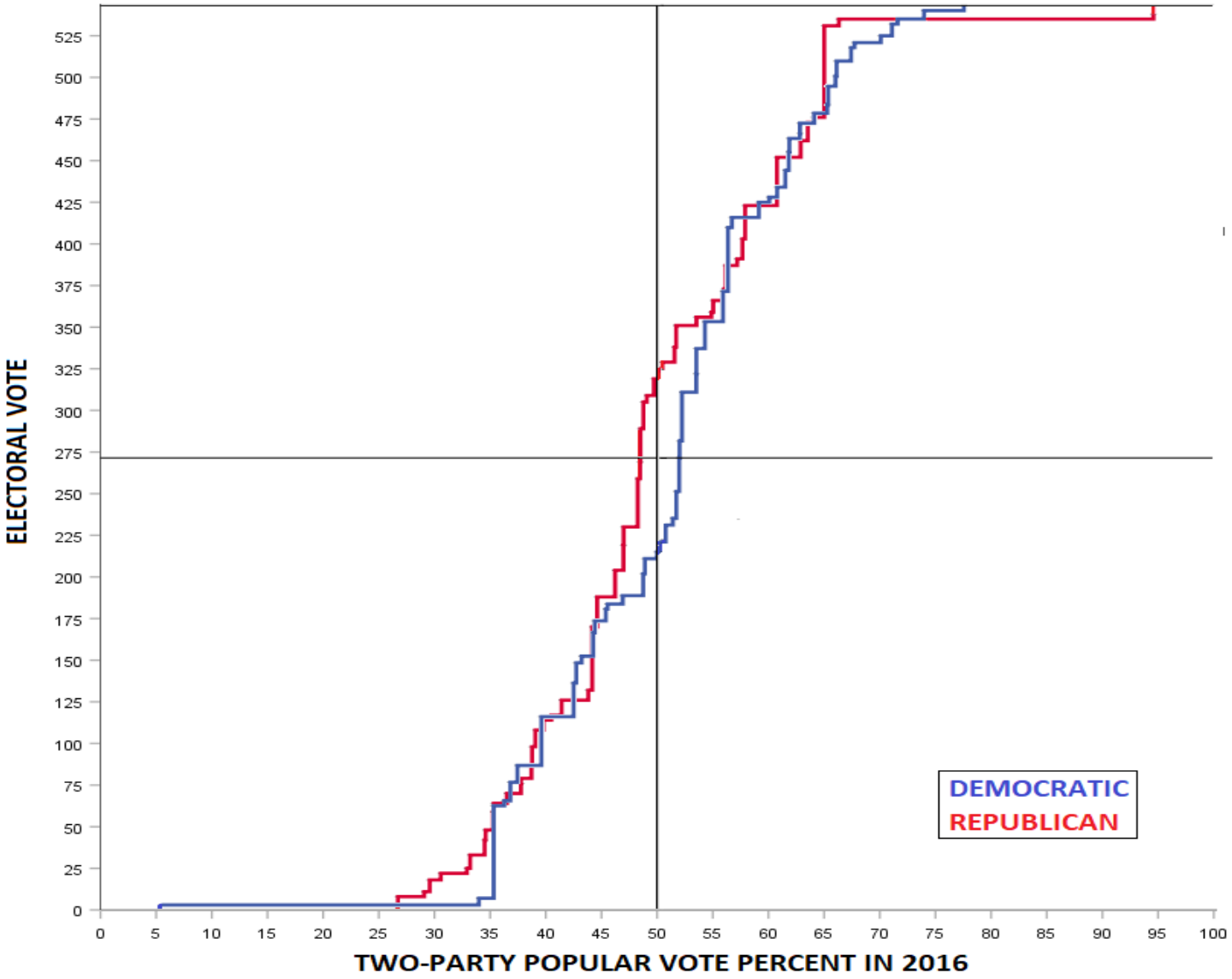


Zooming In on the 2016 Inversion Interval

- The "inversion or tie" interval was 1.53% wide (almost 20 times wider than in 1988).
- Since Clinton lost one CD in Maine, there actually was no tie interval.

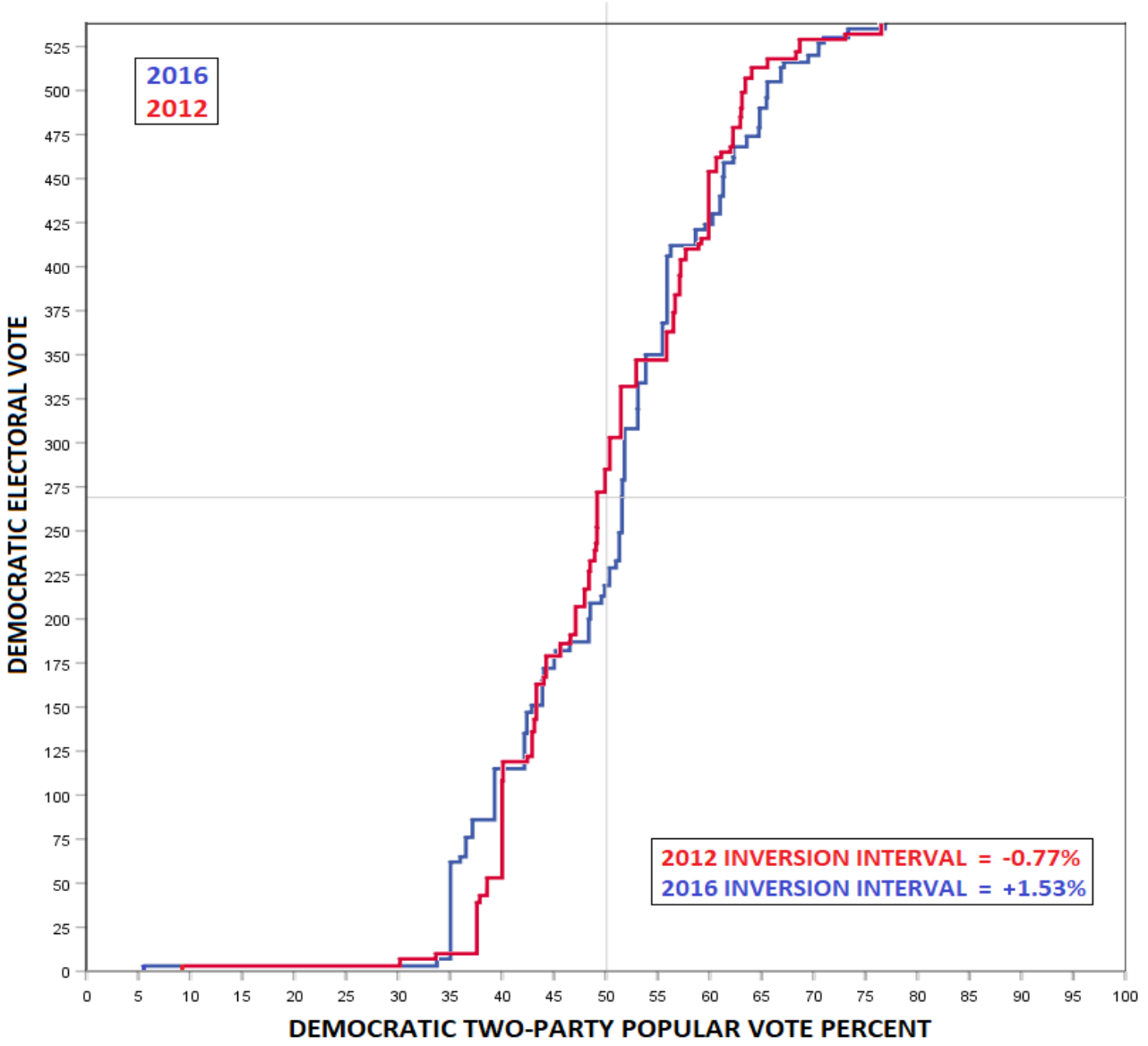


Democratic vs. Republican PVEVs in 2016

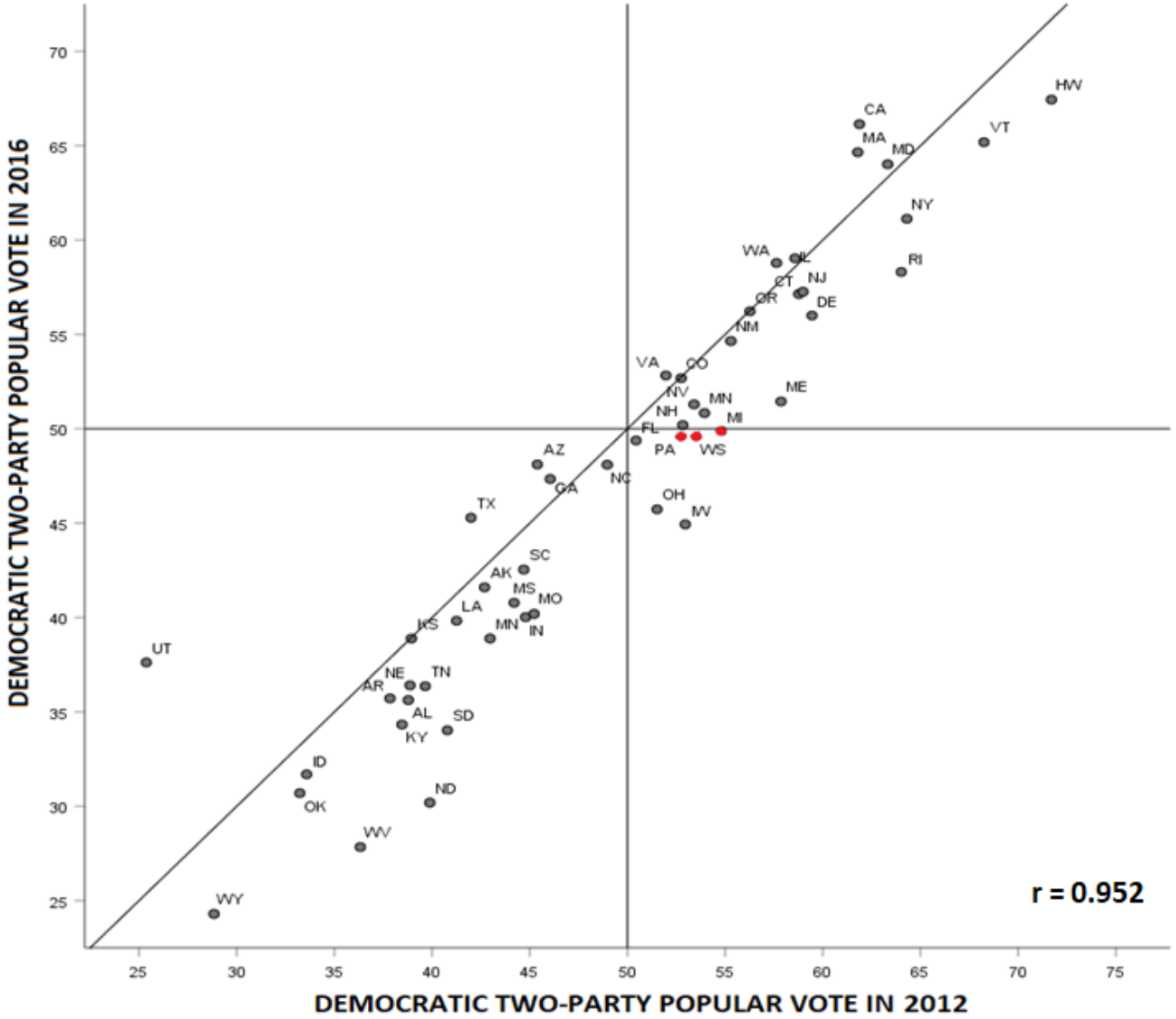


Democratic PVEVs in 2016 vs. 2012

- Note this convention:
- Charts are Democratic-oriented.
- 50% + Inversion Interval = PV% required for Democratic EV majority.

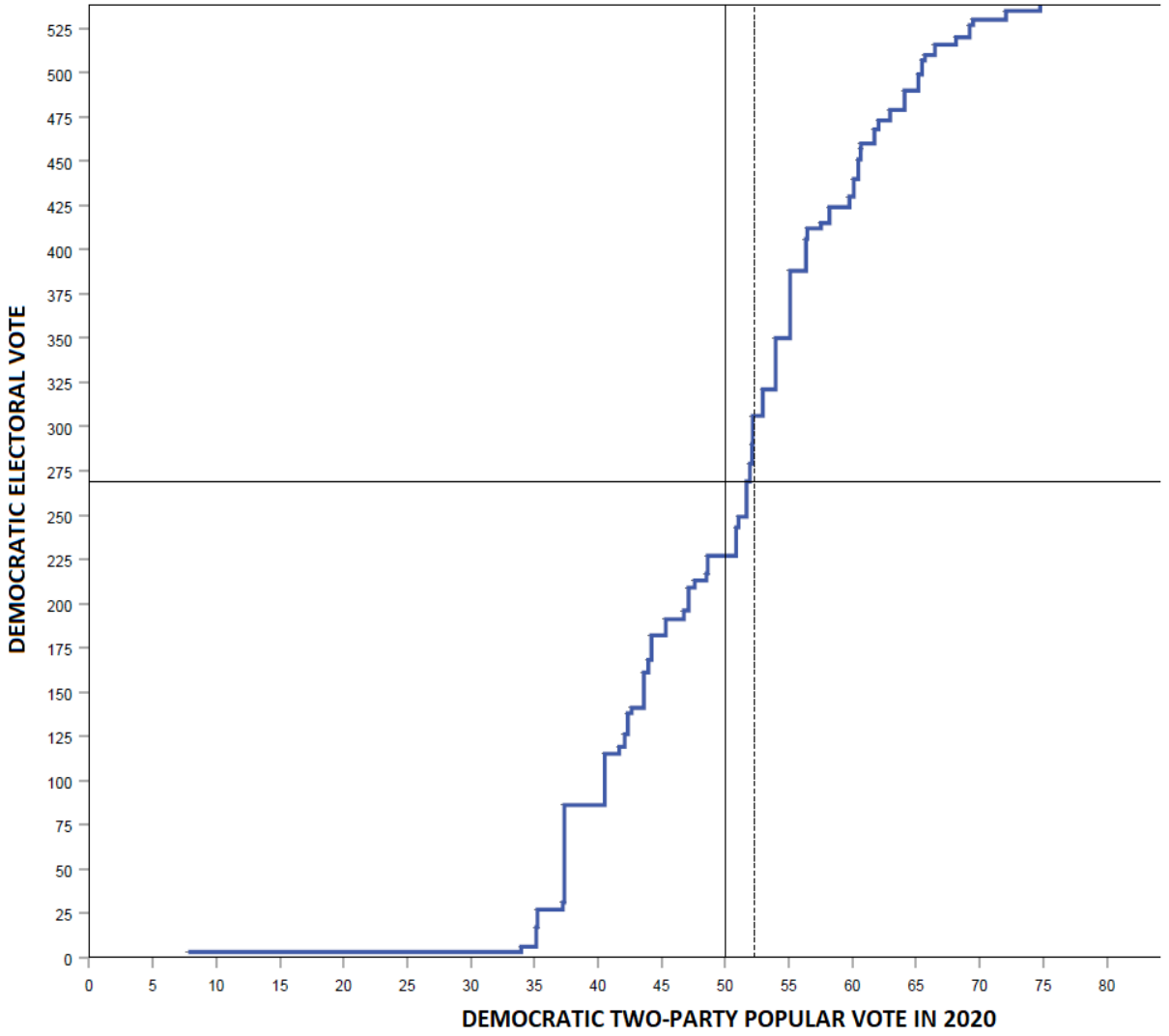


2016 vs. 2012

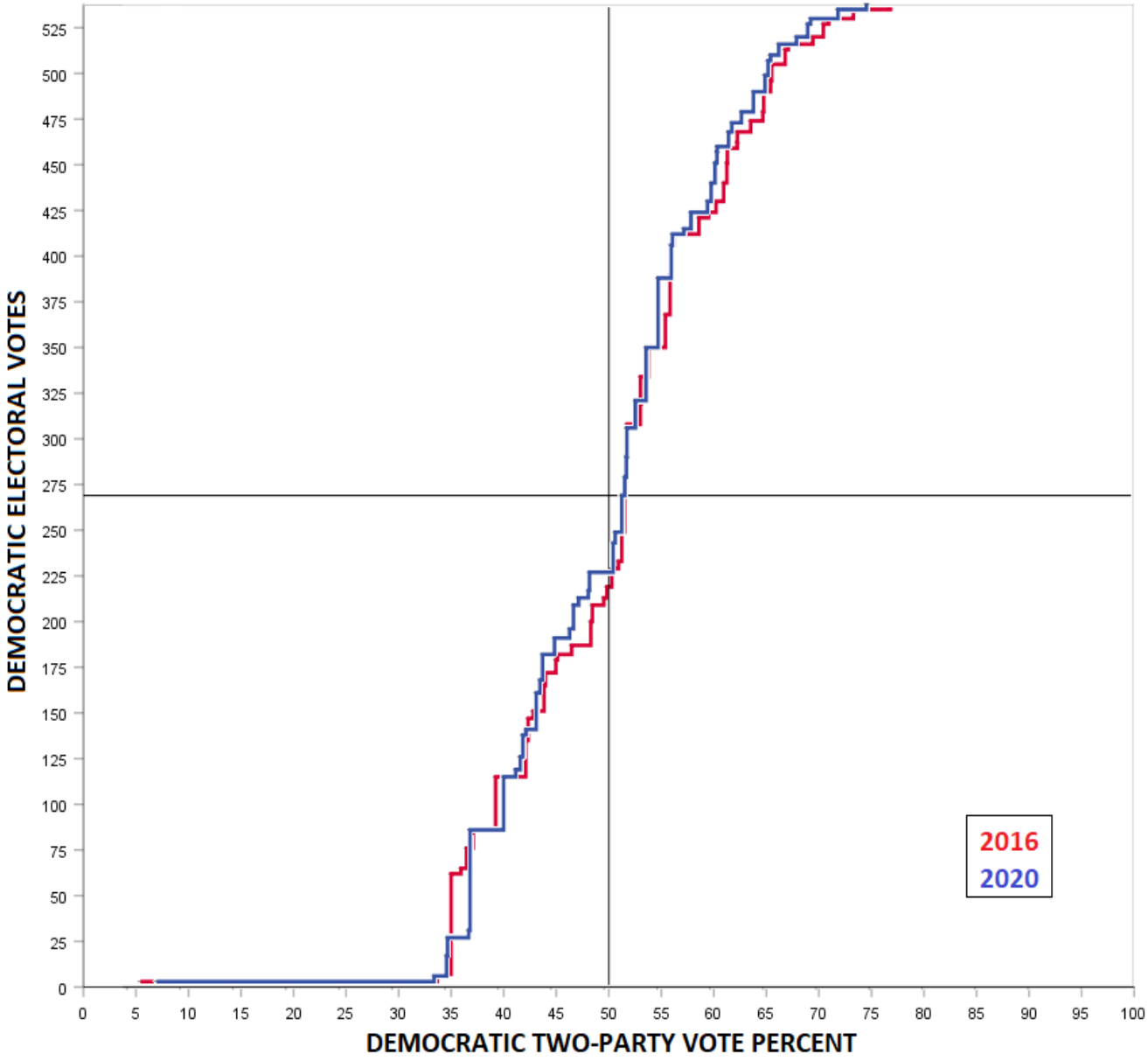

DEMOCRATIC TWO-PARTY POPULAR VOTE IN 2016
DEMOCRATIC TWO-PARTY POPULAR VOTE IN 2012
r = 0.952
HW
CA
VT
MA
MD
NY
WA
IL
RI
CT
NJ
OR
DE
NM
VA
CO
ME
NV
MN
NH
MI
FL
PA
WS
AZ
NC
GA
OH
IW
TX
SC
AK
MS
MO
LA
KS
MN
IN
UT
NE
TN
AR
AL
SD
KY
ID
ND
OK
WV
WY
70
65
60
55
50
45
40
35
30
25
25
30
35
40
45
50
55
60
65
70
75

The Democratic PVEV in 2020

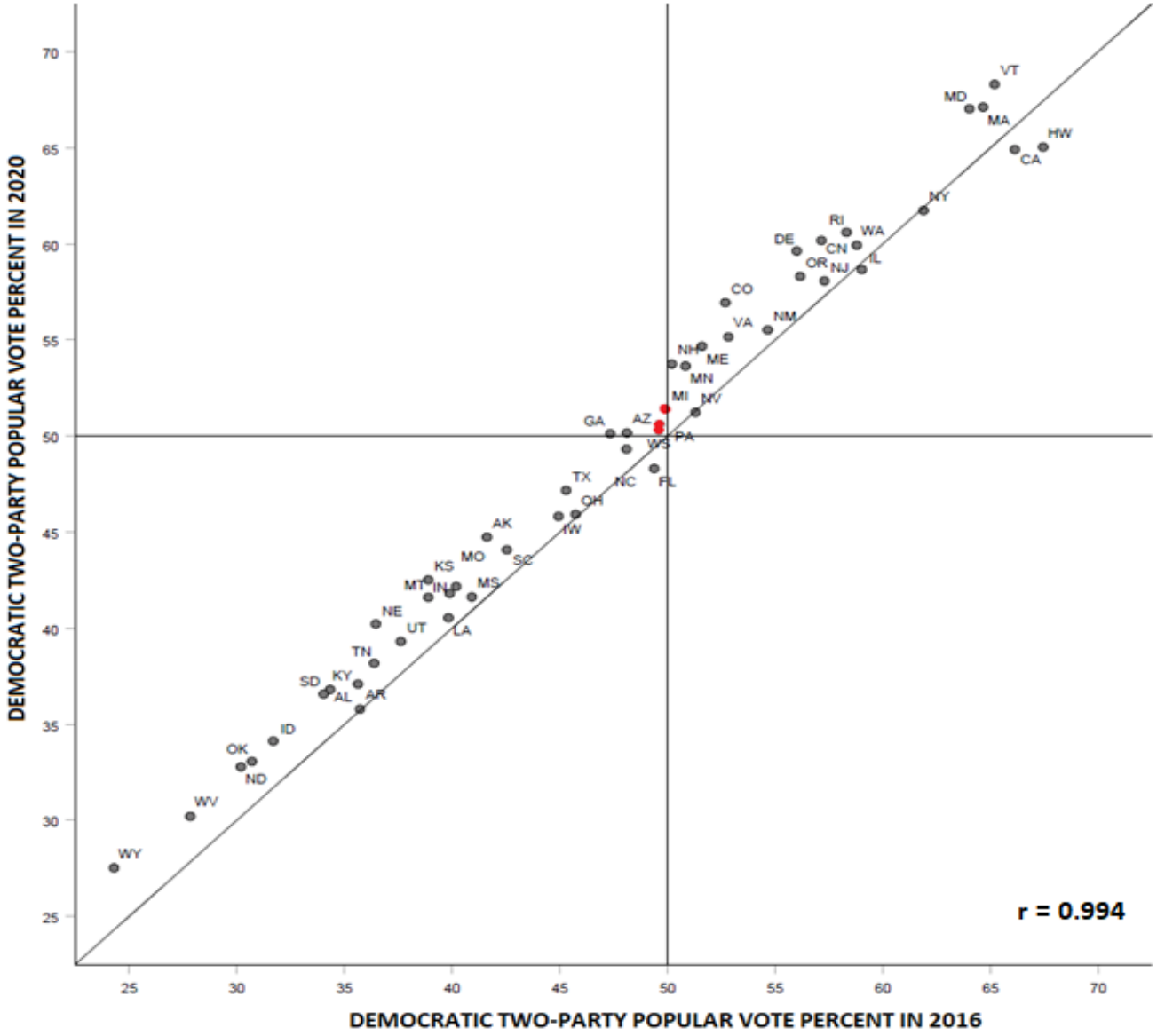
The 2020 election was a close replica of the 2016 election, except that the Democratic 2-party popular vote percent increased by about 1.15 percentage points (from 51.12% to 52.27%).



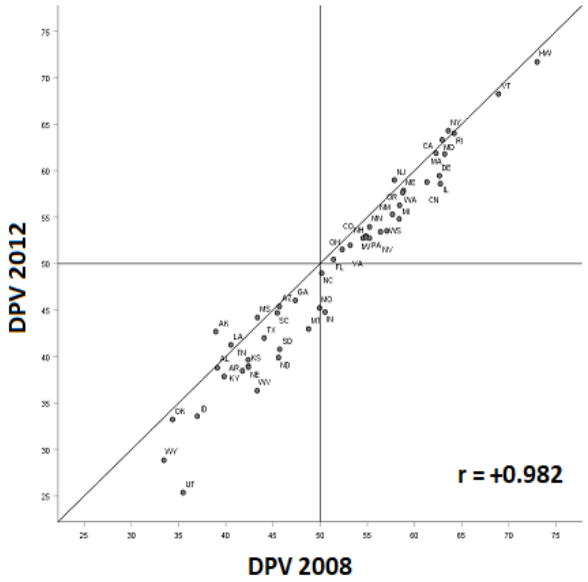
2020 and 2016 PVEVs Compared

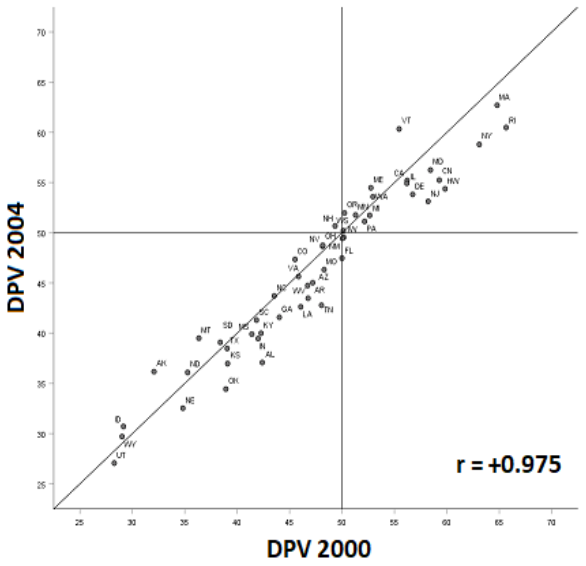


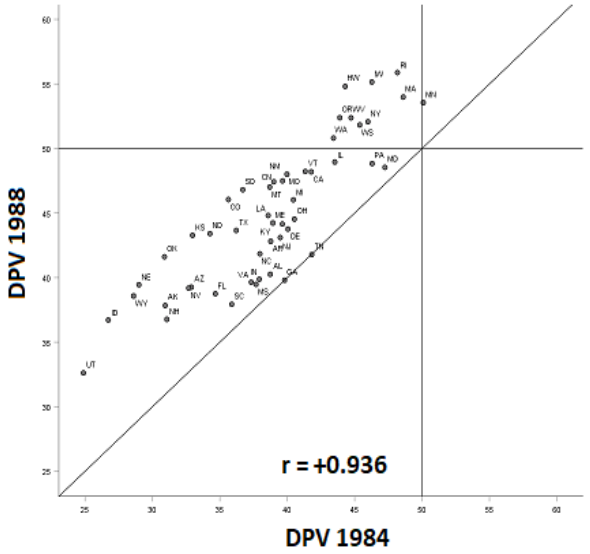
2020 vs. 2016

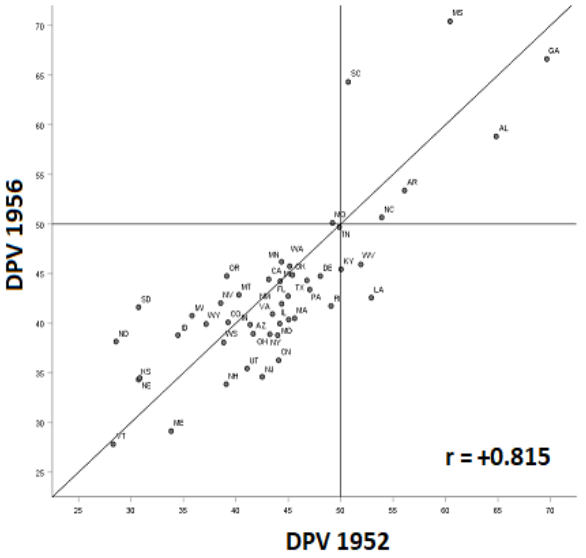

DEMOCRATIC TWO-PARTY POPULAR VOTE PERCENT IN 2020
DEMOCRATIC TWO-PARTY POPULAR VOTE PERCENT IN 2016
r = 0.994
VT
MD
MA
HW
CA
NY
RI
WA
DE
CN
OR
NJ
IL
CO
VA
NM
NH
ME
MN
MI
NV
GA
AZ
PA
WS
TX
NC
FL
OH
IW
AK
SC
KS
MO
MT
IN
MS
NE
UT
LA
TN
SD
KY
AL
AR
ID
OK
ND
WV
WY

Scattergrams and Correlations for Earlier Election Pairs

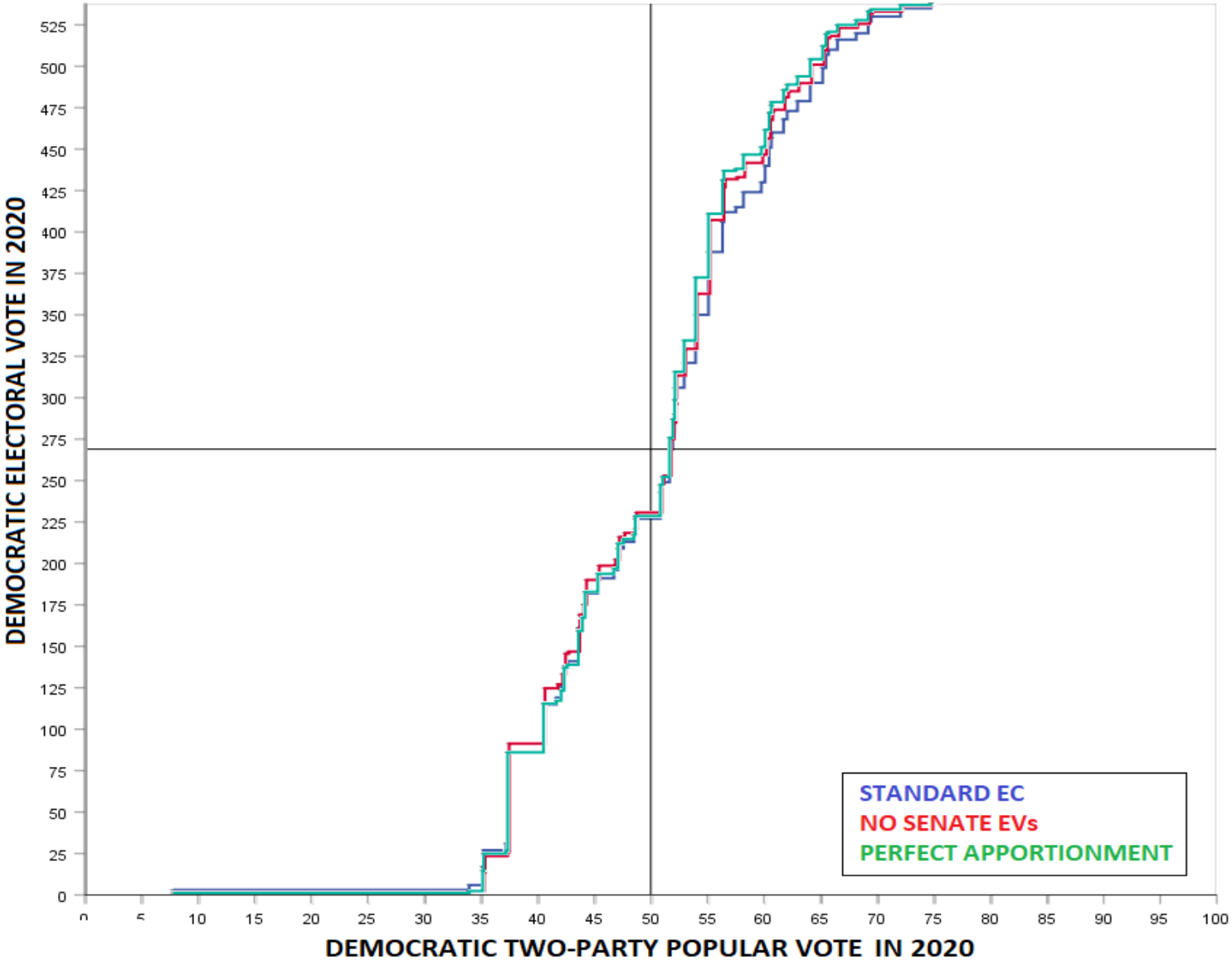








Alternate PVEVs in 2020



The Alternate PVEVs for 2020 in Tabular Form

State	Democratic (2-Party) Popular Vote Percent	Cumulative Electoral Votes		
		Standard	House Only	Perfect Apportionment
DC	7.80	3	1.23	1.162
VT	33.97	6	2.47	2.392
MA	35.15	17	13.57	14.672
MD	35.24	27	23.44	24.917
HW	37.24	31	25.91	26.865
CA	37.36	86	91.31	86.082
NY	40.53	115	124.63	115.469
RI	41.67	119	127.10	117.225
CN	42.10	126	133.27	123.438
WA	42.34	138	145.61	137.118
DE	42.64	141	146.84	138.837
IL	43.61	161	169.05	159.314
OR	43.96	168	175.22	167.267
NJ	44.20	182	190.03	182.806
CO	45.33	191	198.67	193.770
NM	46.75	196	202.37	196.896
VA	47.12	209	215.94	212.035
ME	47.60	213	218.41	214.788
NH	48.52	217	220.88	217.523
MN	48.63	227	230.75	228.598
MI	50.86	243	248.02	247.466
NV	51.05	249	252.96	252.218
PA	51.67	269	275.17	275.882
WS	51.95	279	285.04	287.095
AZ	52.11	290	296.15	298.628
GA	52.15	306	313.42	315.703
NC	52.95	321	329.46	334.534
FL	53.97	350	362.78	372.472
TX	55.10	388	407.20	411.045
OH	56.35	406	426.94	431.228
IW	56.45	412	431.88	436.960
AK	57.53	415	433.11	438.149
SC	58.20	424	441.75	446.717
KS	59.76	430	446.69	451.359
MO	60.11	440	456.56	461.640
IN	60.47	451	467.67	471.922
MS	60.65	457	472.60	476.407
MT	60.67	460	473.83	478.442
LA	61.73	468	481.24	485.748
NE	62.05	473	484.94	488.971
UT	62.96	479	489.88	493.902
TN	64.10	490	500.98	504.268
AL	65.18	499	509.62	512.193
KY	65.47	507	517.02	519.455
SD	65.71	510	518.26	520.879
AR	66.48	516	523.19	524.977
ID	68.15	520	525.66	527.887
OK	69.21	527	531.83	533.160
ND	69.49	530	533.06	534.373
WV	72.07	535	536.77	537.076
WY	74.75	538	538.00	538.000

2020 and 2016 Compared

Comparing the 2016 and 2020 PVEV ("Landscapes")	2016			2020		
	Standard EC	No Senate EVs	Perfect Apportion-ment	Standard EC	No Senate EVs	Perfect Apportion-ment
Democratic Inversion Interval	51.49% (T) 51.53% (W)	51.49%	51.49%	51.67% (T) 51.95% (W)	51.67%	51.67%
Democratic EV at PV=50%	219	223.24	217.89	227	230.75	228.60
Democratic EV at PV=51.12%	233	235.68	232.03	249	252.96	252.22
Democratic EV at PV=52.27%	308	319.36	325.39	306	314.32	315.70

Note 1. The basic analysis assumes the "standard" (i.e, winner-take-all) Electoral College system. Since Clinton lost one CD in Maine, a Democratic popular vote percent within the 2016 tie (T) interval would have actually produced a Trump win. Biden also lost one CD in Maine but won one CD in Nebraska, so the winner-take-all tie interval in 2020 was also an actual tie interval.

Note 2. In the "No Senate EVs" column, electoral votes entries are scaled up by a factor of 538/436 = 1.23394 to make them comparable to the EV entries in other columns.

Note 3. "Perfect Apportionment" as defined in Miller (2012), i.e., state electoral votes proportional to state (two-party) popular votes.

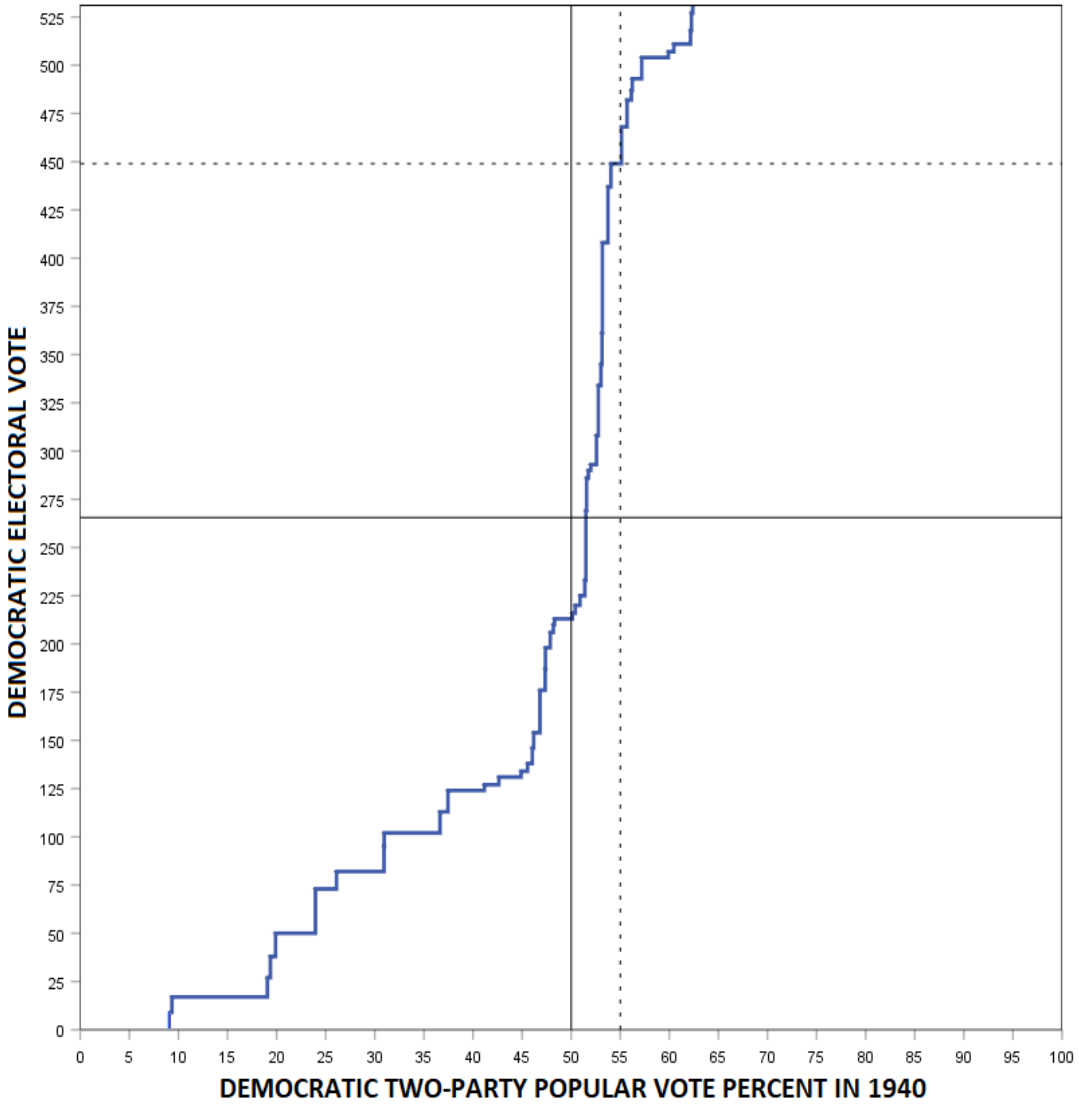
Random Shocks to a Given PVEV Landscape

- Each PVEV is deterministic: a given PV translates into a precise EV.
- Consider that a given PVEV might repeatedly be "jiggled" a bit.
- Specifically, suppose that each state vote PV% is subject to independent random shocks = RN(0,1%)
- The table to the right shows the resulting distribution of EVs in 64,000 simulated elections.
- Only about a dozen states ever tip back and forth between parties but (presumably) all combinations occur in 64K trials.

Year (DPV%)	2016 (51.11%)	2020 (52.27%)
Mode	249	290
Median	253	290
Mean	254.736	287.819
Minimum	200	227
25th Percentile	239	276
75th Percentile	268	299
Maximum	339	372
Standard Deviation	19.407	17.239

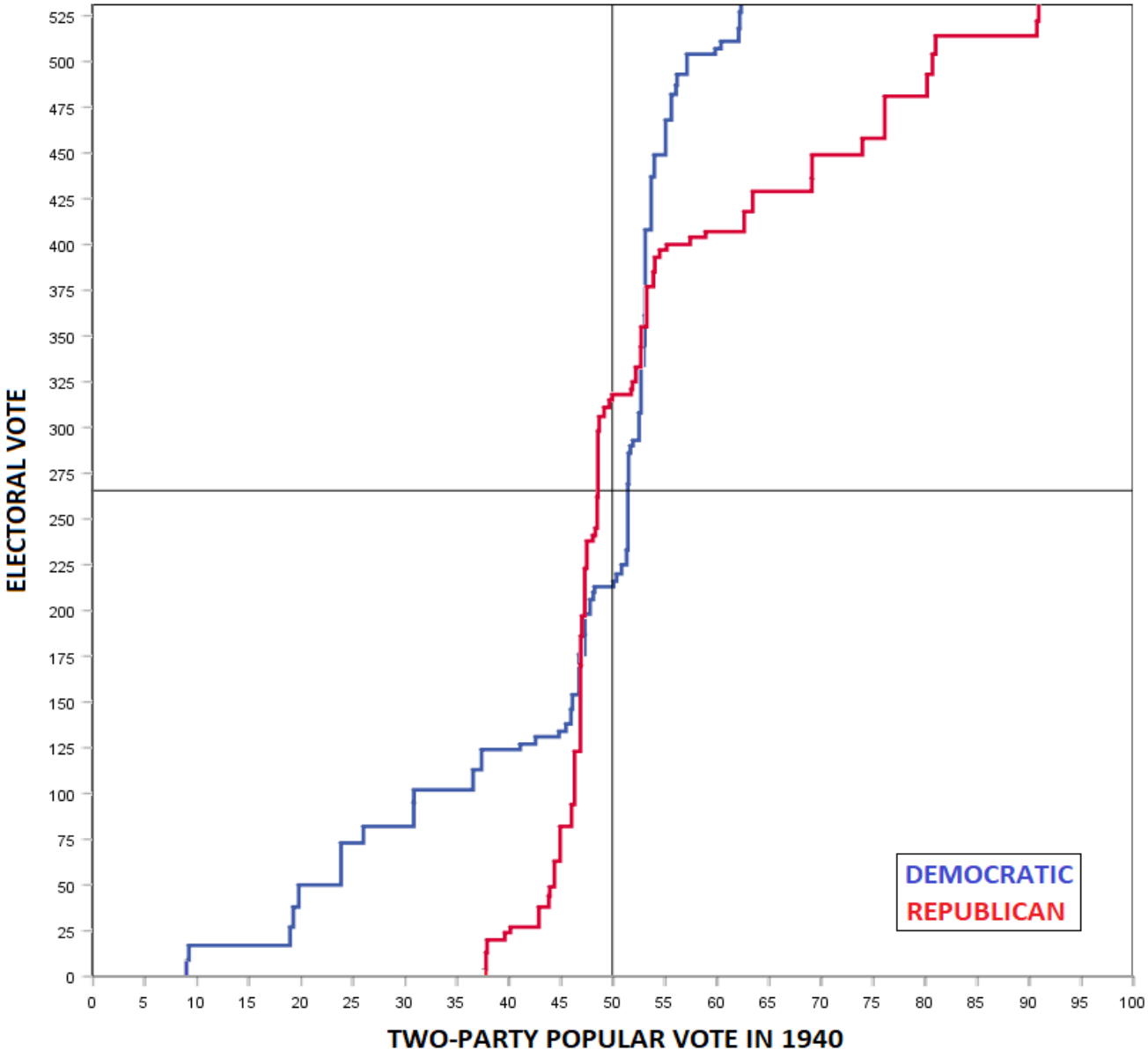
PVEV in 1940

- While the 2016 pro-Rep inversion interval was unusually wide at +1.53%, it certainly was not unprecedented.
- For example, in 1940 the inversion interval was +1.51%.
- But the actual popular vote was well outside the inversion interval.

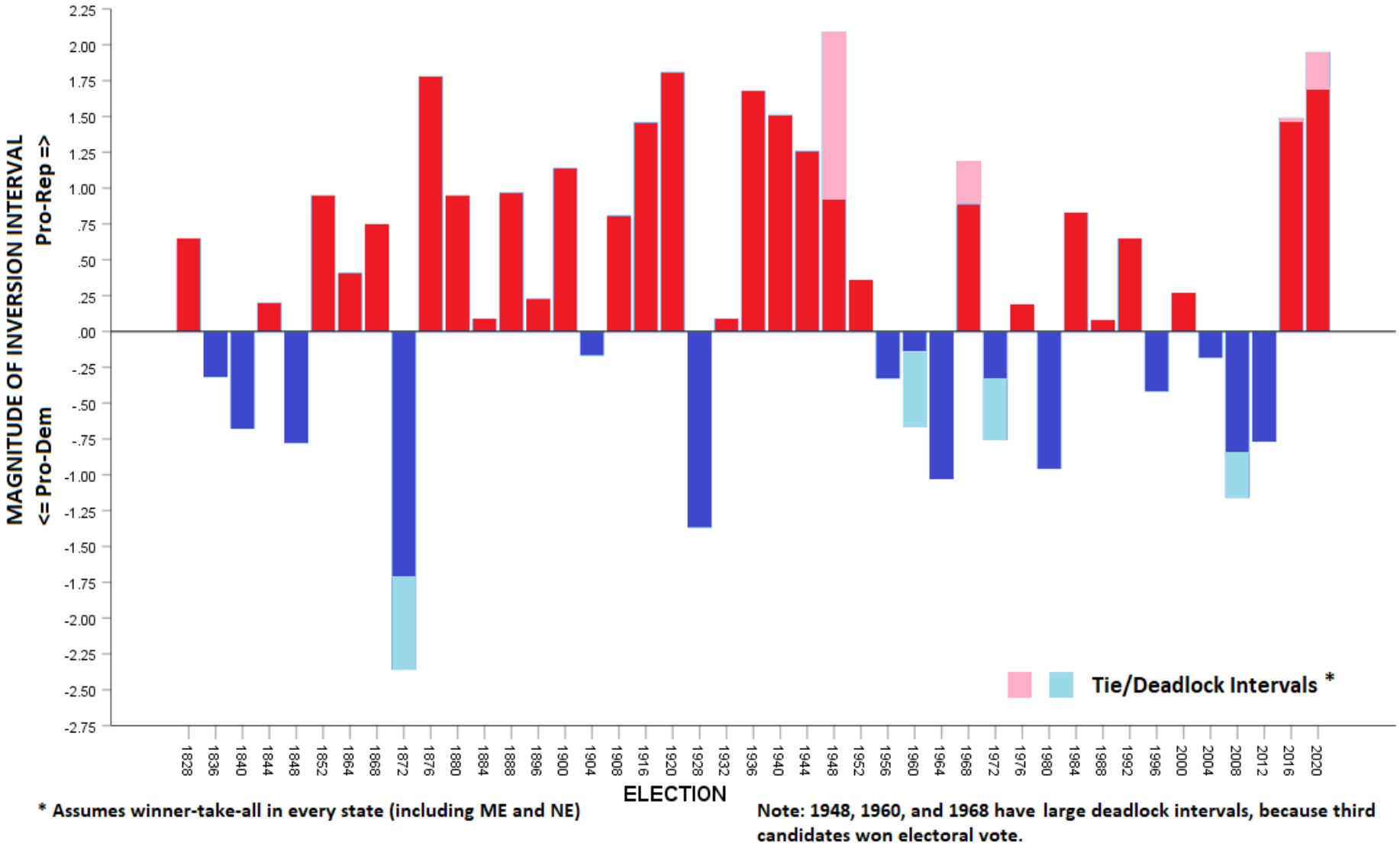


Dramatic Partisan Asymmetry in 1940

- Is this PVEV "biased" in favor of Dems or Reps?
- The Dem PVEV lies above the Rep PVEV over almost all of the PV range.
- But the Rep PVEV lies above the Dem PVEV where it rally matters (especially in presidential elections).



Magnitude and Direction of Inversion (and Tie/Deadlock) Intervals: 1828-2020 (includes 1948, 1960, and 1968)



* Assumes winner-take-all in every state (including ME and NE)

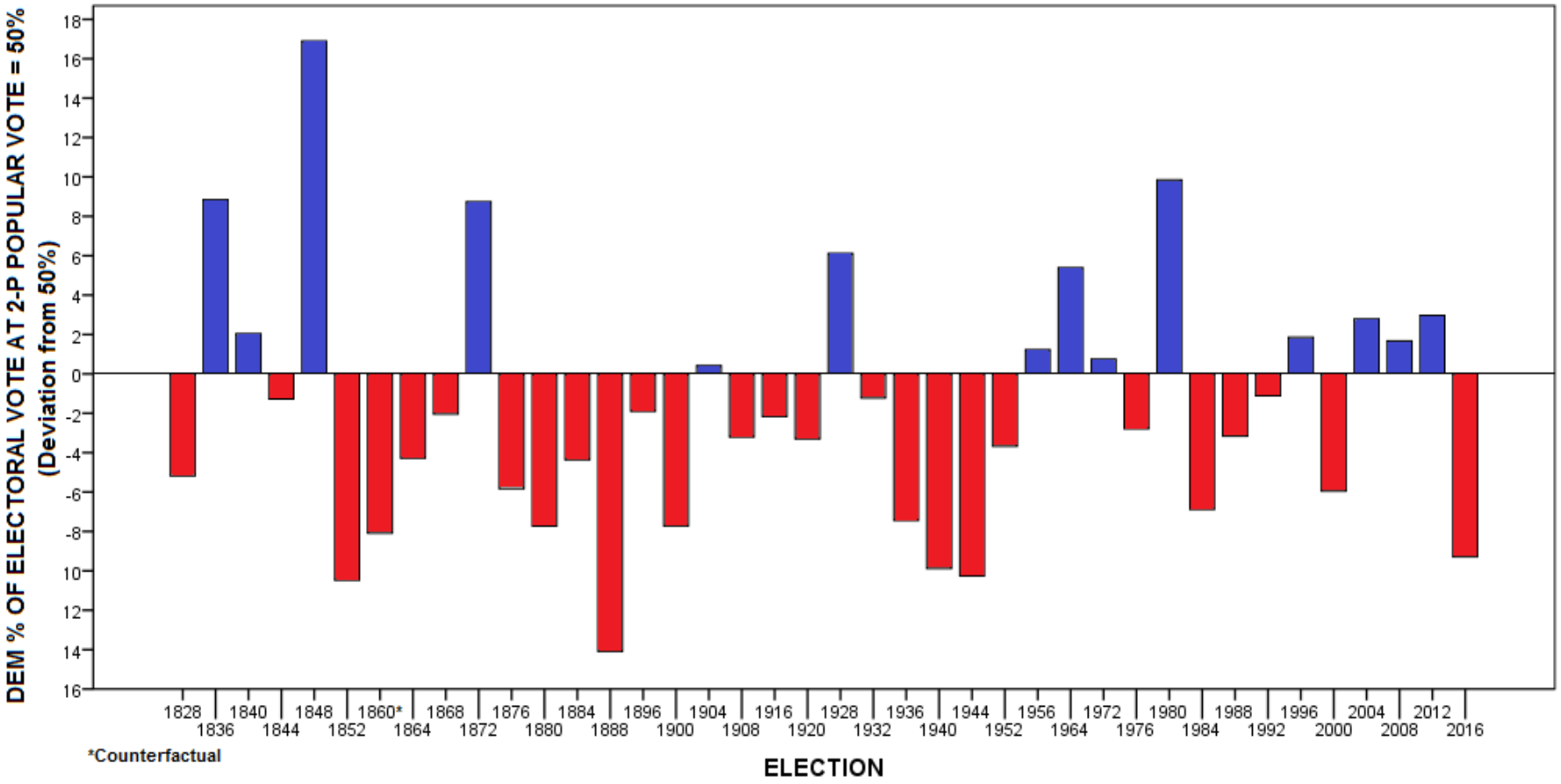
Note: 1948, 1960, and 1968 have large deadlock intervals, because third candidates won electoral vote.

Magnitude and Direction of Inversion (and Tie/Deadlock) Intervals: 1828-2020 (cont.)

- This is the basic story:
- From 1876 to the mid-20th Century, inversion intervals:
  - were often quite large (absolute intervals averaging about 1%), and
  - almost always (14/16) favored Republicans.
  - Overall, actual (positive and negative) intervals averaged about +0.85%.
- From 1952 through 2012 inversion intervals:
  - have been substantially smaller (absolute intervals averaging about 0.6% wide)
  - did not consistently favored either party (9/15 pro-Dem).
  - Overall, actual intervals averaged about -0.1%.
- However, 2016 and 2020 have very large pro-Rep inversion intervals,
  - comparable to those in the earlier period.
- The "Bias 1" (EV at PV = 50%) indicator tells much the same story.

- *Note that the following charts have not been revised to include 2020 or 1948, 1960, and 1948.*

Democratic Percent of Electoral Vote (Deviation from 50%) When Democratic (Two-Party) Popular Vote Is Equal to 50% ("Bias 1"): 1828-2016



Two Sources of Asymmetry (and Large Inversion Intervals) in the PVEV

- The asymmetry or bias in a PVEV that produces large inversion intervals results from either, or more likely both, of two distinct phenomena:
  - *distribution effects*, and
  - *apportionment effects.*
- Either alone can produce bias and election inversions.
- In combination, they can either reinforce or counterbalance each other.
  - It turns out that they typically counterbalance each other,
  - but distribution effects are more powerful (especially in the present era).

Distribution Effects

- Distribution effects result from the "*winner-take-all" rule at the state (or district) level* of the Electoral College (or a FPTP parliamentary system).
- One candidate's or party's vote may be more "efficiently" distributed than the other's, producing an election inversion.
- Here is the simplest possible example:
  - Nine voters are apportioned into three uniform districts, each casting one electoral vote.
  - The individual votes for candidates D and R in each district are as follows: (R,R,D) (R,R,D) (D,D,D).

	Popular Votes	Electoral Votes
D	5	1
R	4	2

  - R's votes are more "efficiently" distributed, so R wins a majority of electoral votes with a minority of popular votes.
- Indeed, Clinton's popular vote margin in 2016 can be attributed entirely to her "inefficiently large" popular vote plurality in CA.

Perfect Apportionment and Apportionment Effects

- The simple 9-voter example is “perfectly apportioned”,
  - that is, each district has the same ratio (3/1) of popular votes to electoral votes, but
  - no actual electoral system (and certainly not the Electoral College) is perfectly apportioned.
- However, we can analytically and retroactively create a uniform ratio of popular votes to electoral votes
  - by (analytically) reapportioning electoral votes (fractionally) among the states so that they are precisely proportional to the total two-party popular vote cast within each state.
- I call this *perfect apportionment*.
- Given perfect apportionment, it follows that a candidate who wins X% of the electoral vote also *carries states that collectively cast X% of the total popular vote*.
- *Apportionment effects* refer to the (net) effects of deviations from perfect apportionment on the PVEV and the inversion interval.

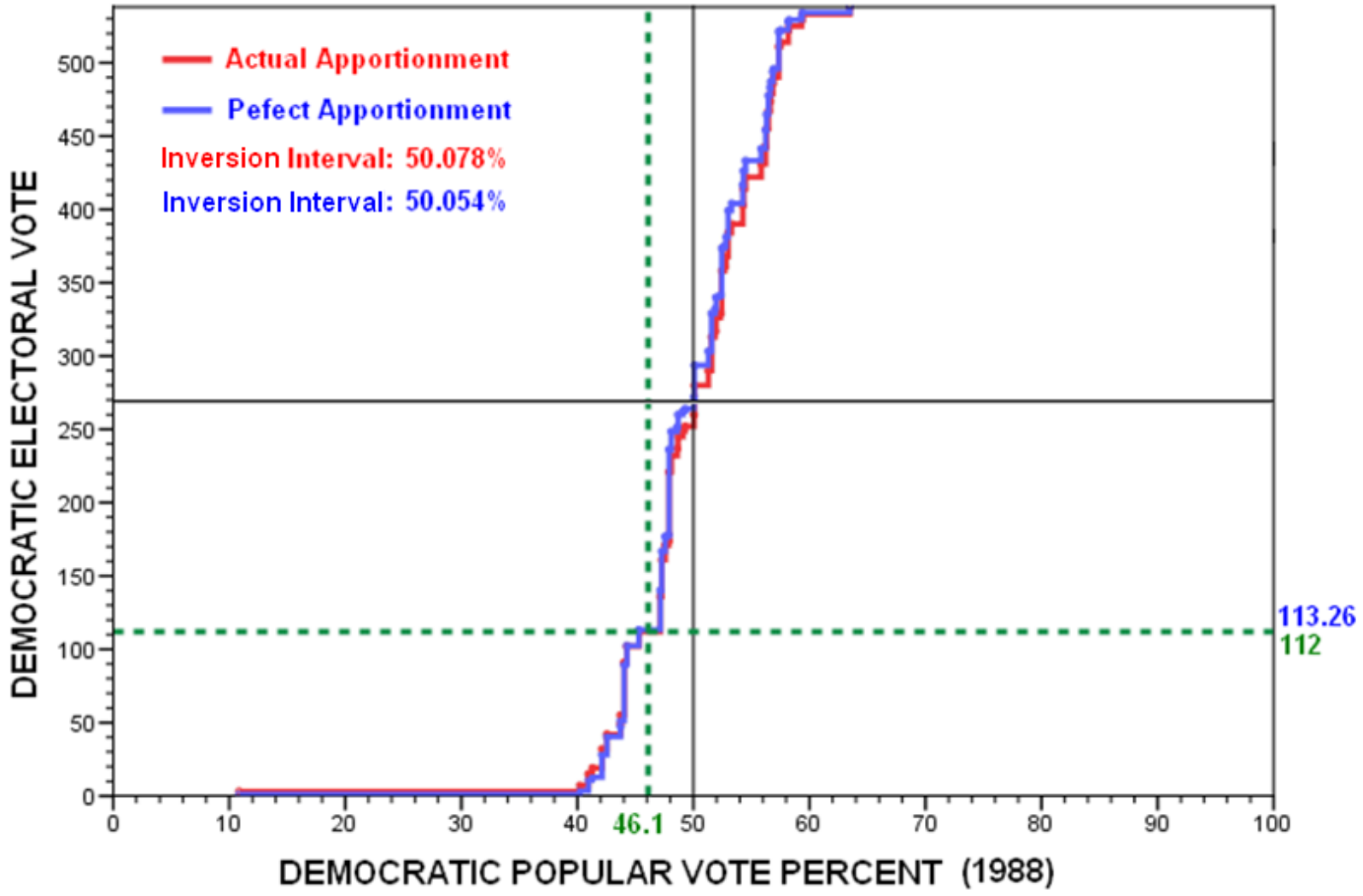
Imperfect Apportionment of Electoral Votes

- The U.S. Electoral College system is (very) imperfectly apportioned, for at least the following reasons:
  - House seats (and therefore electoral votes) must be apportioned in (relatively small) *whole numbers*, and therefore cannot be *precisely* proportional to *anything*.
  - The apportionment of electoral votes is skewed in favor of *smaller states*, as they are guaranteed a minimum of three electoral votes (due to their guaranteed one House seat and two Senate seats); (approximate) proportionality begins only after that.
  - House (and therefore electoral vote) apportionments are based on census information that is anywhere from *two to ten years out-of-date* at the time of a presidential election.
  - The relevant census information is the *total population* of each state and not
    - its *voting-age population*, its *voting-eligible population*, its *number of registered voters*,
    - and certainly not its *actual presidential popular vote* in a given election.

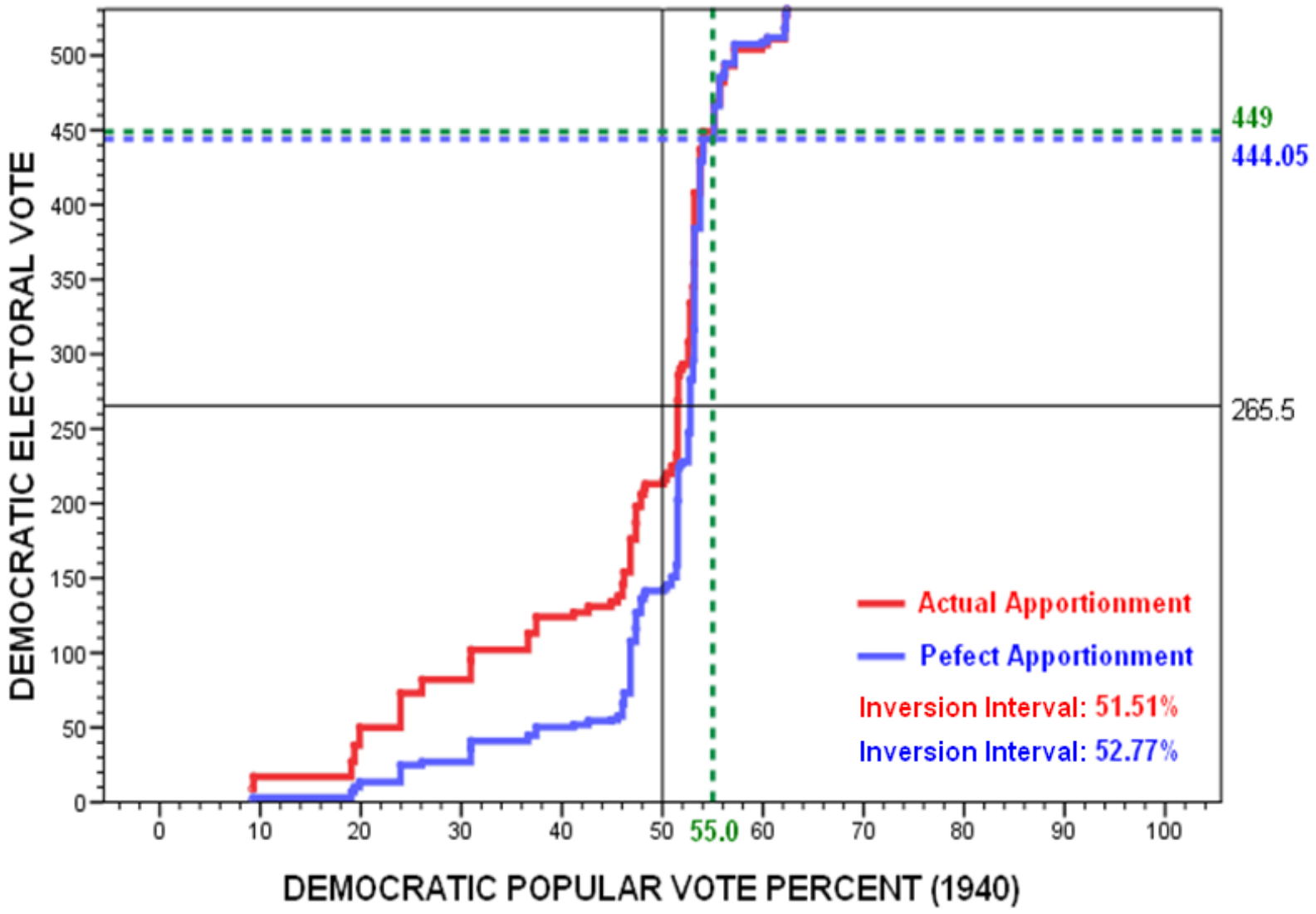
Imperfect Apportionment (cont.)

- Imperfect apportionment may or may not create bias in the PVEV function.
  - This depends on the extent to which state (dis)advantages with respect to apportionment are correlated with their support for the candidates/parties.
- We can separate apportionment effects from distribution effects by plotting the PVEV function given perfectly apportioned electoral votes.
  - Any remaining bias in the PVEV function must be due to distribution effects.
  - If the actual and perfect apportionment PVEVs are similar, apportionment effects are minimal and any substantial inversion interval is due to distribution effects.

In 1988 Apportionment Effects Were Minimal



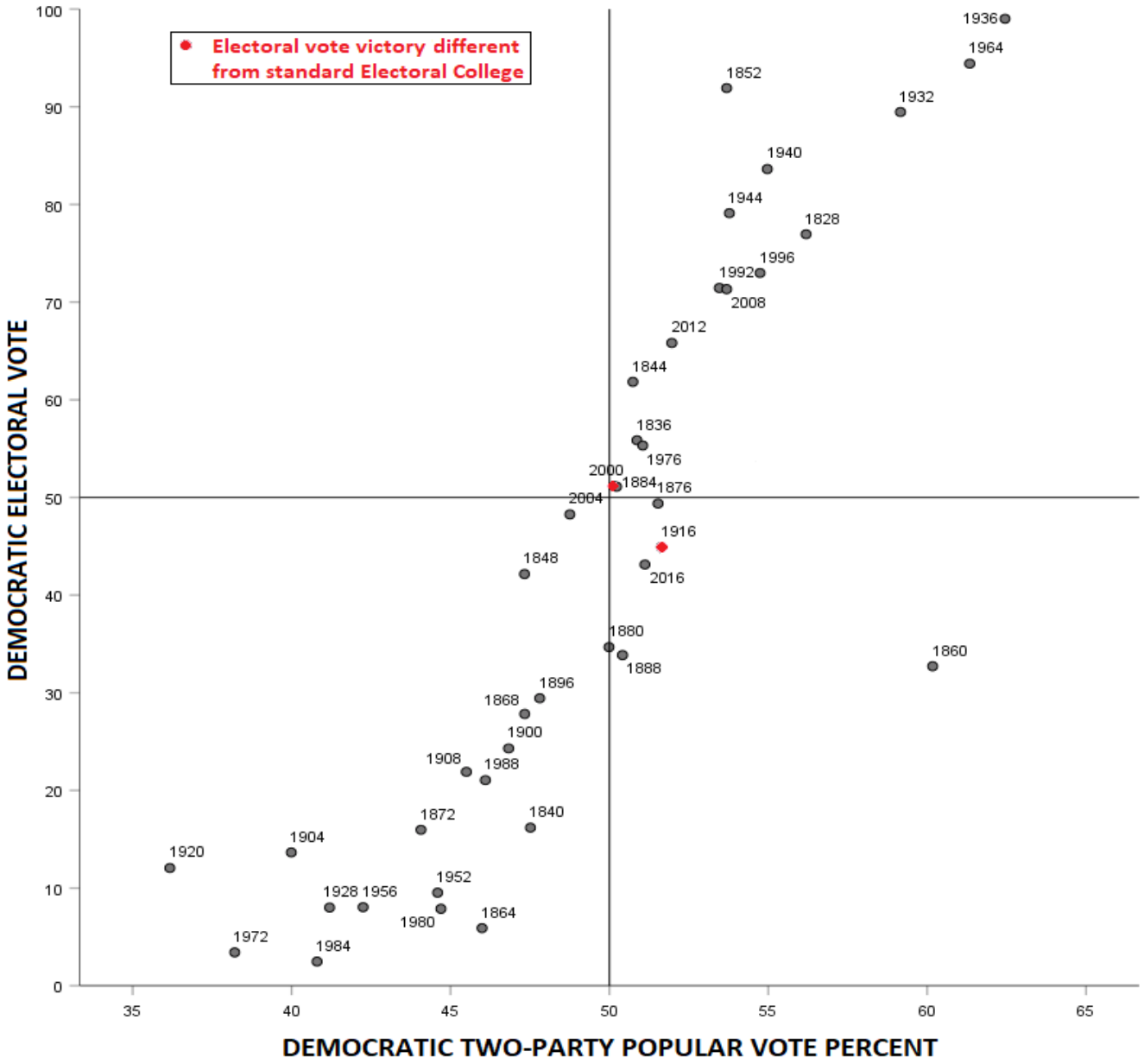
In 1940 Apportionment Effects Were Quite Substantial



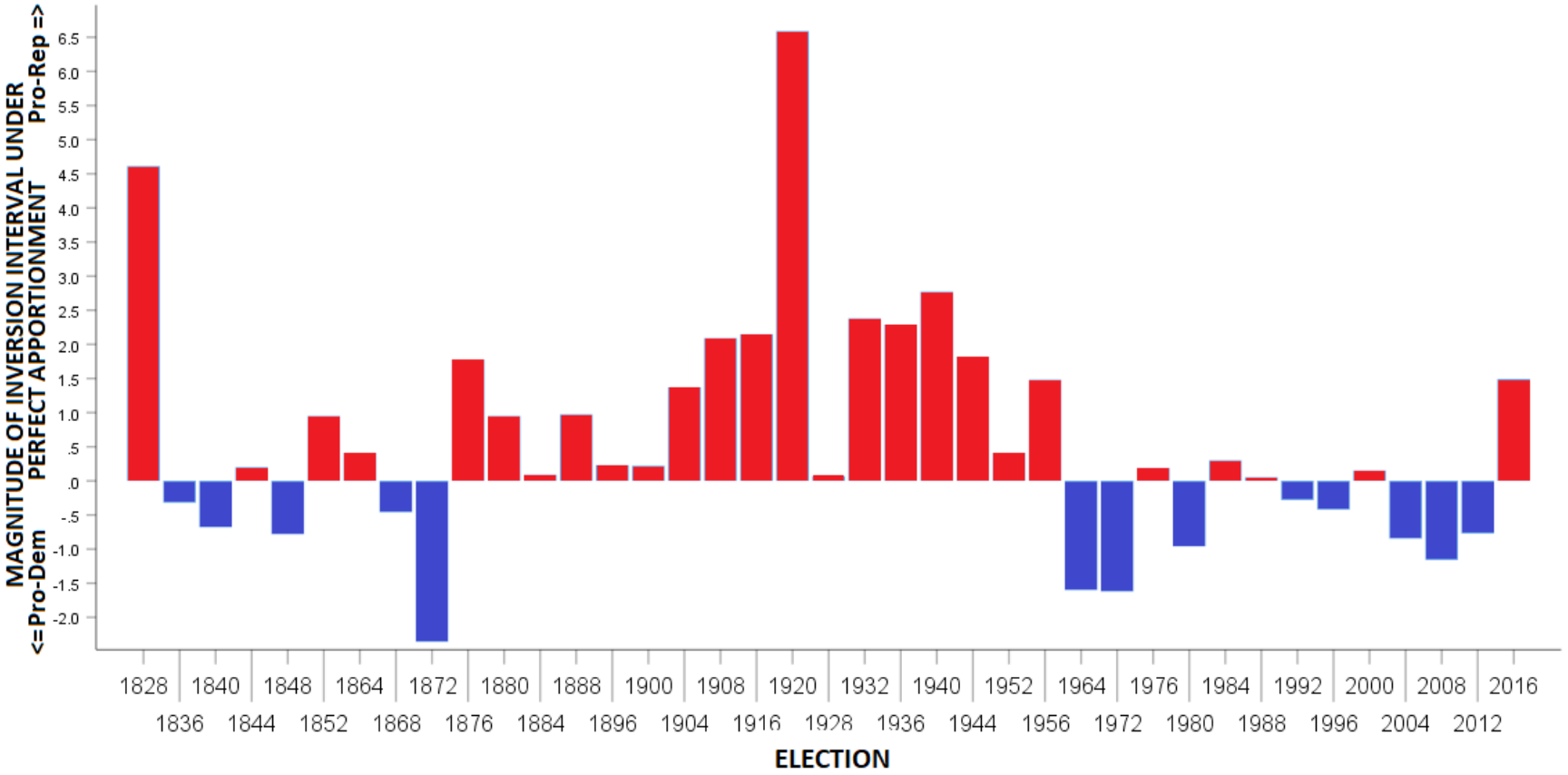
Apportionment Effects (cont.)

- We might expect that perfect apportionment would greatly reduce
  - the frequency of historical election inversions and
  - the average magnitude of inversion intervals.
- In fact, perfect apportionment does not reduce the frequency of historical inversions,
  - though it does reclassify two elections:
    - it “corrects” the 2000 inversion, but
    - it creates a new inversion in 1916.
- Moreover, perfect apportionment actually increases Republican bias on average (so in this respect 1940 is typical), and as a consequence
  - it increases the average magnitude of absolute inversion intervals.

Historical Overview: Perfect Apportionment

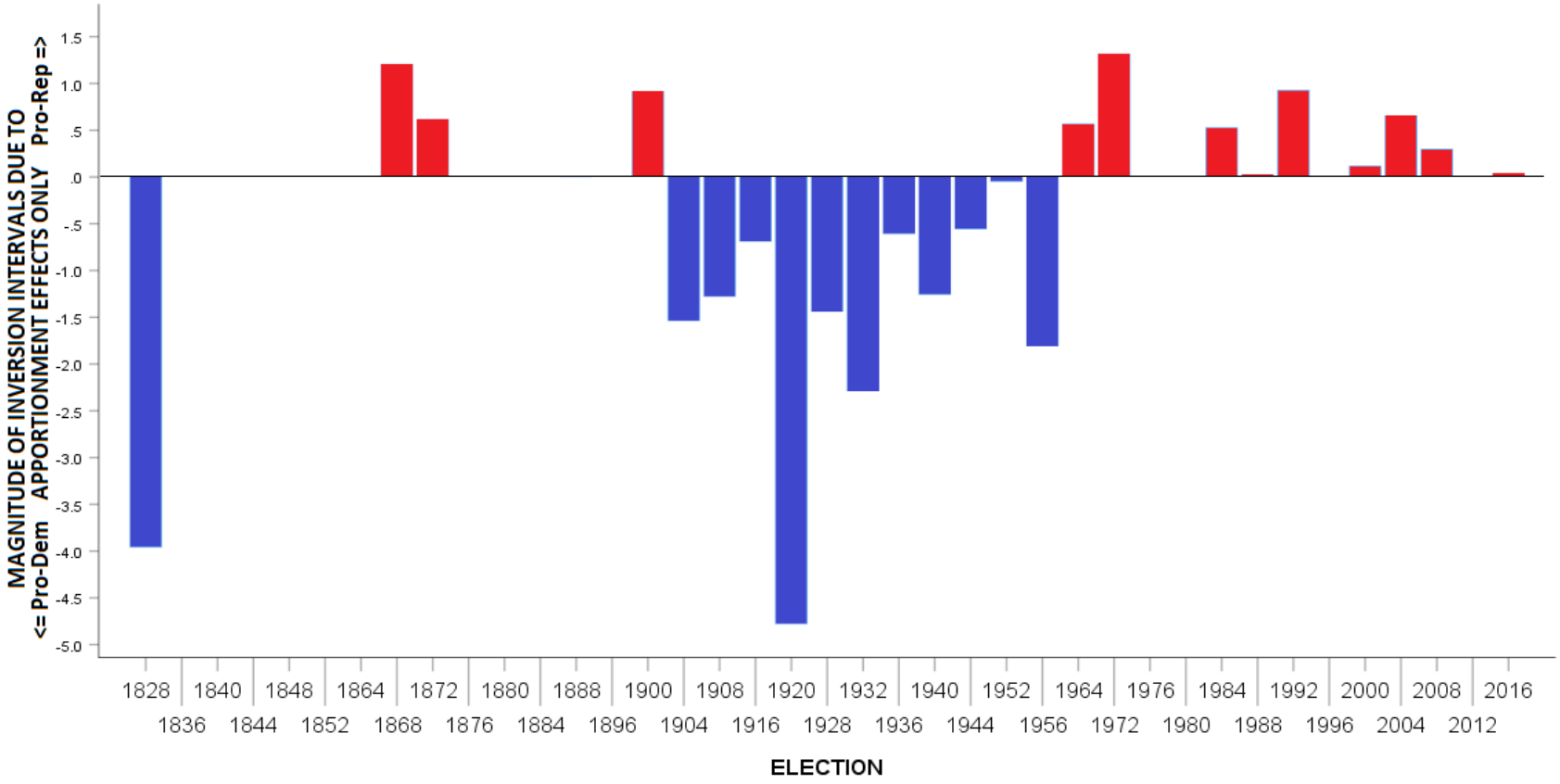


Inversion Intervals under Perfect Apportionment



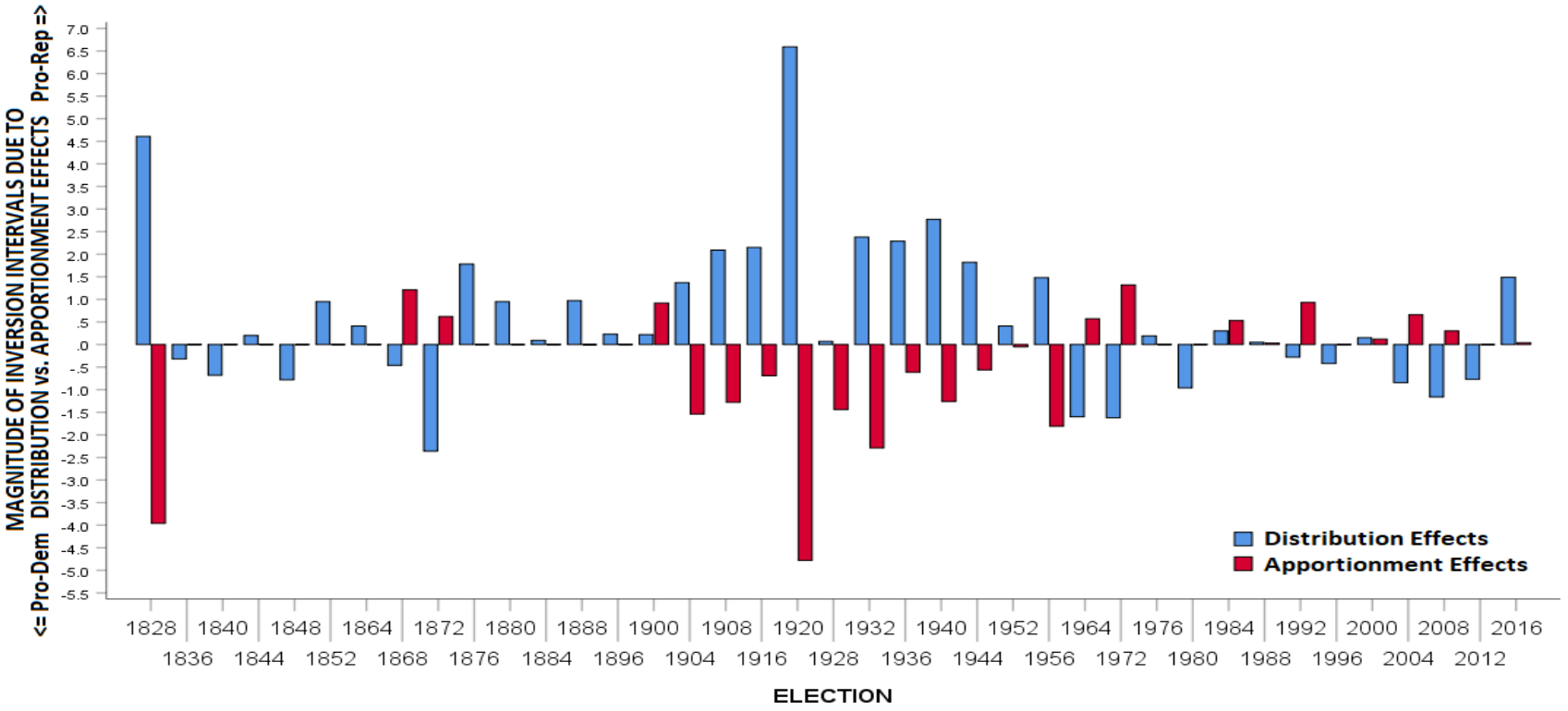
- Given perfect apportionment, the inversion intervals depicted above are due to distribution effects only.
- In the 1876-1956 period, they invariably favored Republicans,
  - though by greatly varying magnitudes.
- Since then they have mostly but modestly favored Democrats,
  - with the notable exceptions of 2016 and 2020.

Inversion Intervals Due to Apportionment Effects Only



- Apportionment effects quite often have no effect on the inversion interval.
- From 1904 through 1956, apportionment effects invariably favored Democrats but since then never have.

Combining Distribution and Apportionment Effects



- "Adding together" the two (usually countervailing) effects for each election gives the earlier graph showing overall inversion intervals.
- It can be observed that distribution and apportionment effects have typically worked in opposition to each other,
  - moderating the overall magnitude of inversion intervals.

Historical Summary

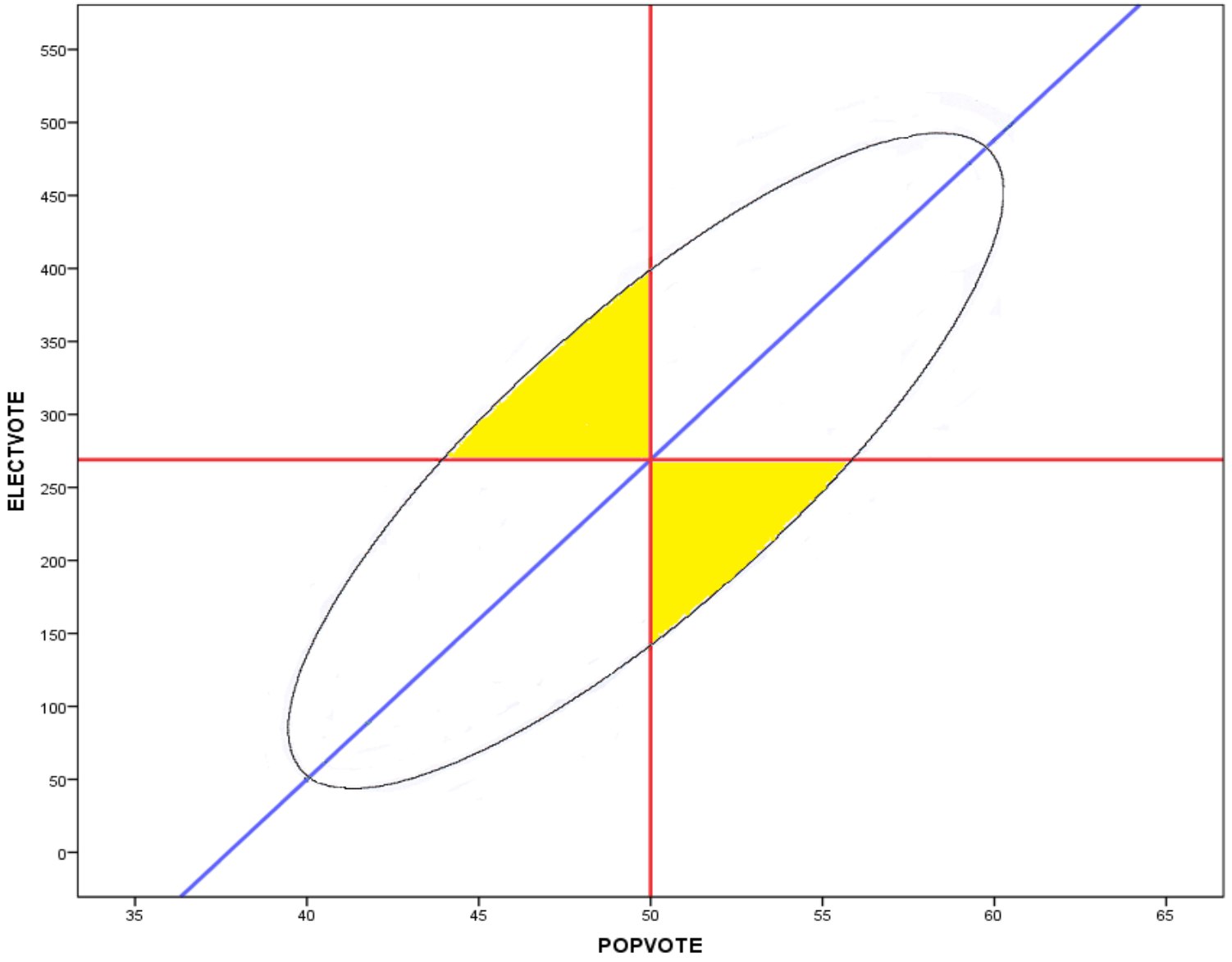
- Over the entire period, apportionment effects have generally favored Democrats and distribution have generally favored Republicans, with the latter effects being somewhat stronger than the former, producing a pro-Republican bias.
- However, throughout the 19th Century, there is no consistent pattern,
  - evidently reflecting relatively loose party ties in the early party systems followed by the disruptive events leading to and following the Civil War.
- The overall pattern is especially clear from 1908 through 1944 (except 1928), reflecting the peculiar character of the Democratic "Solid South" of that era,
  - where Democrats won overwhelming (and thus "inefficient") popular vote margins (producing very strong pro-Rep distribution effects)
  - but on the basis of very low turnout (producing moderately strong pro-Dem apportionment effects).

Historical Summary (cont.)

- From 1952 through 1960, the outer South became more Republican, so the partisan impact of the two effects was reduced.
- Beginning in 1964, the heretofore Democratic “Solid South” began to switch party sides, so the partisan impact of the two effects was reversed.
- As as the Voting Rights Act took effect,
  - (especially black) turnout increased in the South,
    - which provides the basis for a substantial (but rarely winning) DPV% in Southern states.
- Thus both apportionment and distribution effects become relatively small.

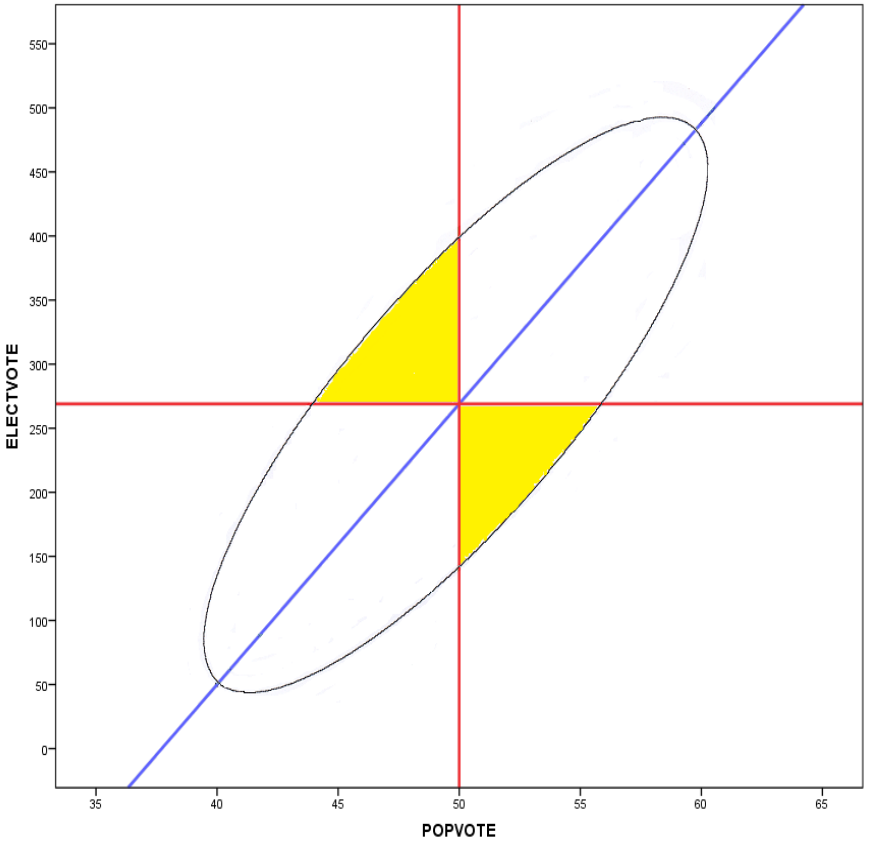
“Are Presidential Inversions Inevitable?”

(Cervas and Grofman, *Social Science Quarterly*, June 2019)



Implications of the Schematic Scatterplot

- The lesson of the schematic scatterplot is that, so long as a given popular vote split can produce varying electoral votes splits, inversions may occur in relatively close elections.
- The frequency of election inversions depends on the closeness of the popular vote.
  - At PV ≈ 50%, the probability of inversion is essentially 50%.
- Holding constant the dispersion of the PV, the frequency of inversions depends on
  - how strongly EV and PV are correlated, and
  - the degree of bias in the relationship between EV and PV.
- Inversions are potentially inevitable unless
  - the correlation between PV and EV is perfect and
  - the relationship is entirely unbiased.



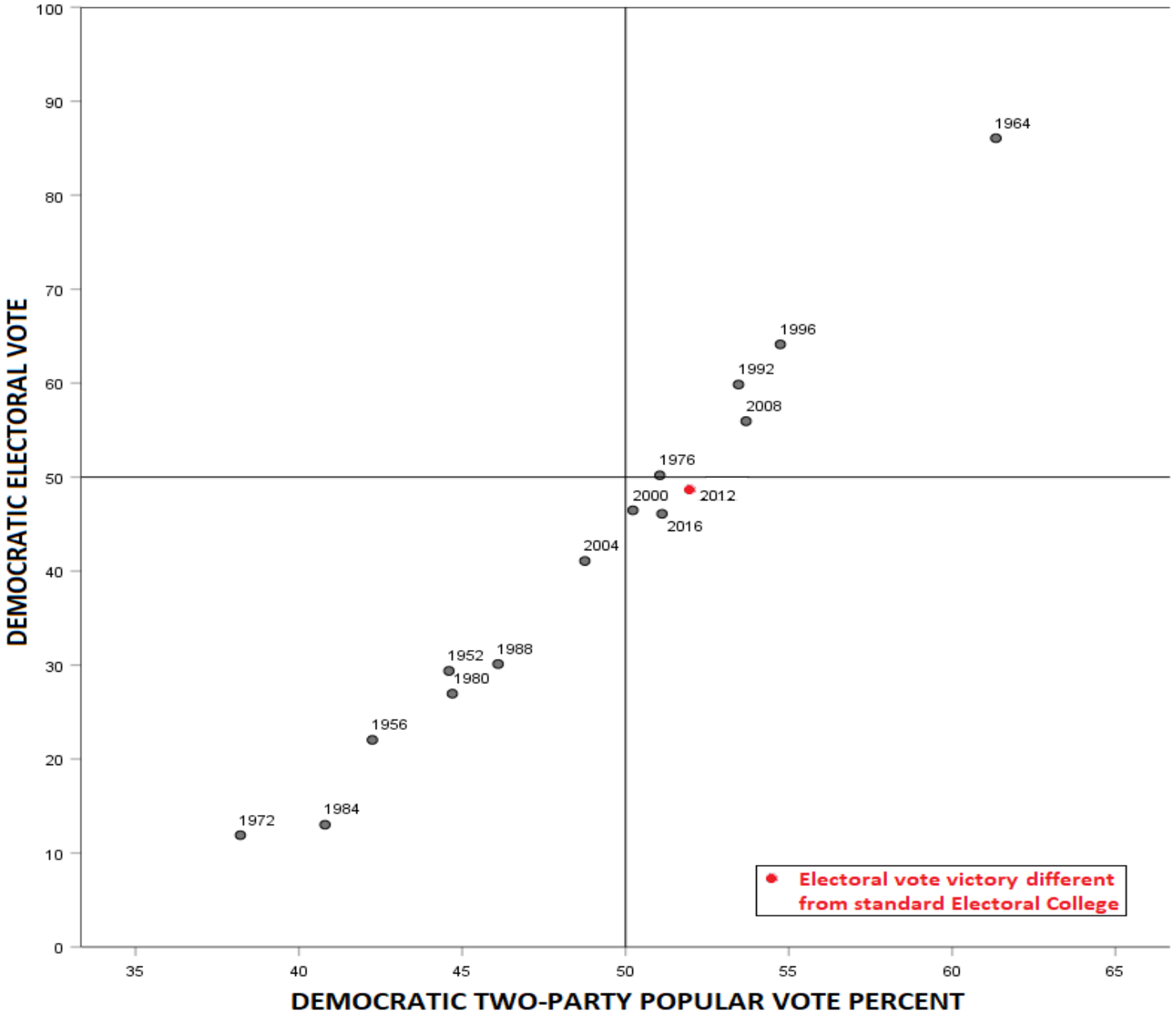
Electoral College EC Variants

- Two commonly discussed variants ("reforms") of the Electoral College are:
  - the *Modified District Plan*
    - as exemplified by present practice in ME and NE (but which would be much more consequential in large states), and
  - the (*Pure*) *Proportional Plan*
    - in which the electoral vote in each state is (fractionally) divided between (or among) the two (or more) candidates in proportion to the popular vote in the state.

The Modified District Plan

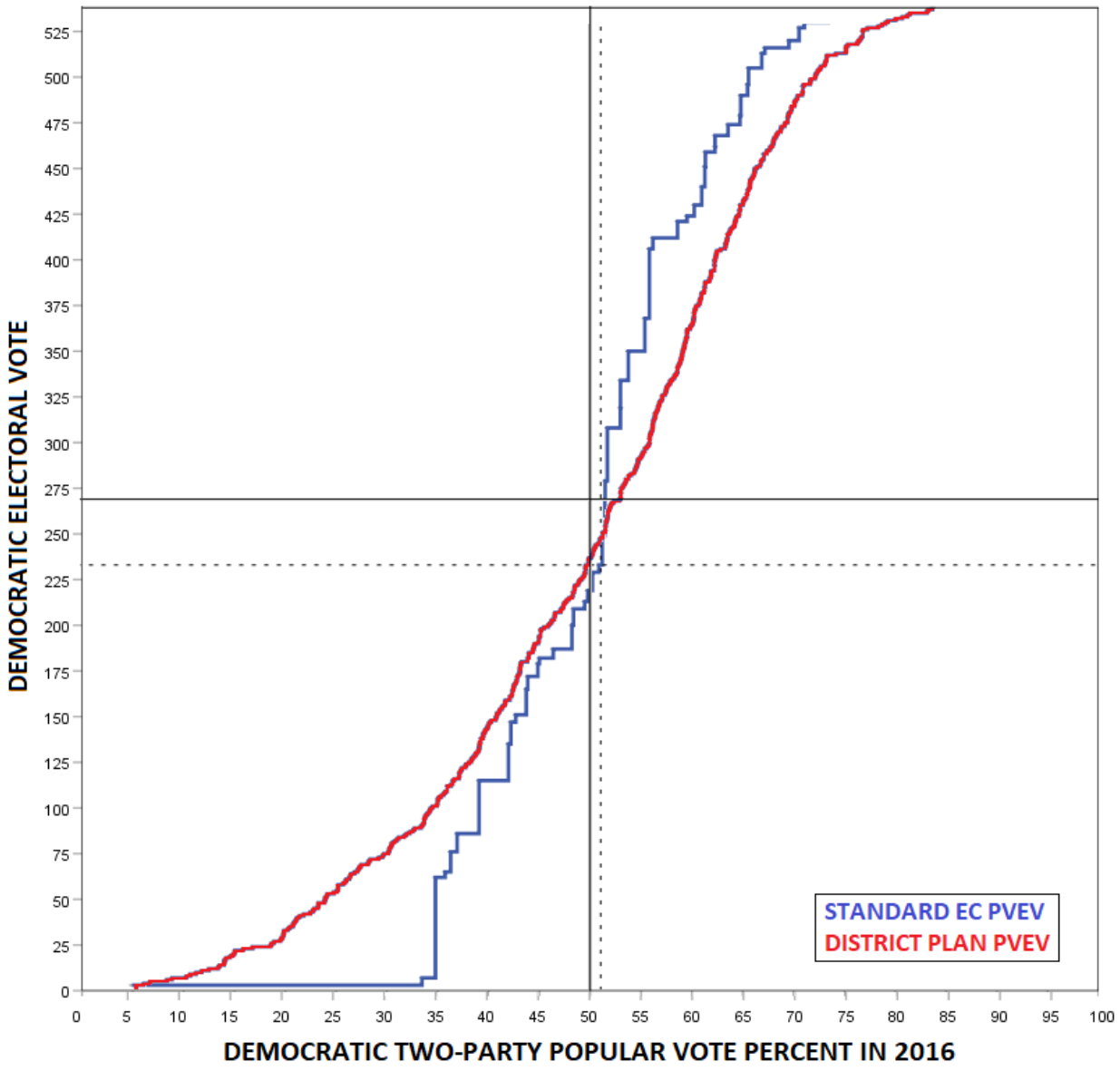
- Data (i.e., presidential vote by CD) needed to examine the district plan in historical elections evidently exists only back to 1952 (and the 2020 is not yet available),
  - and some of the of the earlier data may be somewhat problematic.
- Over the period for which data exists, the District Plan produces a more “proportional” PVEV than the standard EC,
  - and an even higher national PV-EV correlation (+0.992 vs. +0.969),
  - but the relationship exhibits a considerable pro-Rep bias.

Historical Overview: Modified District Plan

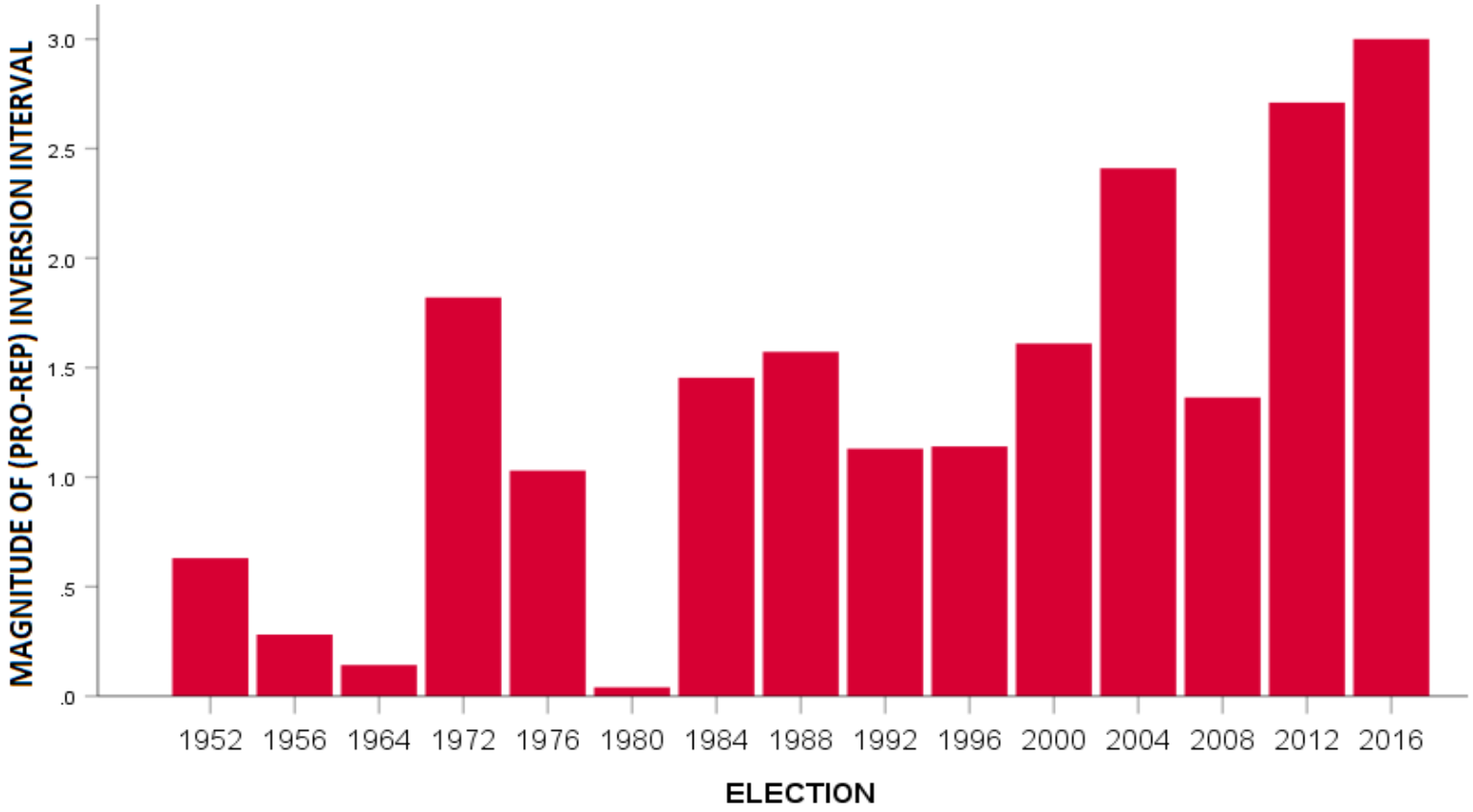


2016: Regular EC vs. District Plan PVEVs

Clinton would have won more EVs at PV=50% (237 vs. 219), and more EVs at the actual PV=51.1% (248 vs. 233); nevertheless, the inversion interval would have been much wider (+3.0% vs. +1.5%)



Inversion Intervals under Modified District Plan: 1952-1956

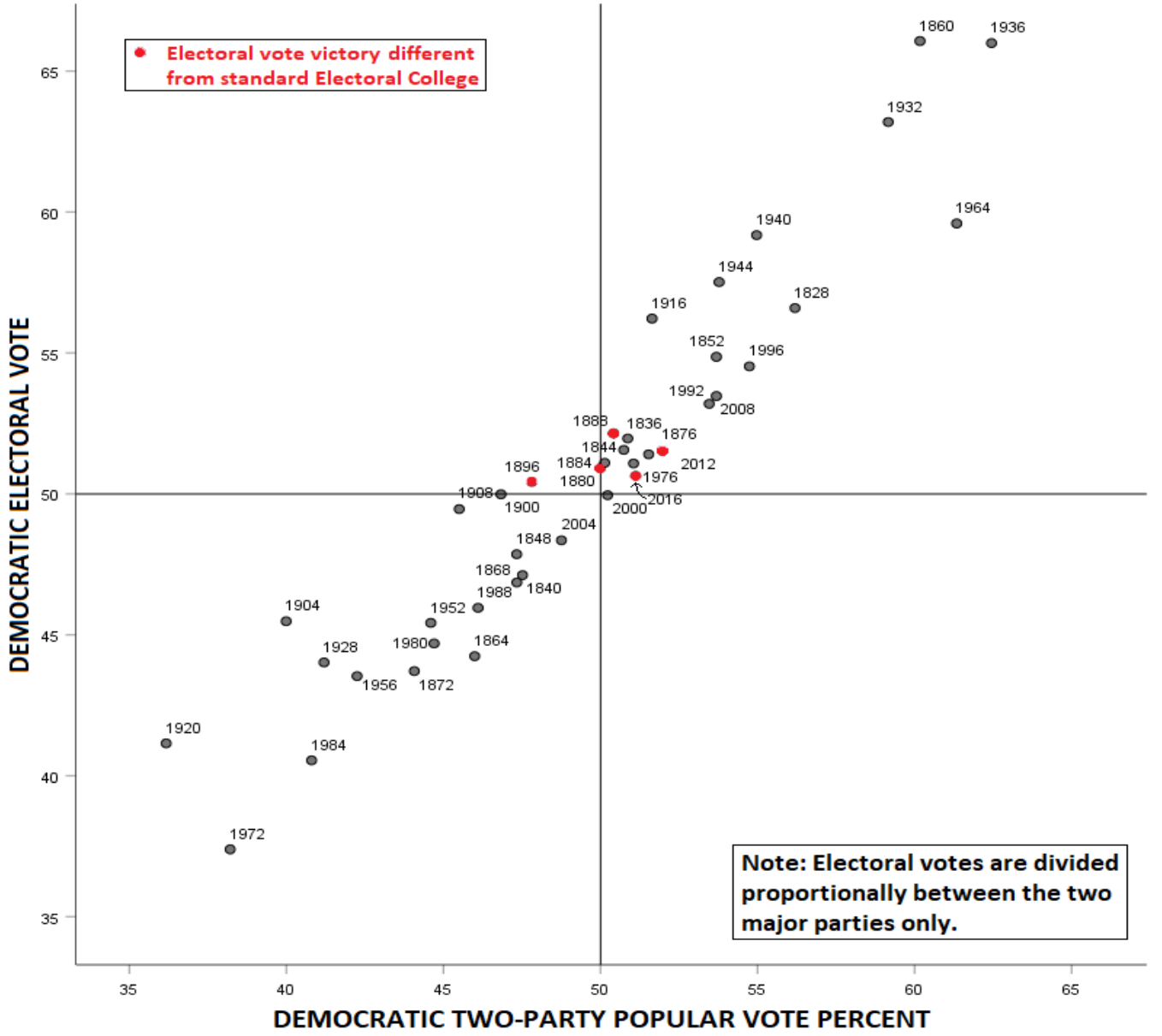


- Since 1952 (though probably not earlier), the Modified District Plan has had a consistent pro-Rep bias,
  - which has increased over time and
  - has become very pronounced recently,
    - presumably because of extensive Republican gerrymandering of CDs in 2010.

The Proportional Plan

- *Note*: this analysis continues to be done on a strictly two-party basis.
  - In particular, electoral votes are proportionately divided between the two major parties only,
    - in contrast to the proposed [Lodge-Gossett] constitutional amendment and its various reinventions.
- Over the whole period, the proportional plan
  - unsurprisingly produces a highly proportional PVEV, and
  - the correlation is much higher (+0.948 vs. +0.785) than under the standard EC, and
  - if anything there is pro-Democratic bias.
- Examining the relationship separately for the 1896-1944 and 1952-2016 periods
  - increases the correlation further (to +0.994 in the earlier period and +0.995 in the latter), but
  - shows that there was huge pro-Dem bias in the earlier and a small pro-Rep bias in the latter period.

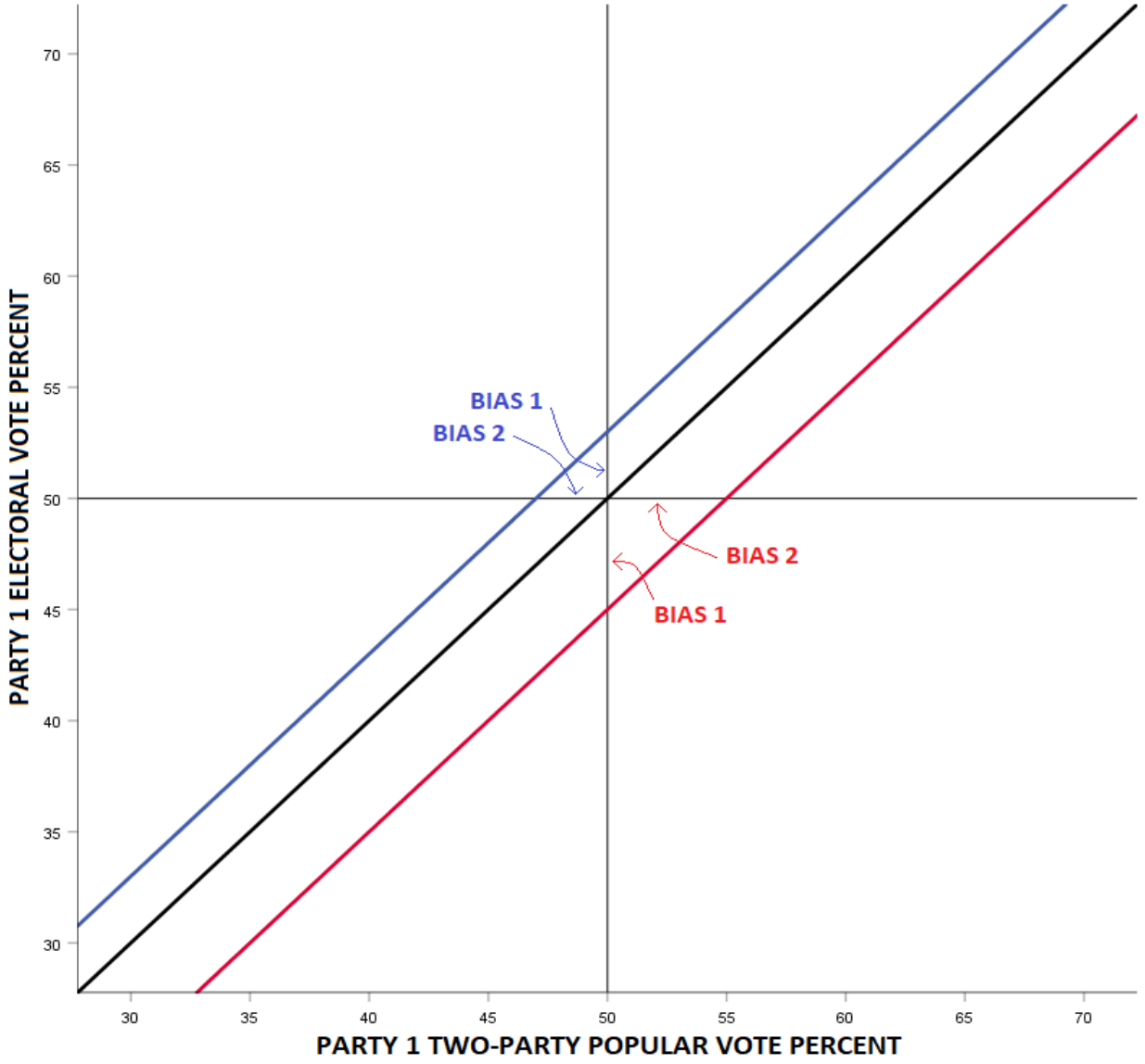
Historical Overview: Proportional Plan



The PVEV under the Proportional Plan

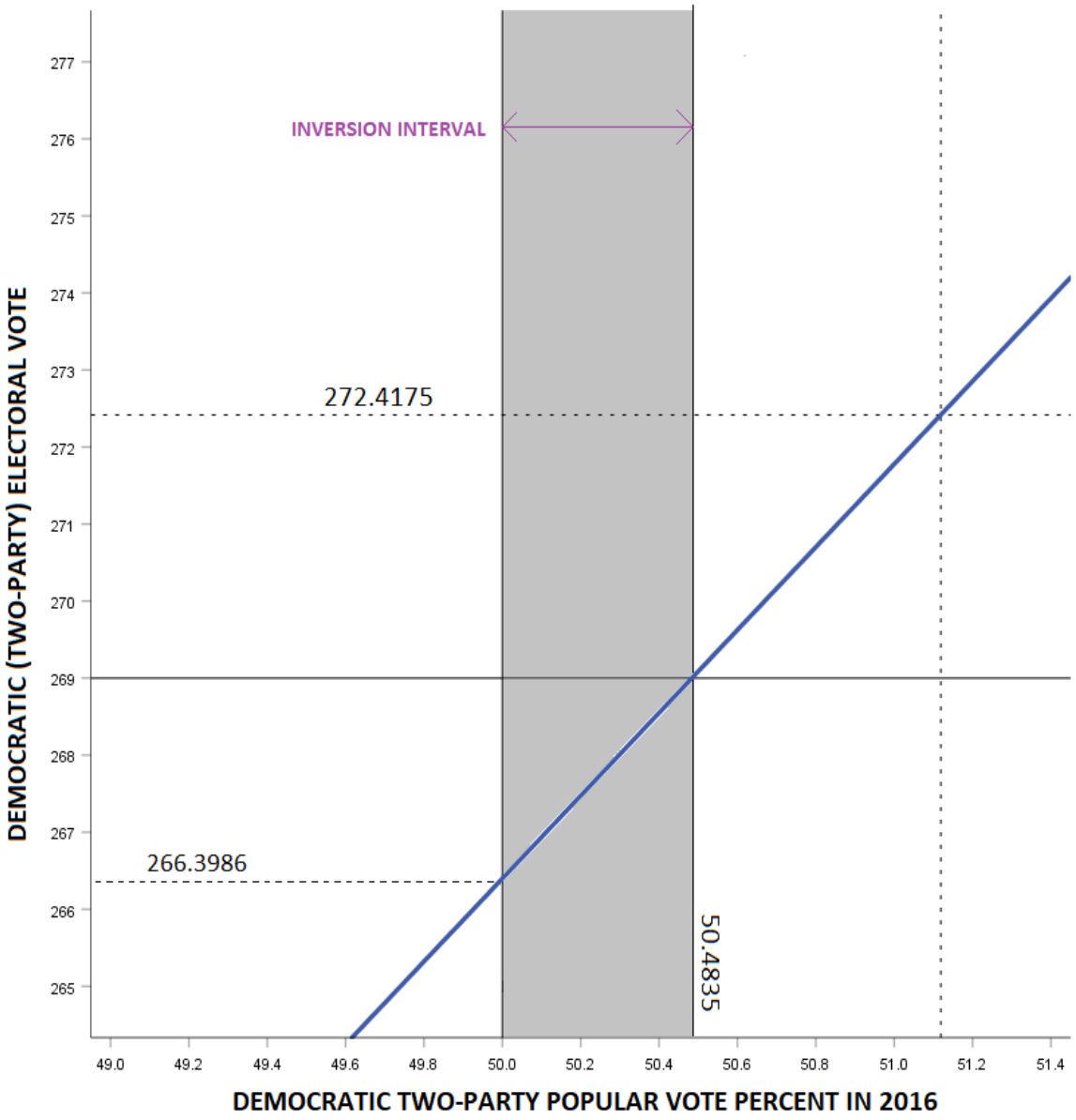
- The Proportional Plan by design
  - eliminates distribution effects but
  - preserves apportionment effects.
- Under the Proportional Plan the PVEV is a straight line with a fixed slope that is equal to:
  - (1) 1 if electoral votes (like popular votes) are expressed as percentages.
  - (2) 5.38 given 538 electoral votes,
  - (3) X/100 given a total of X electoral votes,
- Thus PVEVs vary from election to election only with respect to their *levels* (i.e., intercepts), which indicate the degree and direction of partisan bias,
  - which is constant over the entire PV range and
  - which reflect apportionment effects only.
- It follows algebraically/geometrically that, if EV and PV are both expressed as percentages,
  - Bias 1 = - Bias 2.
- In any case, one can be calculated from the other
  - because the swing ratio/responsiveness is fixed and constant.

Proportional Plan: Bias 1 = -Bias 2



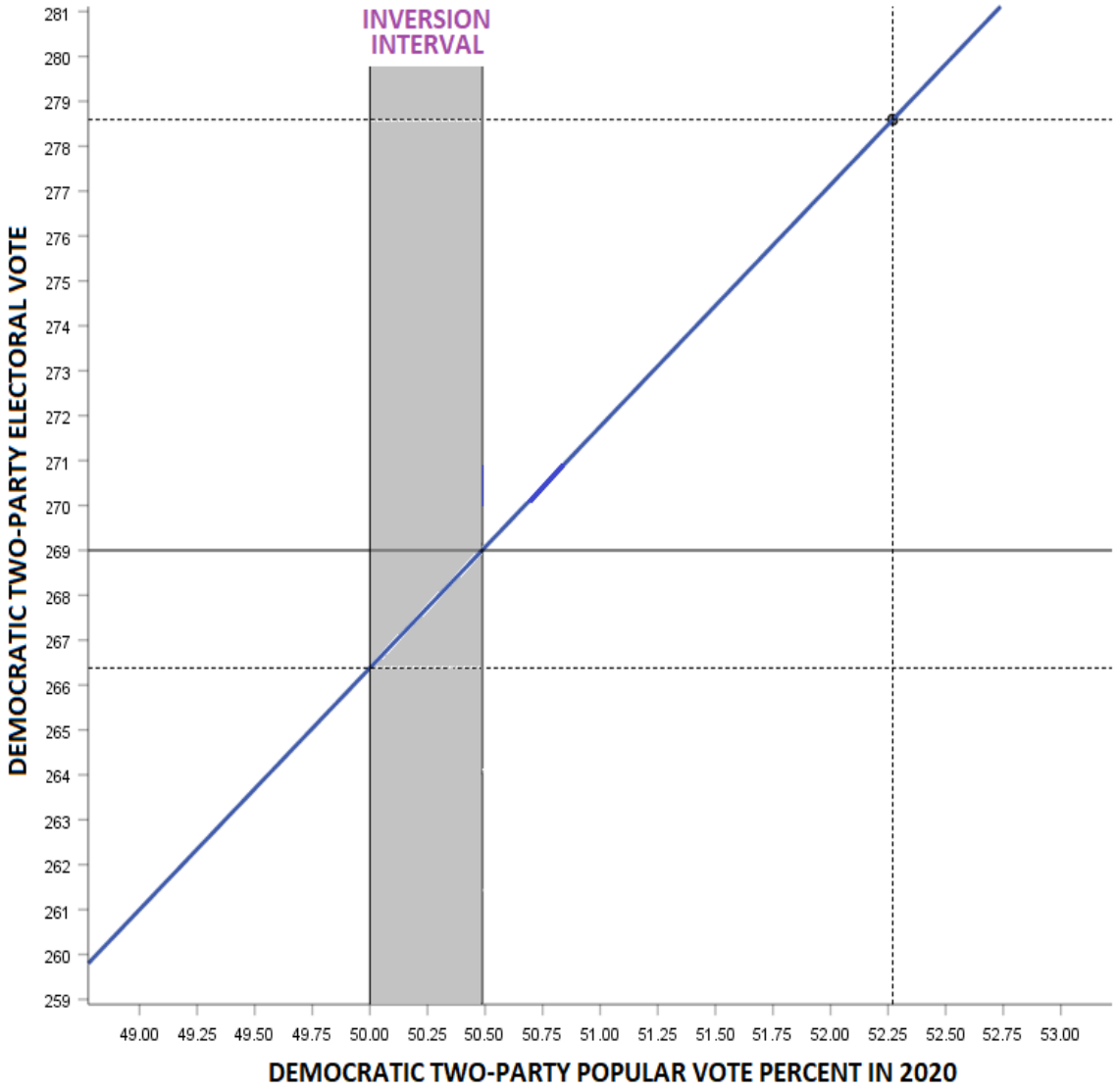
Proportional Plan: PVEV in 2016 (Zoom In)

- Clinton would have won about 266.4 EVs at DPV = 50%.
- Clinton would have needed about 50.5% of the PV to win an EV majority.
- Clinton would have won about 272.4 EVs with her actual PV.

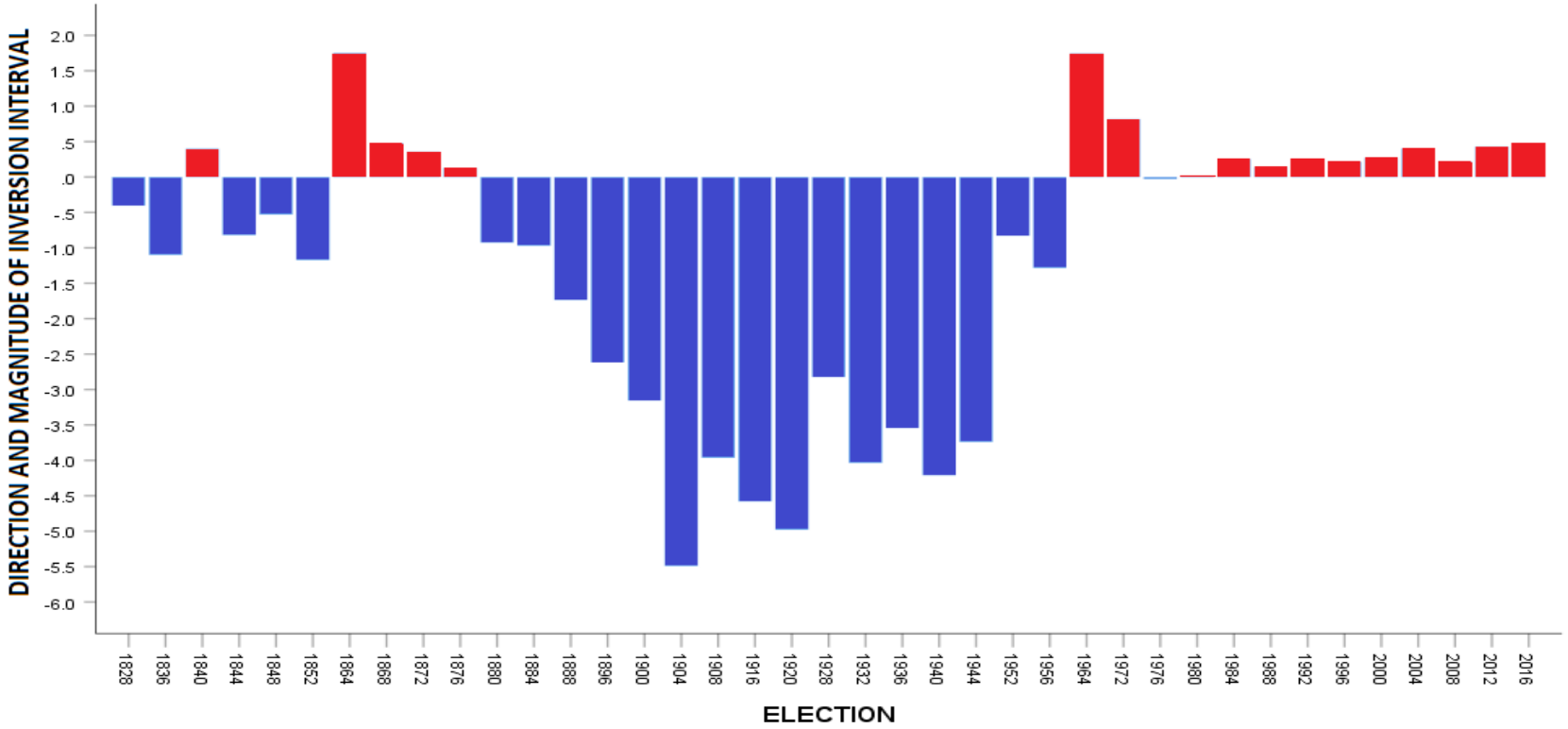


Proportional Plan PVEV in 2020 (Zoom In)

- Biden would have won about 266.4 EVs at PV = 50%.
- Biden would have needed about 50.5% of the PV to win an EV majority.
- Biden would have won about 278.7 Evs with his actual PV.



Inversion Intervals under Proportional Plan



- The Proportional Plan produces
  - a huge and consistent pro-Dem bias in the 1880-1956 period
    - that resulting from elimination of distribution effects and preservation of apportionment effects, and
  - a modest but consistent pro-Rep bias since then.

Simulated Elections

- Another approach to analyzing the propensity of (variants of) the Electoral College to produce inversions is to generate large numbers of simulated elections,
  - that is, profiles of state-by-state two-party popular votes
  - and check whether each simulated election produces an inversion.
- Elections may be simulated by either
  - an *a priori* data-generating process not based on any characteristic "electoral landscape/alignment", or
  - an *empirically based* data-generating process that is based on the electoral landscape/alignment of a given election or the average over a period of elections (or on demographic and survey data, e.g., Geruso, Spears, and Talesara (2019), "Inversions in US Presidential Elections:1836-2016" (http://utecs.org/).
- The former type of simulation can tell us something about the relative propensity of different variants of the Electoral College (or other electoral systems) to produce election inversions, but
  - cannot show whether one or other party is favored by such inversions.
- The purest example of an *a priori* simulation is one based the "impartial culture" assumption.

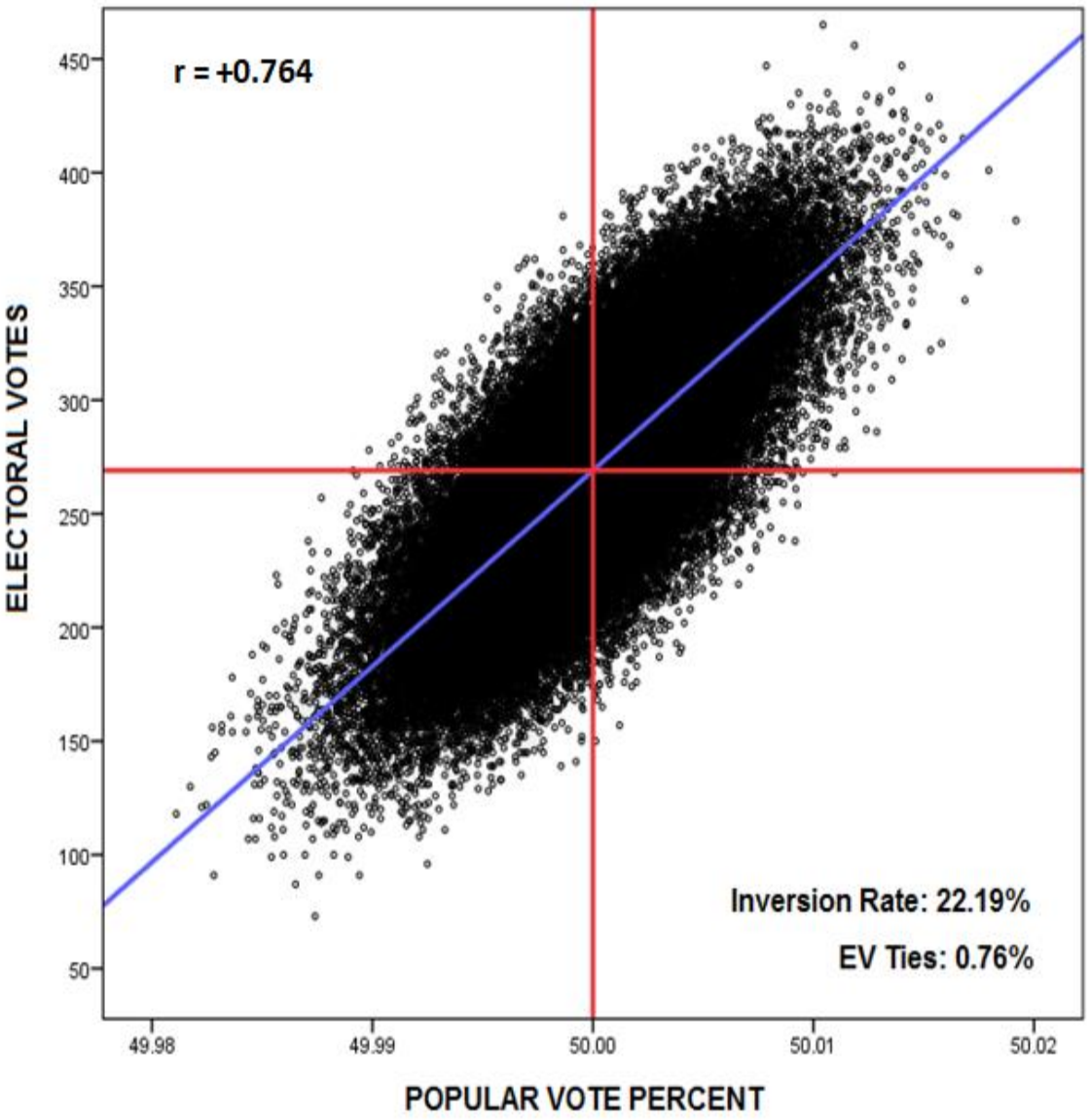
Impartial Culture Simulated Elections

- *Impartial Culture* (IC) elections: everyone votes as if tossing a fair coin.
- IC is a common assumption
  - in social choice theory,
  - and for interpreting the Banzhaf voting power measure.
- The impartial culture implies that essentially all large-scale elections are extremely close.
- It is known that, given *uniform districts* and *perfect apportionment,* the inversion rate in large IC elections ≈ 20.5%

Feix et al., "The Probability of Conflicts in a U.S. Presidential Type Election," *Economic Theory* (2004)

Impartial Culture: Standard EC

- Scattergram of 64,000 IC elections using the 2010 apportionment of electoral votes and uniform turnout across states.
- Note that there is very small dispersion in PV but substantial dispersion in EV.
- Non-uniformity of districts evidently increases the propensity for inversions somewhat.

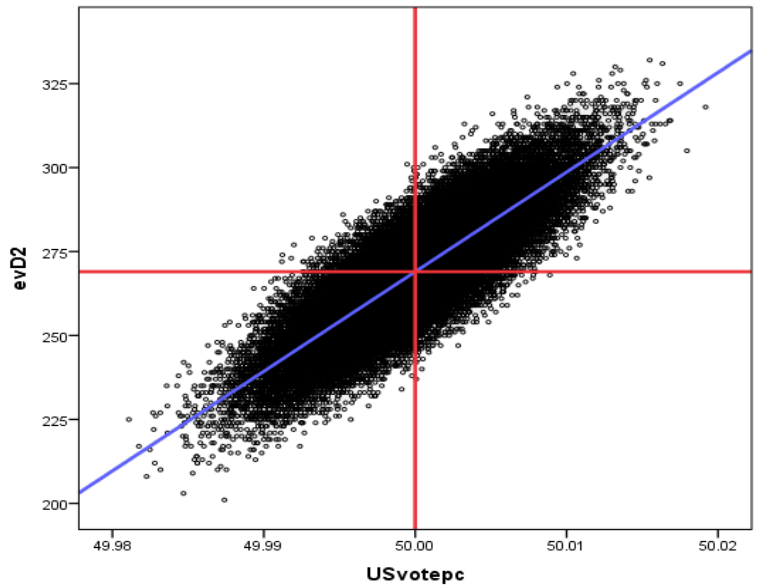


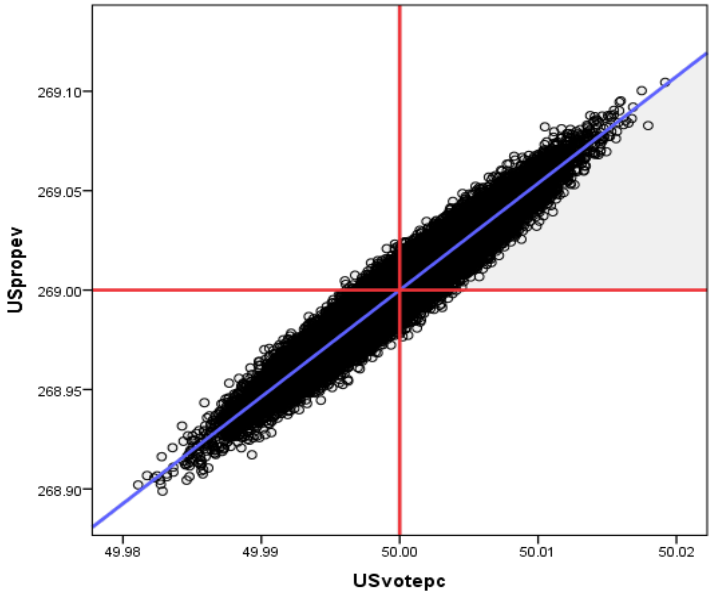
Impartial Culture Election Statistics

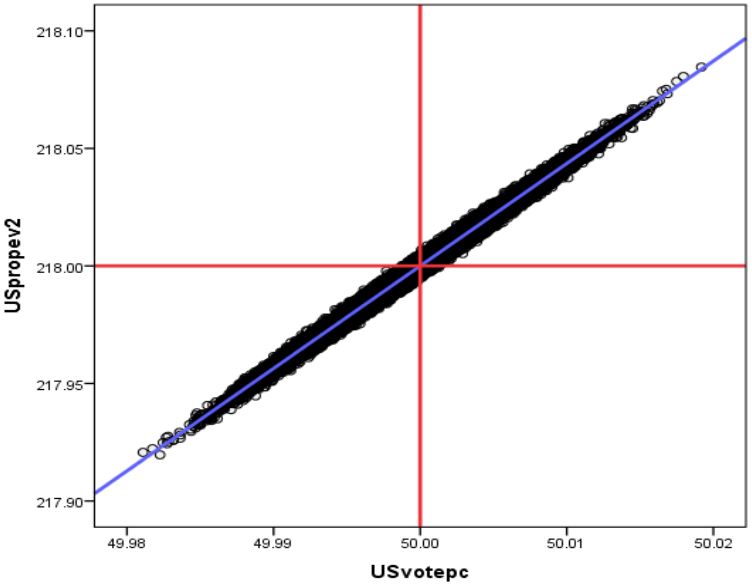
- Non-uniform districts and imperfect apportionment increase the rate of inversions but only slightly.
- *Note*: district plans entail state-level inversions,
  - which are mitigated but not eliminated under the modified plan.

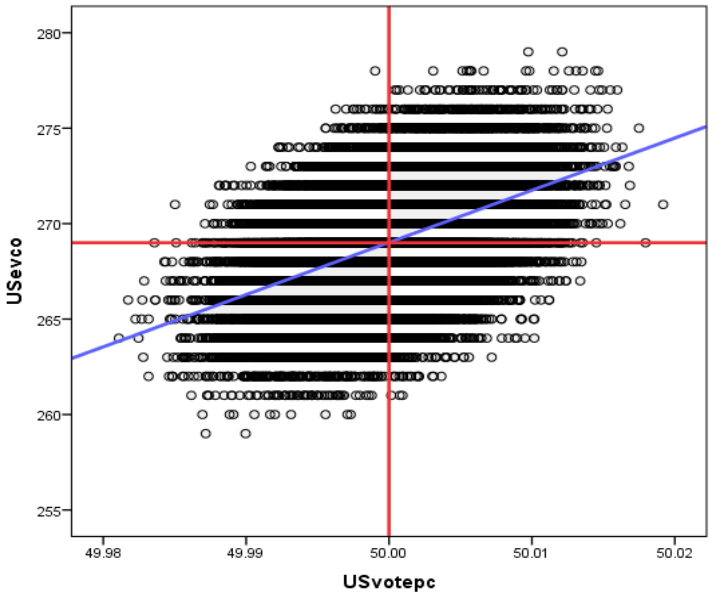
Electoral College Variant	*Inversions*	*EV Ties*
Existing EC	22.19%	0.76%
EVs Based House Seats Only	23.49%	0.76%
EVs Perfectly Proportional	23.80%	–
EVs Perfectly Proportional +2	22.51%	–
Equal EVs	24.87%	–
Pure District Plan	19.25%	3.76%
Modified District Plan	16.99%	2.52%
Pure Proportional Plan	8.45%	–
Pure Proportional (House Seats Only)	2.87%	–
Whole-Number Proportional Plan	26.63%	15.34%

Modified District; Proportional; Proportional (House only); Whole-Number Proportional









More Realistic Random Elections

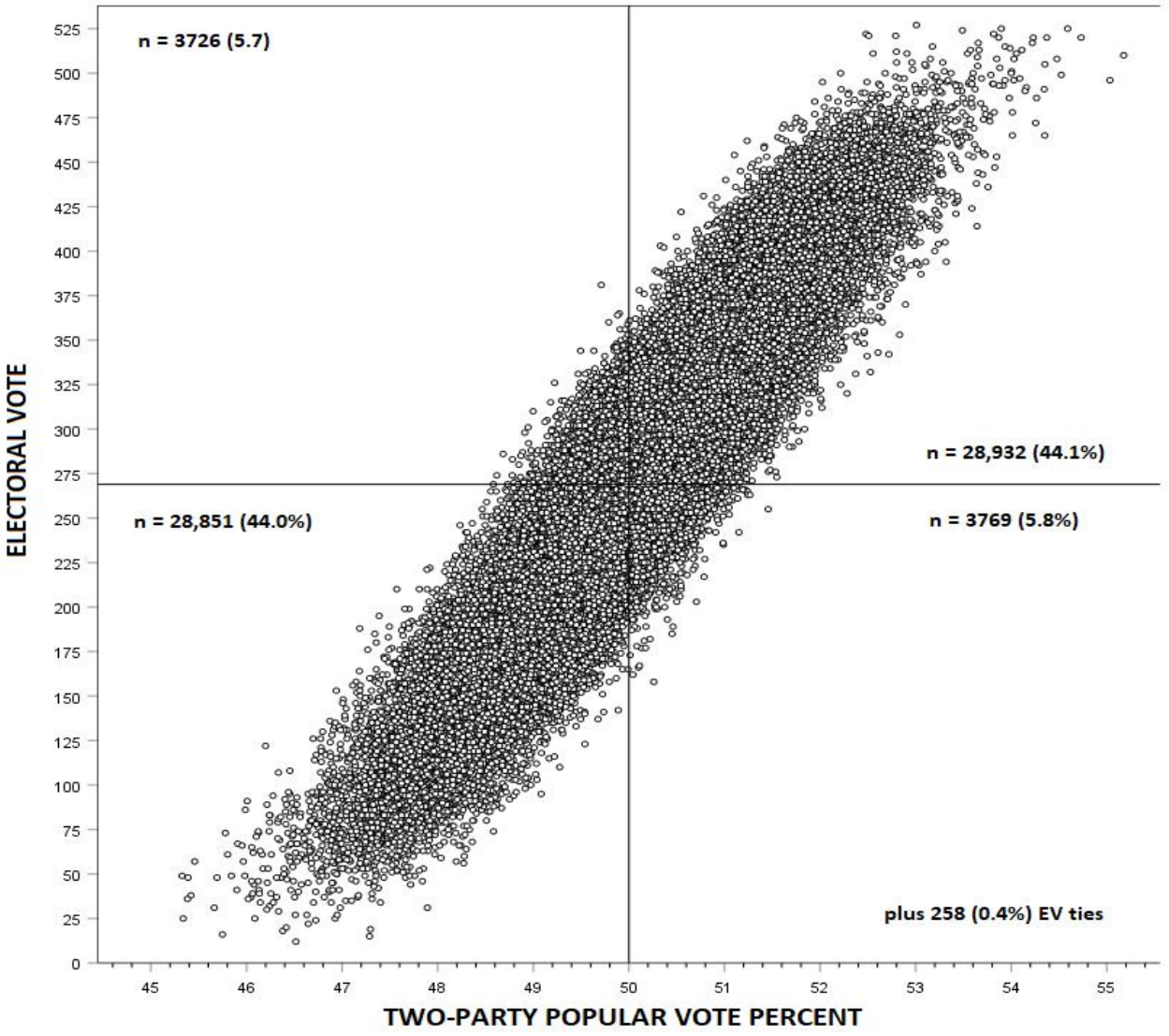
- The election generating formula for each election:

  ExpV + EV x STATESWING + EV x NATIONALSWING, where

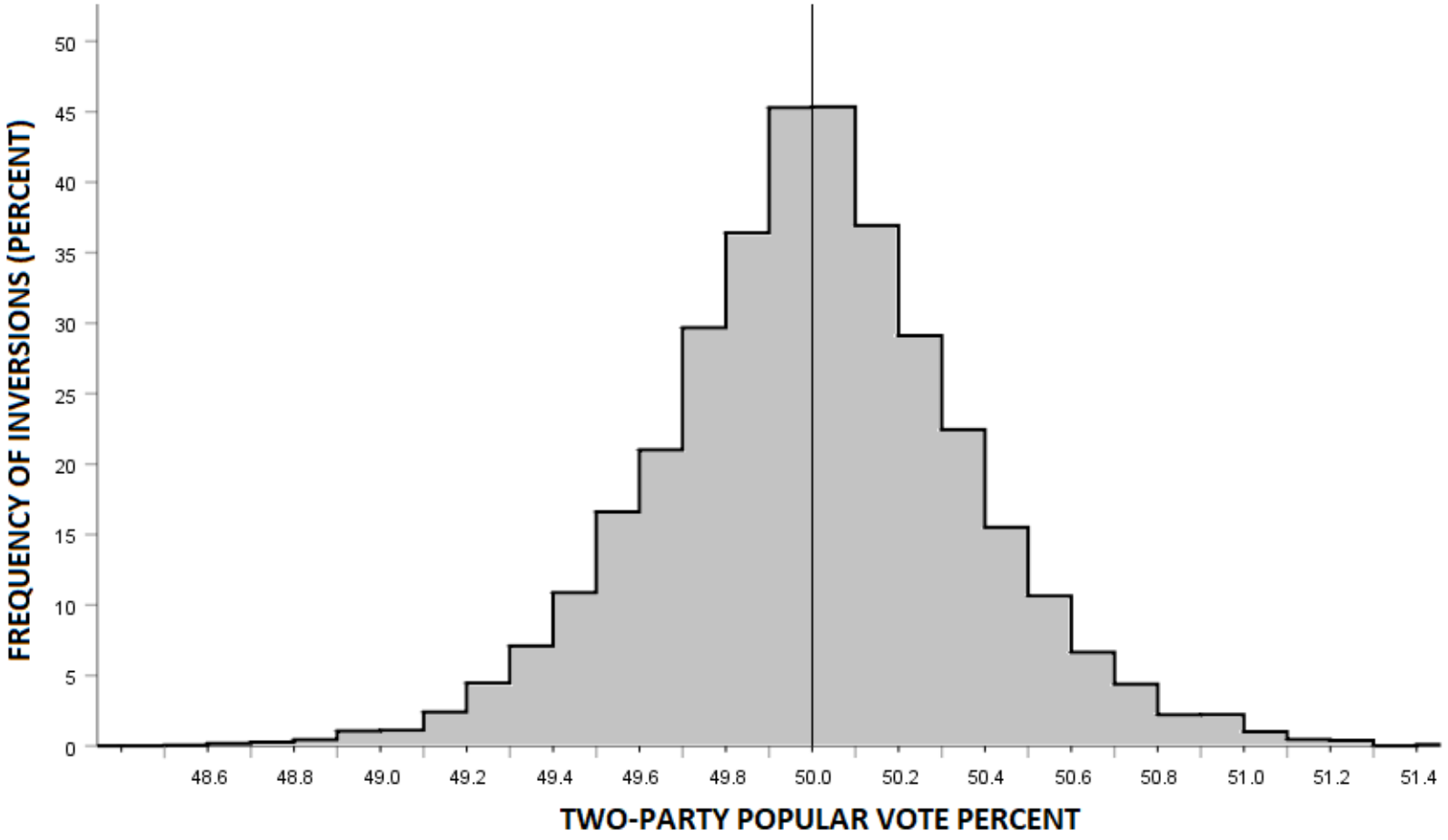
  - ExpV is the EXPECTED VOTE and is equal to half the number of voters in each STATE,
  - STATESWING is a normal random variable with mean = 0 and SD = 0.02, and
  - NATIONALSWING is also a normal random variable with mean = 0 and SD = 0.02.
- These random elections are very roughly similar to "Impartial Anonymous Culture" (IAC) elections.
- Since these simulations have been done at the state (not CD) level, there are no results for district plans.

Scattergram of 65K+ Random Elections

- The popular vote range is quite narrow: 45% < PV < 55%, though the EV range is wide.
- Election inversions occur in the range 48.5% < PV < 51.5%.



Standard EC: Inversions by PV Closeness in Random Elections



Random Election Statistics

EC Variant	*Inversions*	*EV Ties*	*EV by PV Correlation*
Standard EC	11.436%	0.394%	+0.927
EVs Based on House Seats Only	12.427%	0.542%	+0.915
EVs Perfectly Proportional	12.663%	–	+0.916
EVs Perfectly Proportional + 2	11.578%	–	+0.927
Equal EVs	14.340%	–	+0.891
Pure Proportional	2.316%	–	+0.997
Pure Proportional – 2	2.724%	–	+0.996
Whole Number Proportional	7.851%	5.614%	+0.947

- There are fewer inversions (and ties) than in the impartial culture because
  - popular vote percent has greater spread,
  - which produces a higher correlation.

Simulated Elections Based on the Contemporary National Electoral Alignment

- The election generating formula is based on the EXPECTED VOTE (EV) in each state and CD, which is based on
  - the average of state-by-state popular votes in 2004, 2008, and 2012, and
  - the *Partisan Voting Index* (PVI) [of *The Cook Political Report*] for each Congressional District for the 113th Congress [CDs within each state are assumed to have equal absolute turnout],
  - Swung up or down so that that the national popular vote is tied.
- For each election, the Dem vote % in each CD is:
  - EV ± EVxSTATESWING ± EVxREGSWING ± EVxCOMPSWING[RN(0,1%)] ± EVxNATSWING[RN(0,2.5%)], where
    - STATESWING: RN(0,1.5%); NATSWING: RN(0,1.5%)
    - REGSWING: swing [RN(0,1.5%)] common to one of four regions;
    - COMPSWING: swing [RN(0,1%)] common to competitive or "battleground" states
- Electoral votes are those based on the 2010 Census.

Summary: Simulated Elections Based on 2004-2012 Landscape

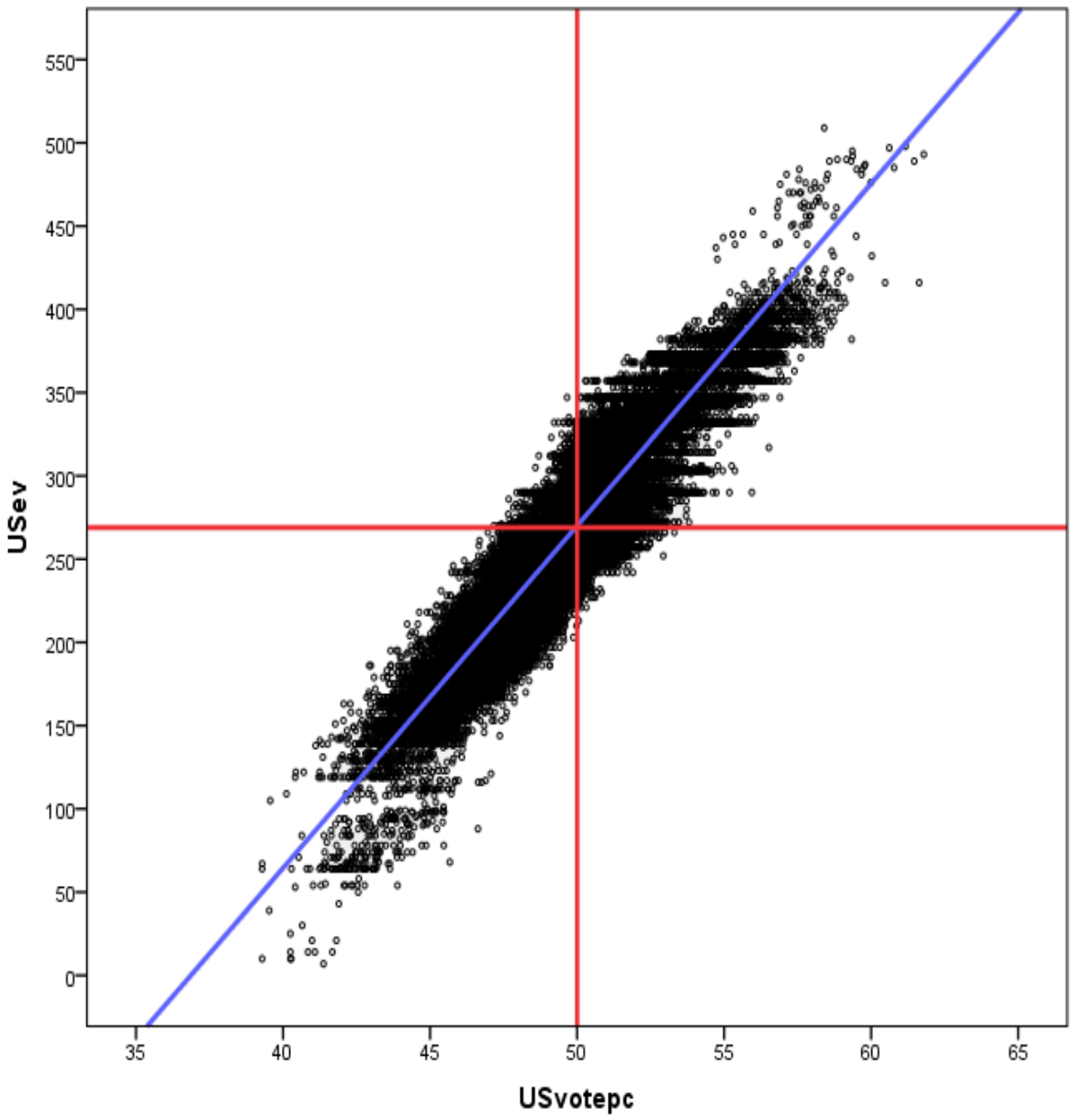
- The 2004-12 EC had a very small pro-Dem bias that would be increased by more proportional apportionment of EVs, reversed by proportional plans, and dramatically reversed by either district plan.

Electoral College Variant	*Dem. EV Wins*	*Inversions*		*EV Ties*	*EV by PV Correlation*
		Pro-Dem	*Pro-Rep*		
Standard EC	50.492%	5.620%	5.168%	0.127%	+0.934
EVs Based on House Seats Only	54.147%	7.530%	3.256%	0.689%	+0.932
EVs Perfectly Proportional	54.131%	7.516%	3.493%	–	+0.932
EVs Perfectly Proportional + 2	50.497%	5.625%	5.238%	–	+0.935
Equal EVs	29.452%	0.231%	20.889%	0.652%	+0.932
Pure District Plan (436 EVs)	12.397%	[0]	37.061%	0.656%	+0.970
Modified District Plan	14.561%	[0]	34.892%	–	+0.980
Pure Proportional	43.506%	[0]	6.603%	–	+0.999
Pure Proportional –2	47.809%	[4]	2.306%	–	+1.000
Whole Number Proportional	39.672%	[17]	7.622%	2.933%	+0.992

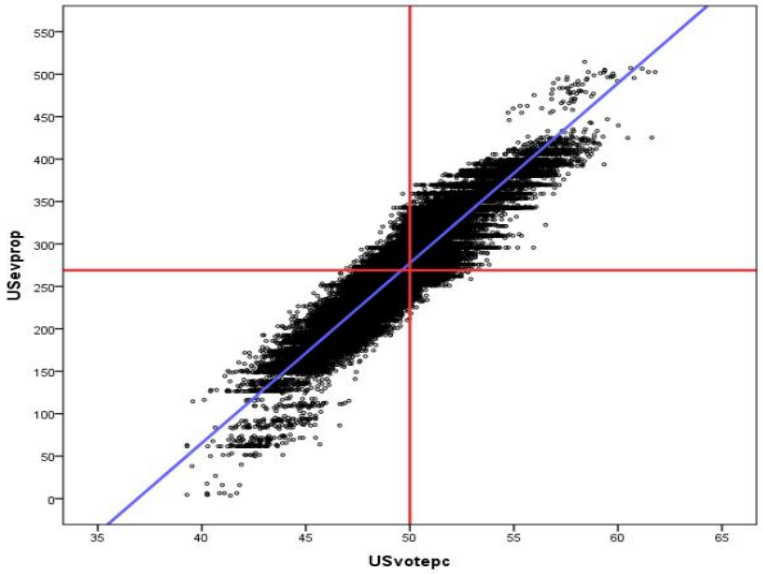
Contemporary Alignment: Existing EC

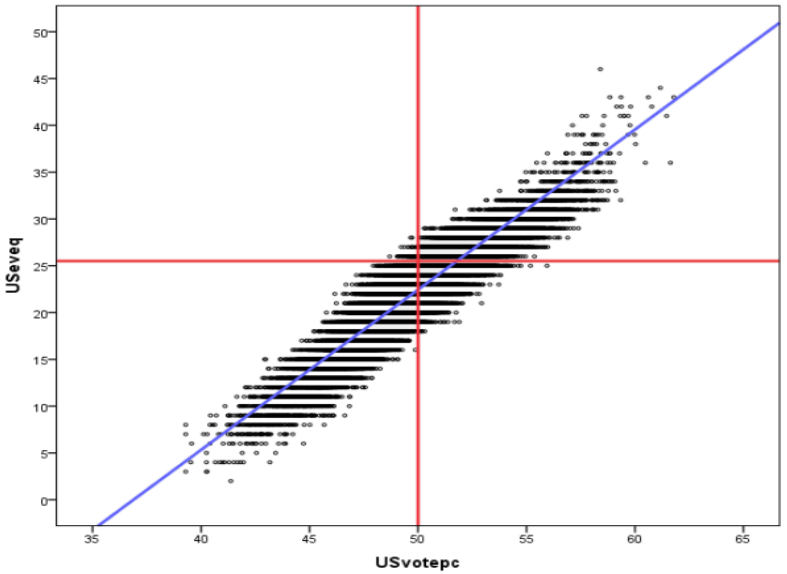
(n = 64,000)

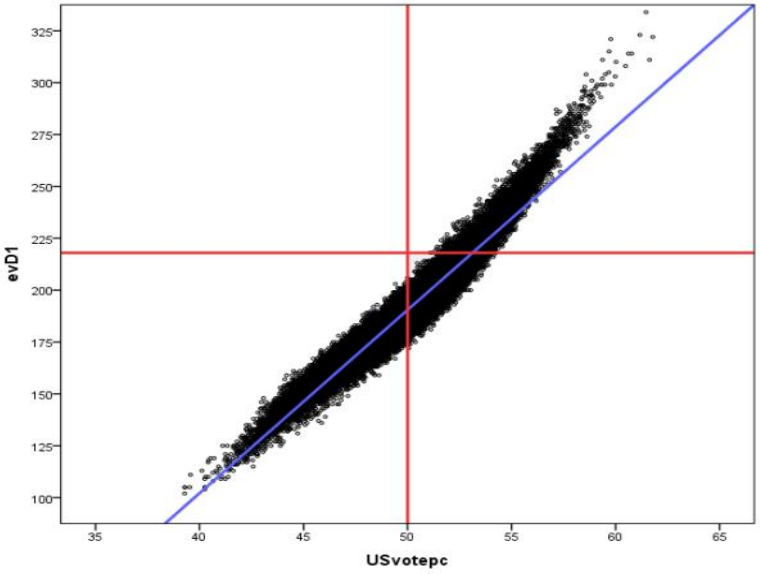
Virtually no bias

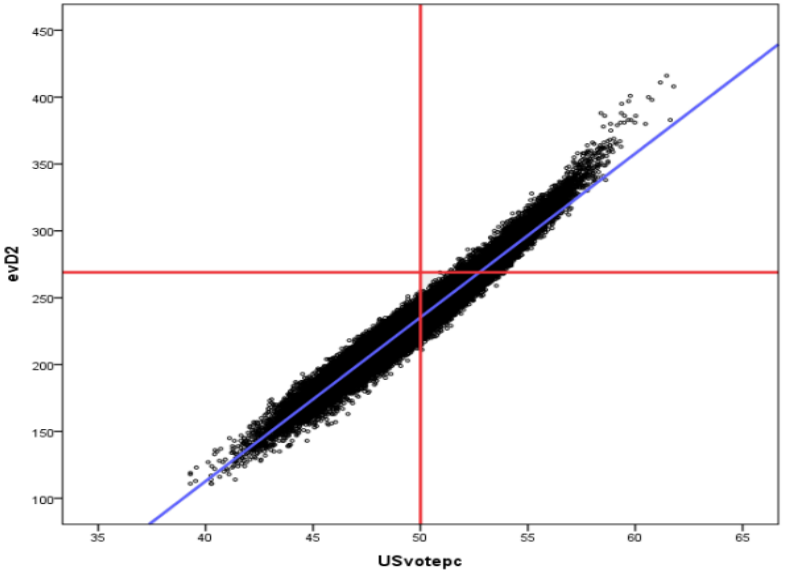


Prop EV; Equal EV; Pure District; Modified District









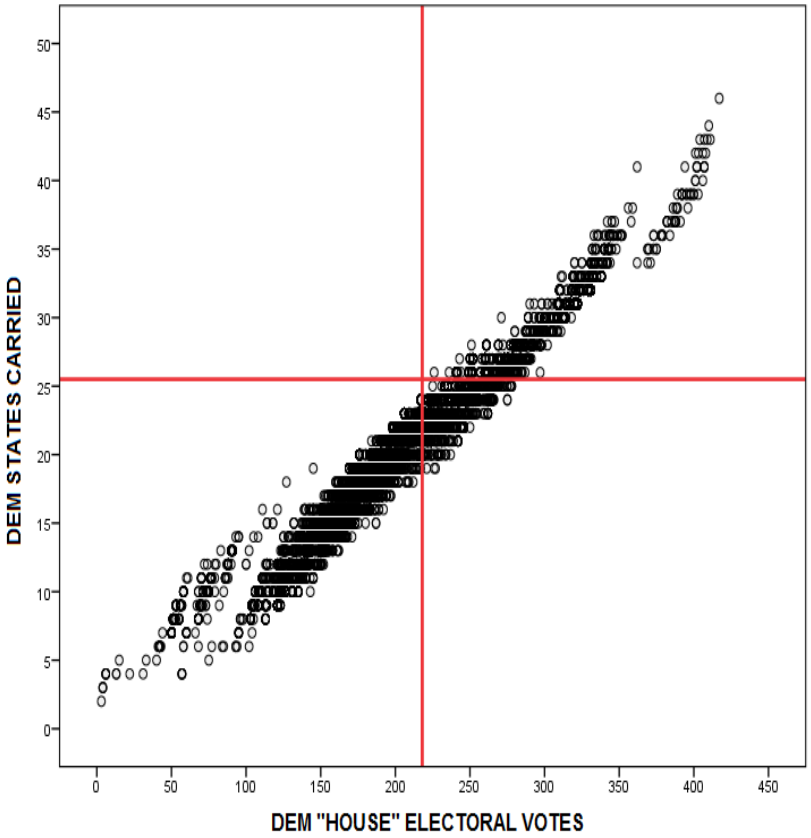
Side Point: The House Size Effect

- The 2000 election, in addition to producing an inversion, was subject to the "House size effect."
  - Gore, who lost with HS = 435, would have won had House size been sufficiently larger.
  - The (almost but not quite) necessary and the (strictly) sufficient condition for this effect is that one candidate wins a majority of "House" electoral votes and the other a majority of "Senate" electoral votes.
  - Usually (almost 90% of the time in historical elections) the same candidate wins a majority of both types of EVs.
  - But evidently the 2000 exception was entirely typical of exceptions in the contemporary alignment.
    - Almost 25% of the simulated elections were subject to the House size effect and in every case the Dem candidate would have benefitted from a larger House size.
    - This does not imply that in every such case (or even most) the Dem candidates lost with HR=435..

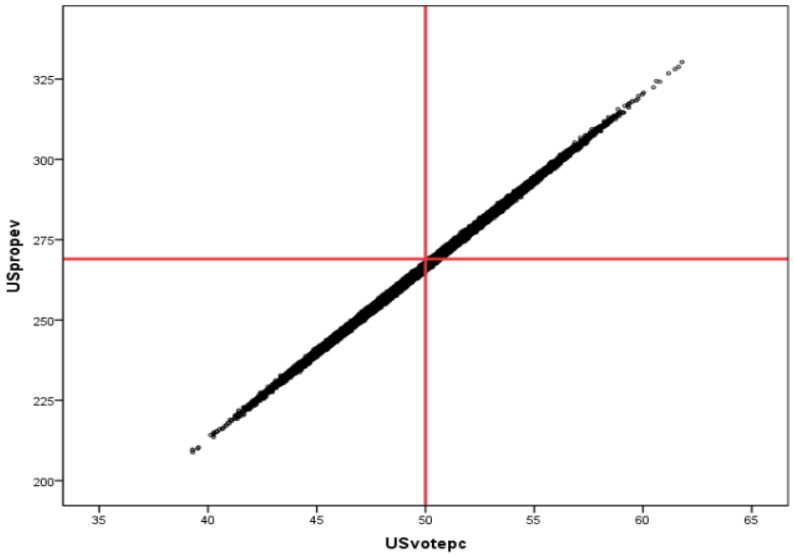
Neubauer and Zeitlin, "Outcomes of President Elections and the House Size," *PS* (2003)

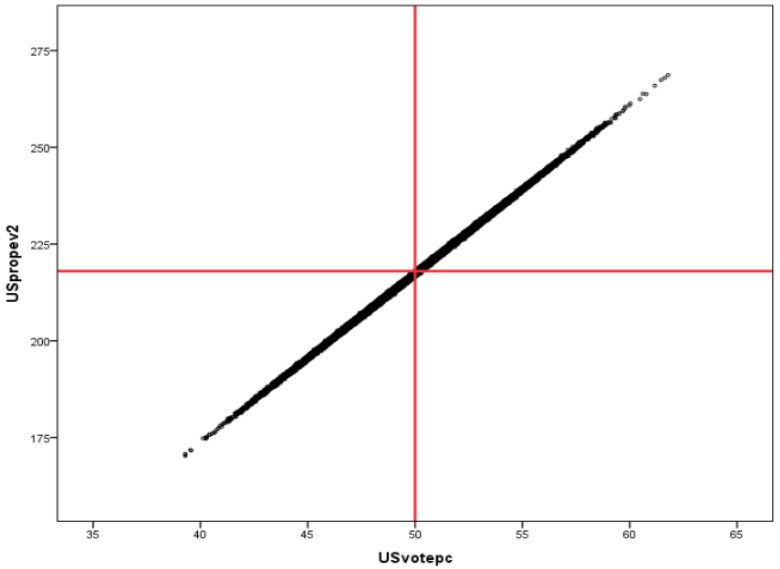
N. R. Miller, "The House Size Effect and the Referendum Paradox in U.S. Presidential Elections," *Electoral Studies* (2014)

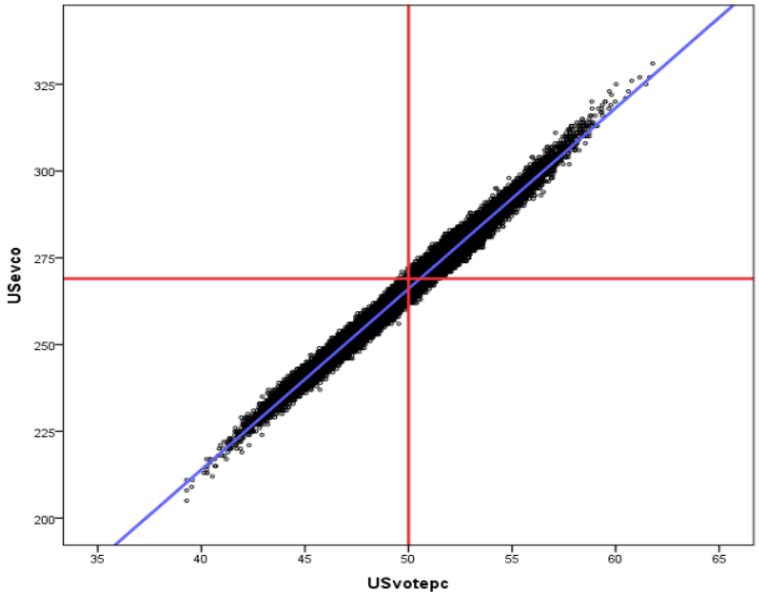
Gore-Bush EVs	*"House" EVs*	*"Senate" EVs*	*Total EVs*
HS = 435	225-211	42-60	267-271
HS = 830	429-403	42-60	471-463

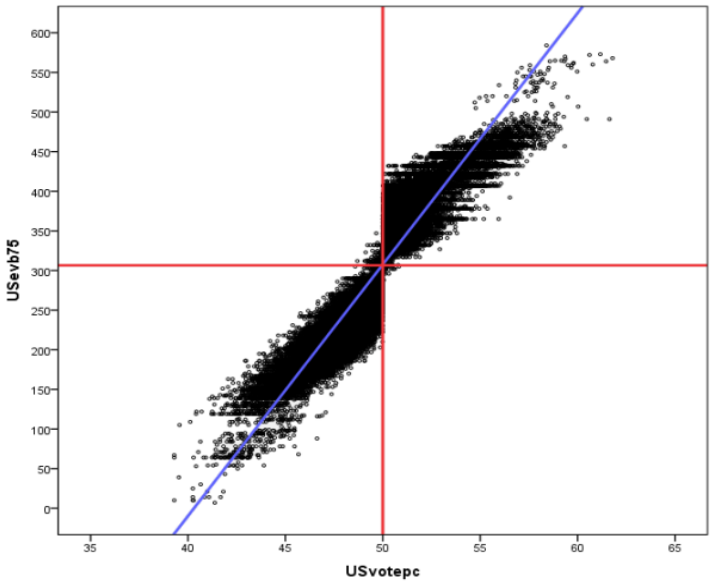


Prop Plan; Prop Plan (House only); Whole-Number Prop.; Nat. Bonus = 75

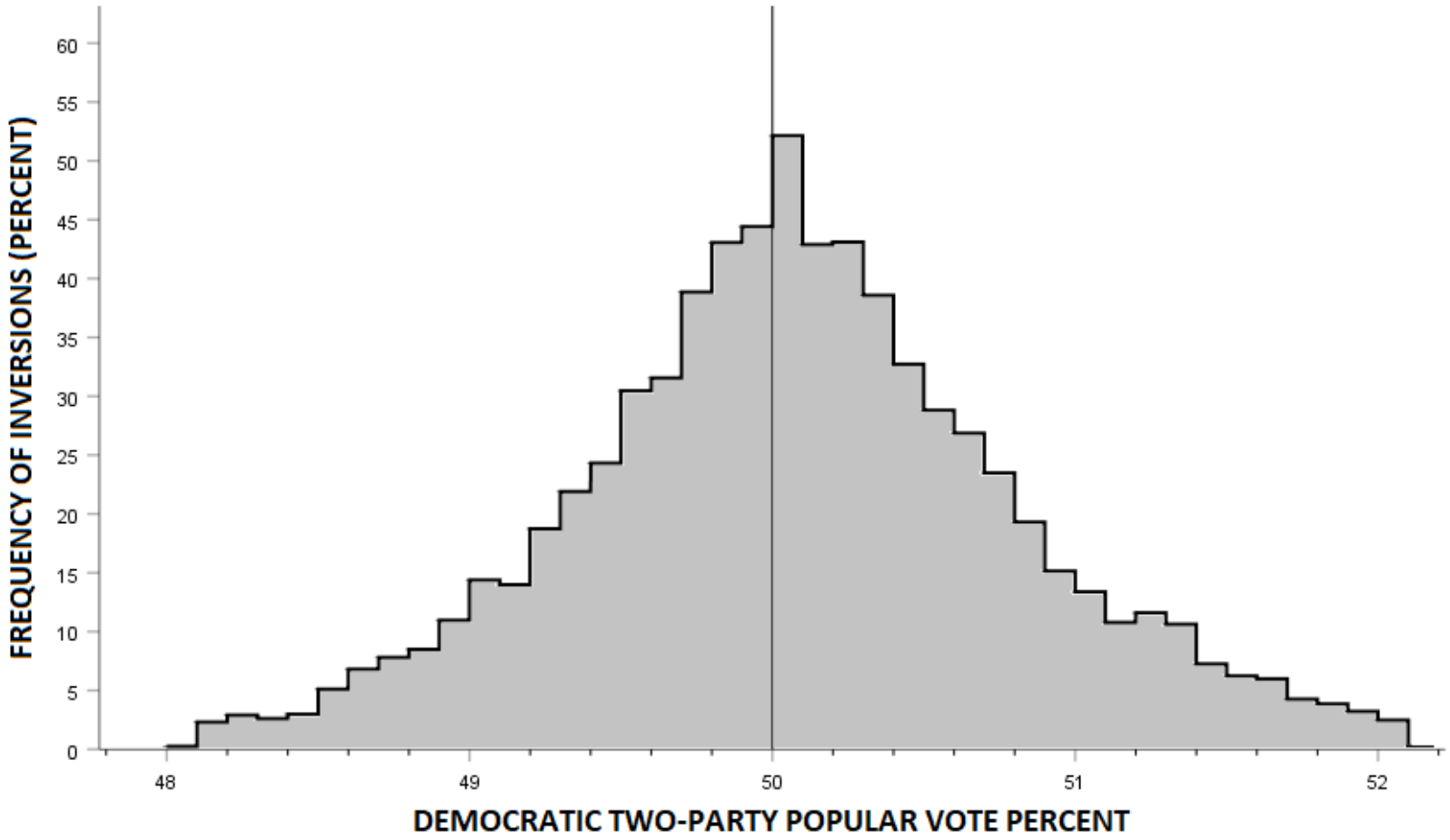








Frequency of Inversions by Democratic PV



Simulated Elections Based on the New Deal Electoral Alignment

- The election generating formula is based on
  - the average of state-by-state popular votes in 1936, 1940, and 1944,
  - adjusted so that that the national popular vote is tied.
  - *Note*: data on Presidential vote by Congressional District is not available, so there are no results for the district plans.
- For each election, the Dem vote % in each state is:

  EV ± EVxSTATE SWING ± EVxREGSWING ± EVxNATSWING, where
    - STATESWING: RN(0,1.5%)
    - REGSWING: [RN(0,1.5%)]
    - NATSWING: [RN(0,2.5%)]
- Electoral votes are those based on the 1940 Census.

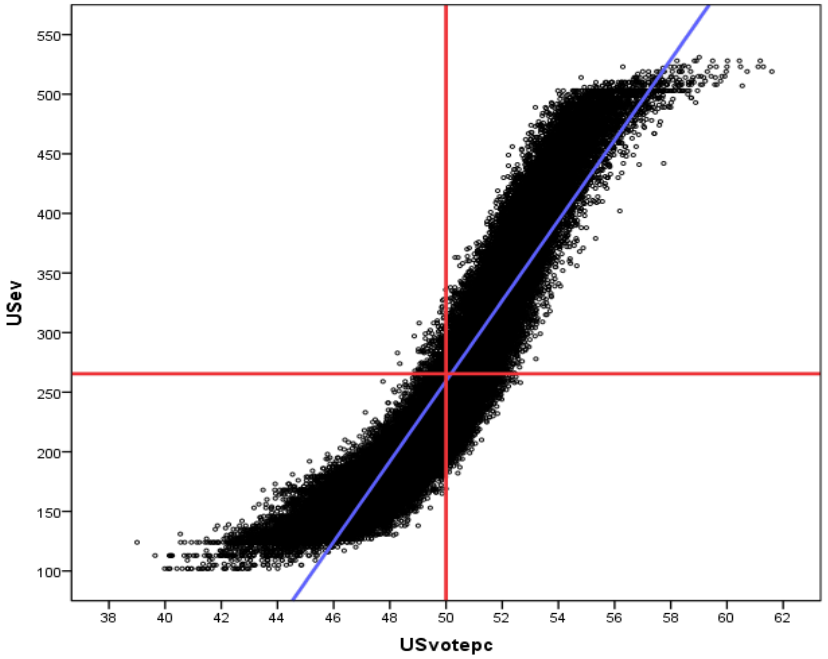
Summary: New Deal Alignment and All EC Variants

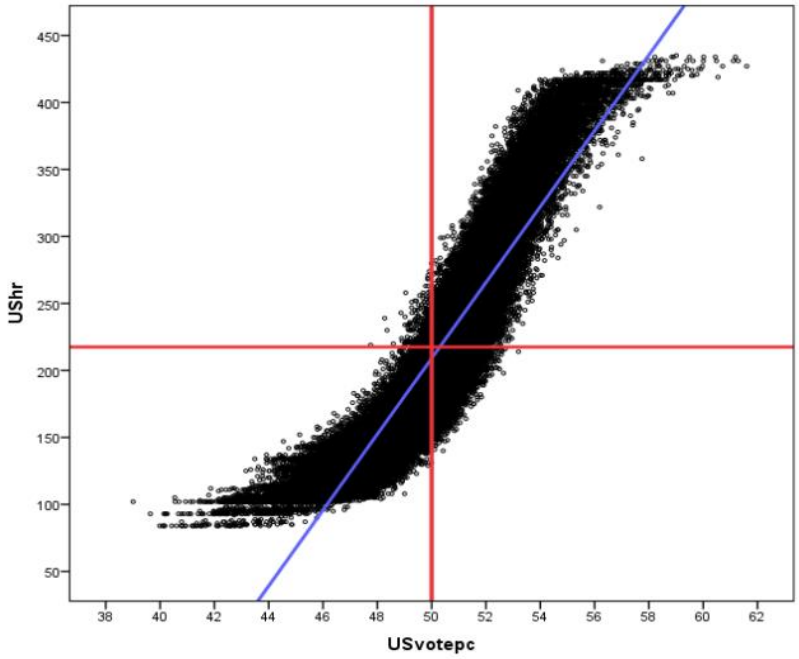
Existing EC had a substantial pro-Rep bias that would have been eliminated by state equality of EVs, and dramatically reversed by any kind of proportional plan.

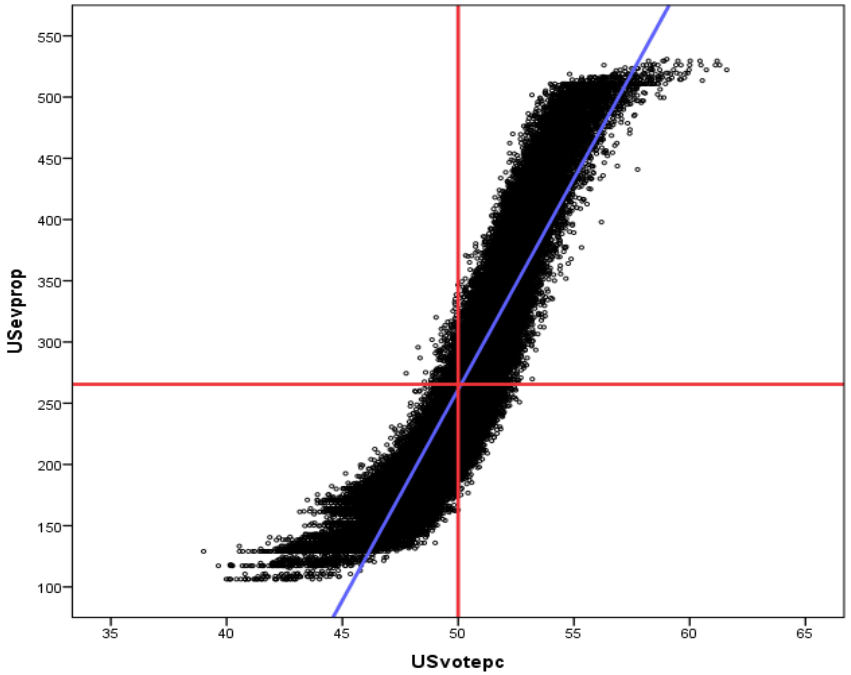
Electoral College Variant	*Dem. Wins*	*Inversions*	*Dem.*	*Rep.*	*EV Ties*
Existing EC	39.0%	12.1%	0.60%	11.54%	–
EVs Based House Seats Only	37.2%	13.9%	0.57%	13.31%	--
EVs Perfectly Proportional	39.1%	12.4%	0.80%	11.63%	–
EVs Perfectly Proportional +2	40.9%	10.7%	0.90%	9.85%	–
Equal EVs	49.3%	4.8%	2.97%	1.71%	4.786%
Pure Proportional Plan	89.7%	39.8%	39.77%	[0]	–
Pure Proportional (House Seats Only)	89.5%	39.6%	39.60%	[0]	–
Whole-Number Proportional Plan	84.1%	34.2%	34.23%	[0]	1.652%
National Bonus of 25	40.8%	9.5%	0.21%	9.26%	–
National Bonus of 50	43.7%	6.1%	[46]	5.99%	–
National Bonus of 75	46.6%	3.3%	[14]	3.29%	–
National Bonus of 100	48.5%	1.4%	[5]	1.39%	[35]
National Bonus of 150	49.8%	[23]	[0]	[23]	–
National Bonus of 200	49.9%	[0]	[0]	[0]	–
National Bonus of 250	49.9%	[0]	[0]	[0]	–

New Deal Alignment:
Existing EC
House Apportionment
Proportional Apportionment

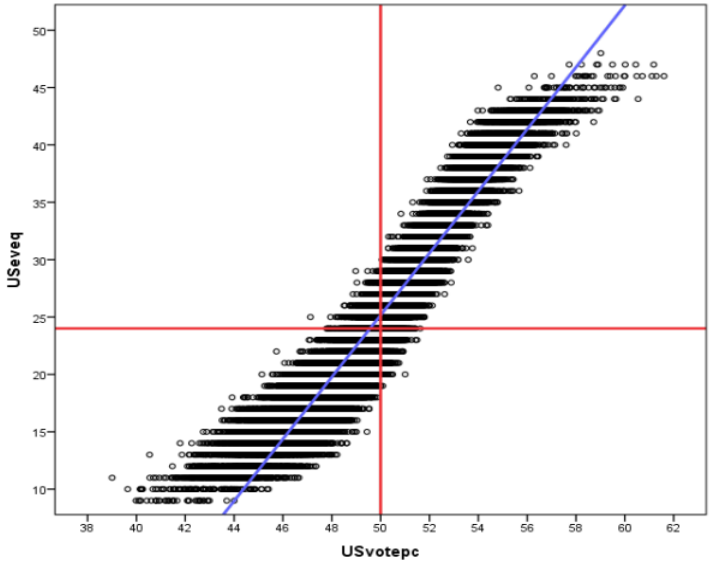
(n = 64,000)

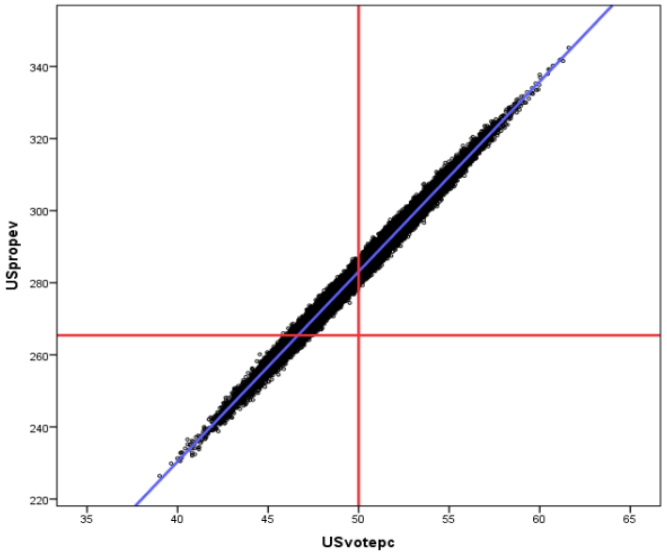


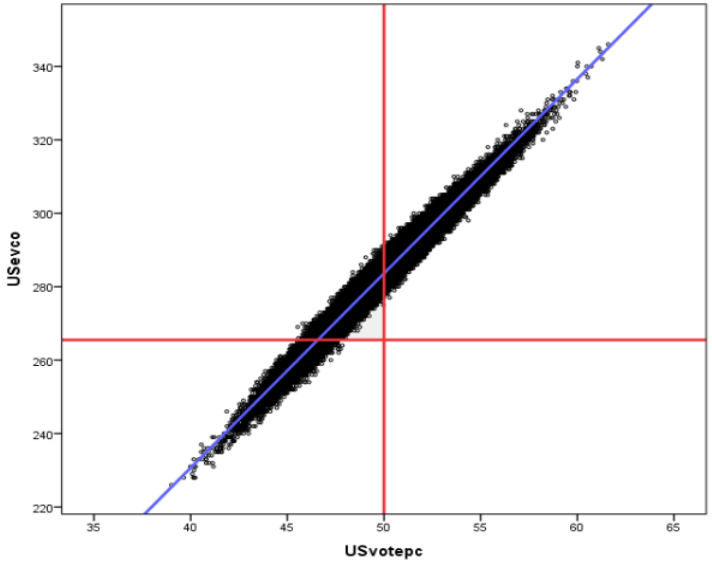


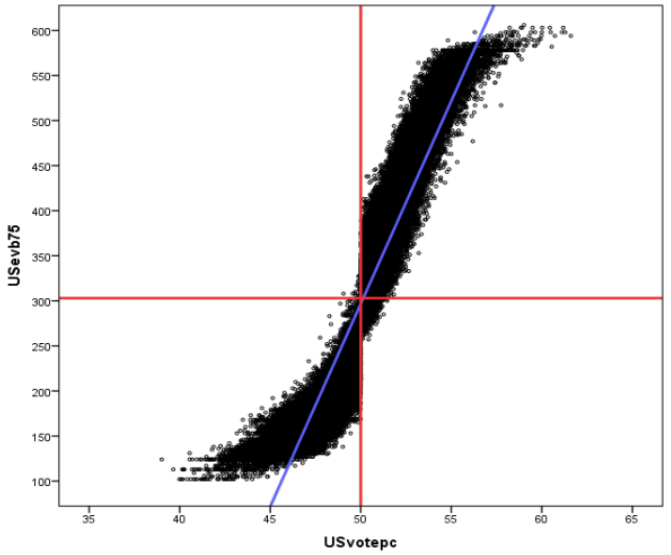


New Deal Alignment: Equal Evs; Proportional; Whole-Number Proportional; National Bonus = 75









Frequency of Inversions by Democratic PV